Title: General Boundaries and Transition Amplitudes in Quantum Gravity

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Abstract:

GENERAL BOUNDARIES AND TRANSITION AMPLITUDES IN QUANTUM GRAVITY

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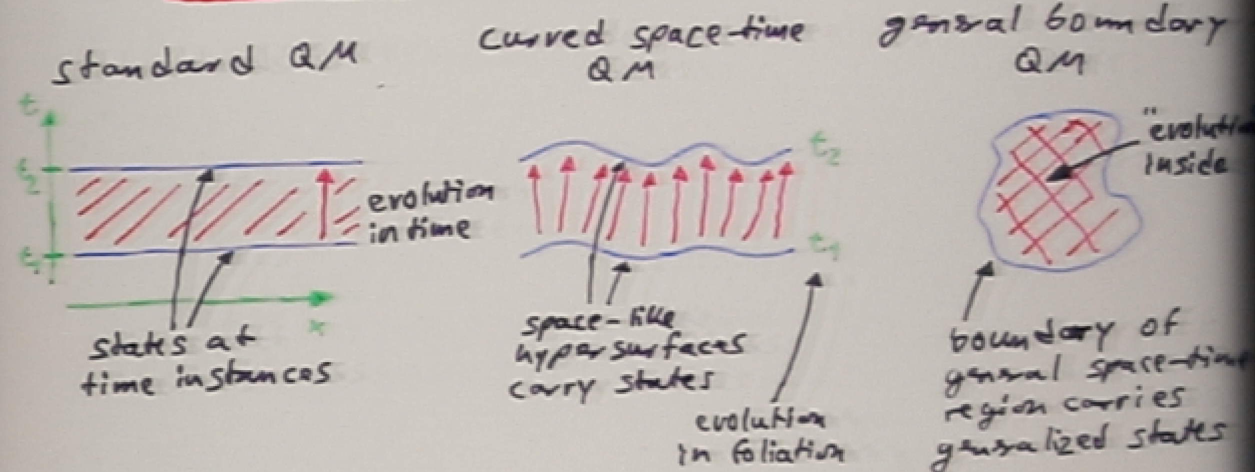
a few related papers:

gr-qc/0306007
hep-th/0306025
gr-qc/0307118
gr-qc/0312081

THE GENERAL BOUNDARY FORMULATION

on axiomatic level

$\boxed{\text{QM + TQFT = general boundary QM}}$



▷ associate generalized state spaces to boundaries of regions of space-time

▷ associate "transition" amplitudes to regions ~~them~~selves

features

▷ avoid interpretational problems of combining GR with standard QM (notably problem of time)

▷ preserve standard QM where applicable

▷ local description of measurement process

▷ distinction between "in" and "out" states and between "preparation" and "observation" disappears

▷ interpretation: "collapse of wavefunction" is delocalized in time

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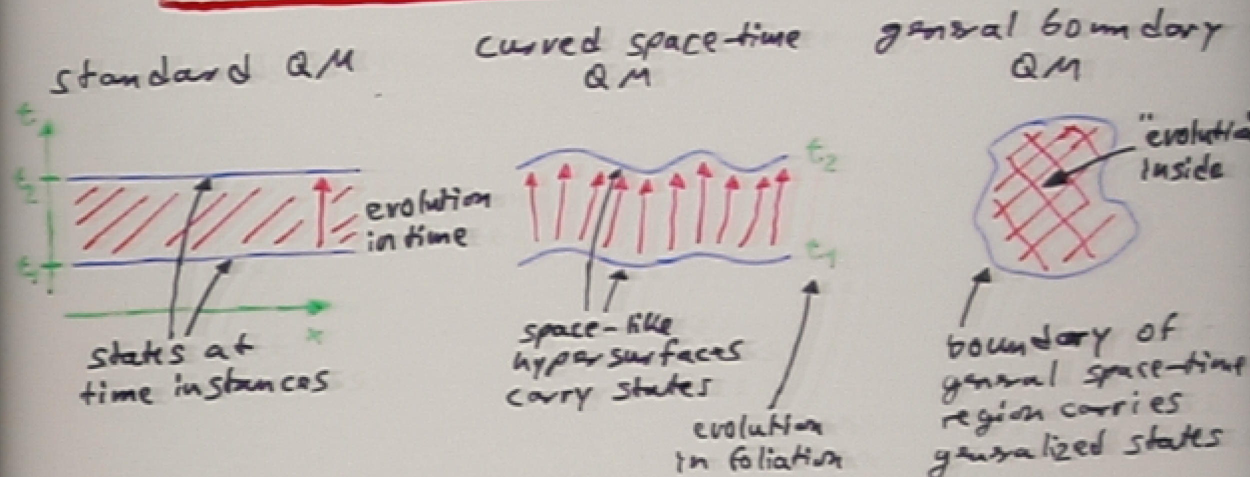

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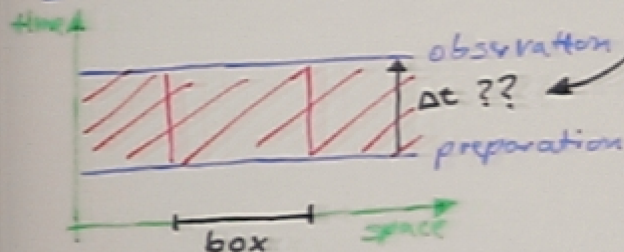
AVOIDING THE PROBLEM OF TIME

what is the analogue of time evolution in a quantum theory of gravity ?

▷ consider a typical quantum mechanical experiment in a box:

① prepare initial state
② wait for time Δt
③ observe final state

standard QM

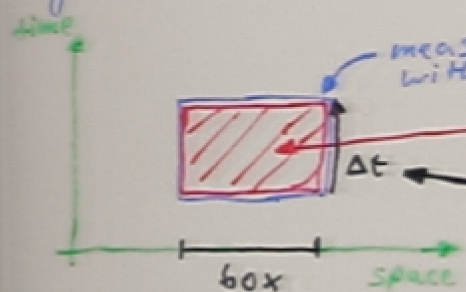


space-time here is supposed to be quantum mechanical
no classical Δt !

②

general boundary QM

measurement associated with closed boundary



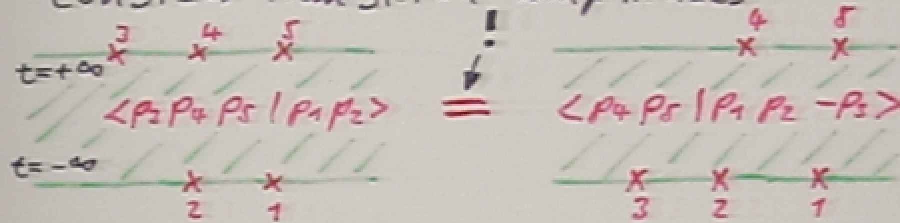
space-time quantum mechanical

almost classical nature of space-time encoded in state including time duration Δt !

If gravitational component in boundary state is (almost) classical it encodes classical geometry of the boundary (and thus Δt)

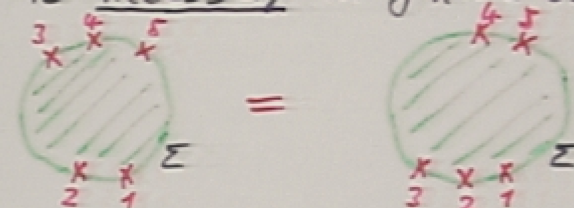
ⓐ compatibility from LSZ reduction

consider transition amplitudes

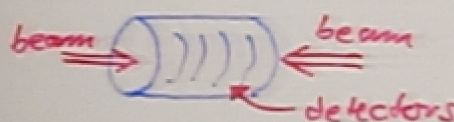


equality is <u>necessary</u> in general boundary formulation

since

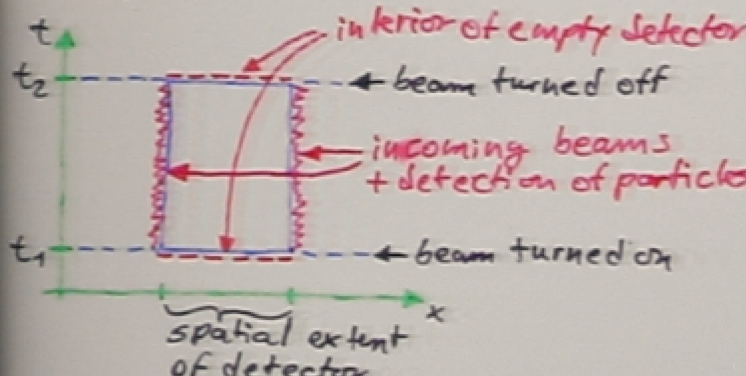


ⓑ typical measurement setup



collision experiment at accelerator

space-time diagram

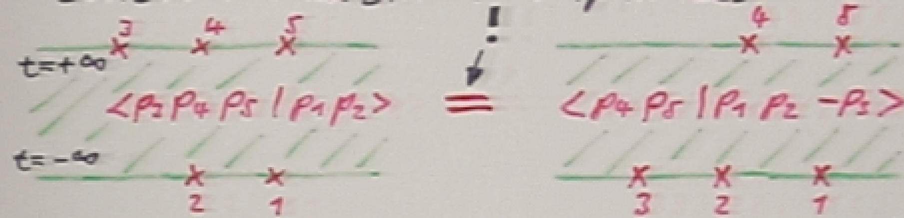


interesting stuff happens at <u>time-like</u> boundaries

MOTIVATION FROM QFT

ⓐ compatibility from LSZ reduction

consider transition amplitudes



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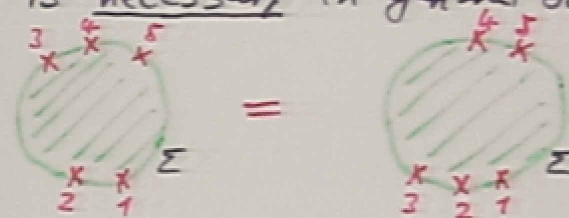

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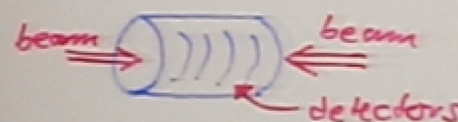

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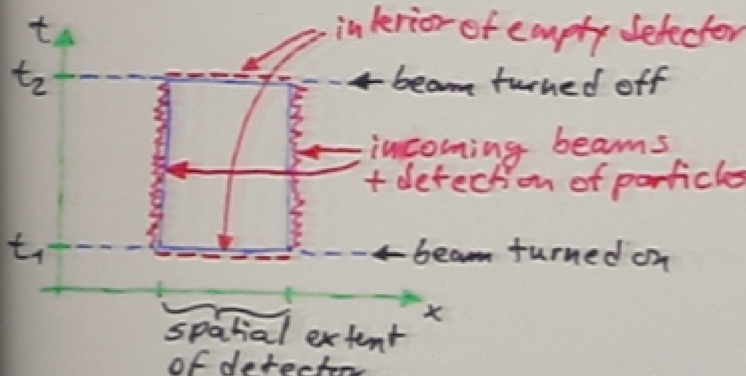
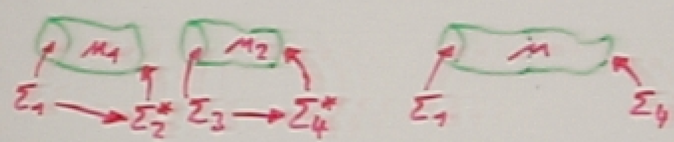


interesting stuff happens at time-like boundaries

FORMALIZATION

- state spaces $\mathcal{H}_\Sigma$ associated to boundaries Σ of space-time regions M
- if $\Sigma = \Sigma_1 \dot\cup \Sigma_2$ with Σ_1, Σ_2 disconnected then $\mathcal{H}_\Sigma = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}_{\Sigma_2}$
- Σ^* the same hypersurface as Σ but with opposite orientation $\rightarrow$ $\mathcal{H}_{\Sigma^*} = \mathcal{H}^*_\Sigma$
- associated with M is the amplitude function $\rho_M : \mathcal{H}_\Sigma \rightarrow \mathbb{C}$ where $\Sigma = \partial M$
- composition rule: given $M = M_1 \cup M_2$



$\Sigma_2^* = \Sigma_3$

ρ_{M_1} $\quad$ ρ_{M_2} $\quad$ $\boxed{\rho_M = \rho_{M_2} \circ \rho_{M_1}}$

TQFT

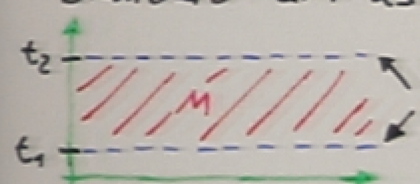
- probability rule: $\rho_M : \mathcal{H}_\Sigma \rightarrow \mathbb{C}$

$P = |\rho_M(\eta)|^2$

probability (density) $\quad$ (generalized) state $\eta \in \mathcal{H}_\Sigma$

▶ standard QM as special case

$M = \mathbb{R}^3 \times [t_1, t_2]$



Σ is the union of two components $\Rightarrow \mathcal{H}_\Sigma = H_{t_1} \otimes H^*_{t_2}$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \psi \quad\quad \psi$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad |\psi\rangle \quad \langle\varphi|$

$$\rho_M(|\psi\rangle \otimes \langle\varphi|) = \langle\varphi| U(t_1, t_2) |\psi\rangle$$

final state $\quad$ time evolution operator $\quad$ initial state

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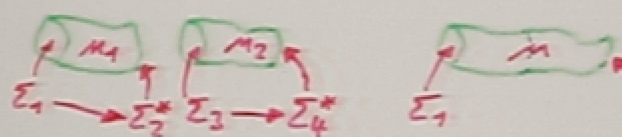

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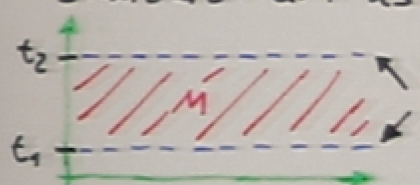

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INTERPRETATION OF AMPLITUDES

in the setup of standard QM we can resort to the standard interpretation – but in general?

PROPOSAL

- encode information about what is known/fixed/prepared in experiment through choice of subspace $\mathcal{H}_S \subset \mathcal{H}$
- the probability density for $\alpha \in \mathcal{H}_S$ to occur/be observed is given by $|\rho(\alpha)|^2$ with respect to a measure on $\mathcal{H}_S$
- if not all information is collected, then subspaces of $\mathcal{H}_S$ corresponding to the "same outcome" are integrated over with appropriate probability measures

---

→ standard QM: $\mathcal{H} = H_{in} \otimes H_{out}^*$

$\psi \in H_{in}$ in-state $\quad \mathcal{H}_S = \psi \otimes H_{out}^*$

→ in perturbative QFT proposal works already more generally

essentially $H = H_{in/out} = \bigoplus_{n=0}^{\infty} H_n$ ← n-particle state

$\mathcal{H} \simeq H \otimes H \simeq H$

e.g. fix $\psi_{in,part} \in H_{in}$ and $\psi_{out,part} \in H_{out}$

(know <u>some</u> in/out particles)

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$\mathcal{H}_S := \{ \psi_{in,part}\, \eta_{in} \otimes \psi_{out,part}\, \eta_{out} \mid \eta_{in/out} \in H_{in/out} \}$

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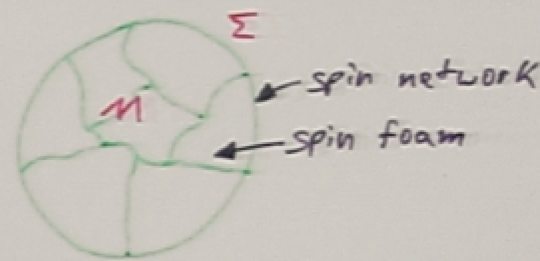
then above interpretation applies.

APPLICATION TO LQG / SPIN FOAMS

general picture



implications of general boundary proposal:

→ states on a compact Σ (spin networks) have direct interpretation in terms of measurement

→ Σ has generally time-like components encoded
(a) either with a "causal structure" (a kind of background specifying space-like / time-like / null parts of Σ)
(b) or "internally" in spin-network state (e.g. using connection of Lorentz group)
⇒ need new "boundary LQG"

LQG & spin foams

→ variables in the interior are not directly related to observations ⇒ no observables associated to "internal" degrees of freedom

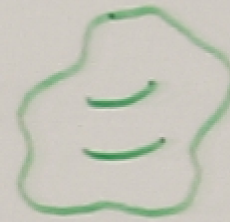
→ provides a renormalization fixed point equation for "statistical field theory" interpretation of spin foams

spin foams

BACKGROUND STRUCTURES

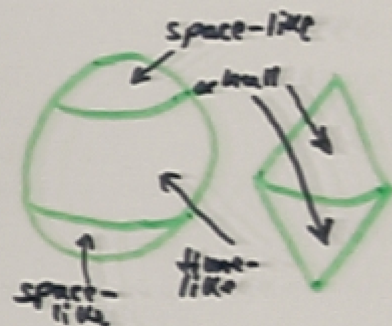
TQFT TYPE — **THEORIES**

METRIC
metric on bdy. and interior



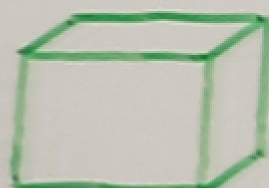
- standard QFT
- QFT on curved space-time
- perturbative QG

CAUSAL
boundary divided into regions



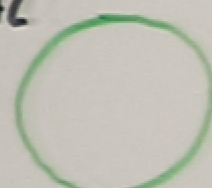
- "causal QG":
- causal spin foams
- causal dynamical triangulations

DIFF
differentiable structure with "corners"



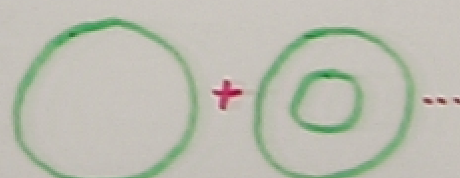
- "natural" setting for quantum gravity?
- Euclidean Quantum Gravity
- generalized LQG

TOPOLOGICAL
topological manifold



- spin foam models

SUM OVER TOPOLOGIES
sum "all topologies in the interior



- spin foam models
- GFT