Title: Statistical Aspects of Semi-Classical Quantum Gravity

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Abstract:

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CONTEXT

Discrete-Based Theory	Continuum Theory
Full QG theory	
$\downarrow$	
Kinematical Semiclassical Sector	
$\downarrow$	
Physicalstates and Dynamics	
	Effective QFT on curved spacetime
	$\downarrow$
	Physical Predictions

CONTEXT

Discrete-Based Theory		Continuum Theory
Full QG theory		
$\downarrow$		
Kinematical Semiclassical Sector	$\rightarrow$	Effective Geometry (and fields)
$\downarrow$		$\downarrow$
Physical states and Dynamics		
		Effective QFT on curved spacetime
		$\downarrow$
		Physical Predictions

Main points

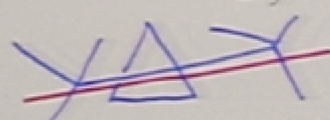
- Two "transitions" need to be addressed ("continuum limit" not enough)
- Need to adapt definition of semiclassical state.
- (Can do in the Lorentzian case as well)

SEMICLASSICAL STATES

For a (constrained) theory with a finite number of degrees of freedom, choose a complete family of Dirac observables $\mathcal{O}_\alpha$, and tolerances. Then Ψ is semiclassical if

$$|\langle \hat{\mathcal{O}}_\alpha \rangle_\Psi - \bar{\mathcal{O}}_\alpha|, \quad \langle \hat{H}_a \rangle_\Psi \quad \text{small}$$
$$(\Delta \hat{\mathcal{O}}_\alpha)^2_\Psi, \quad (\Delta \hat{H}_a)^2_\Psi \quad \text{small}$$

The situation in quantum gravity is different. Candidate semiclassical states are peaked with respect to quantities related to a discrete structure, not to a unique $p \in \bar{\Gamma}$.



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Here, we want to analyze the issue of exactly how much information such a discrete structure gives us about the continuum.

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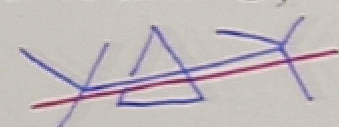

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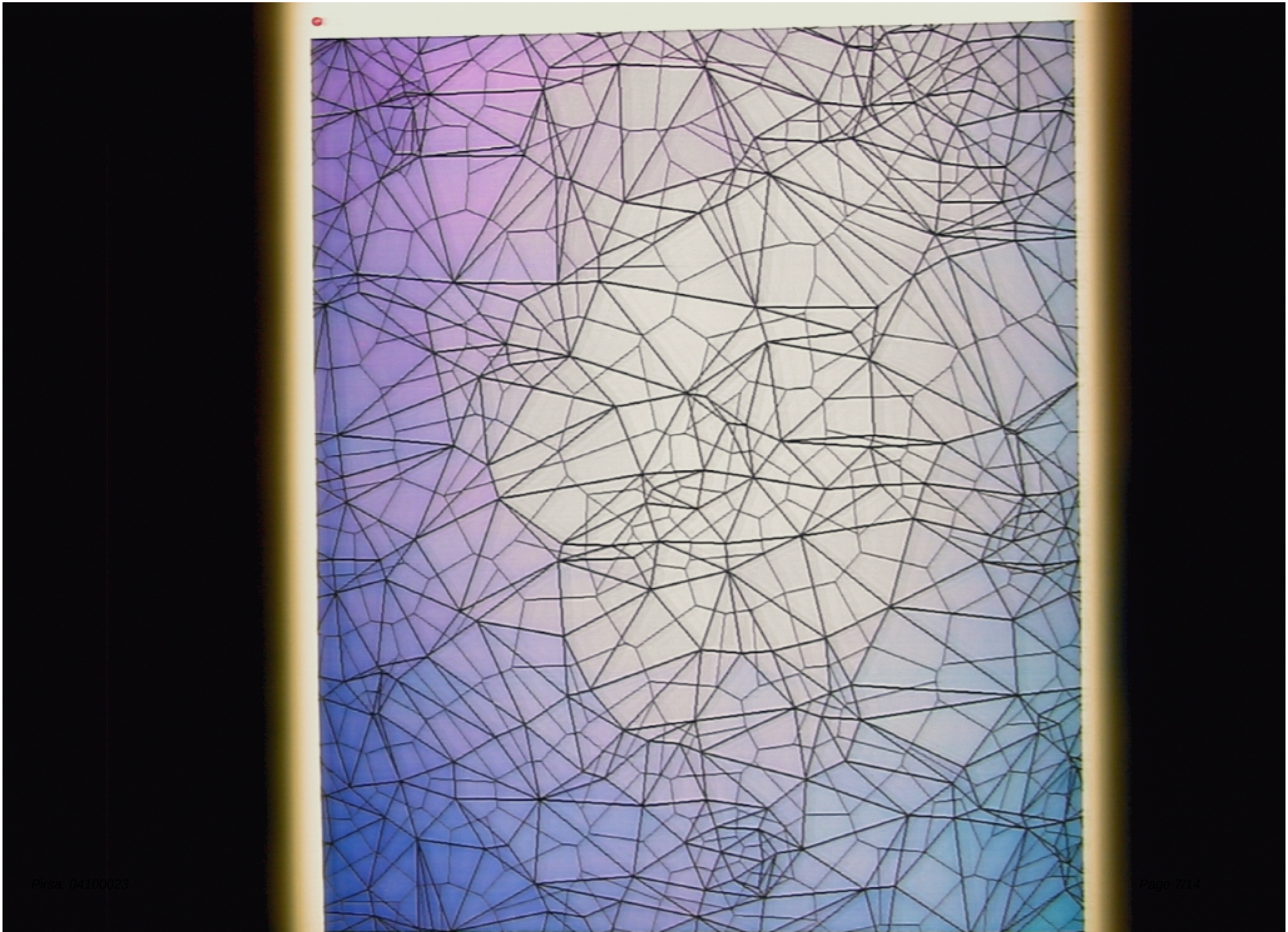
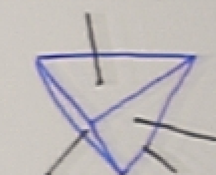


HOW TO PROCEED

If (Ω, E, Φ) is to be associated with a region of classical phase space, first find out what range of (Ω, E, Φ) a given $p \in \bar{\Gamma}$ would give, then invert the procedure.

For example, consider a simple manifold (S^3, g_{ab}), sprinkle a random set of points, construct the cell complexes, and calculate the mean and variance of the distribution of triangle areas, and directions.

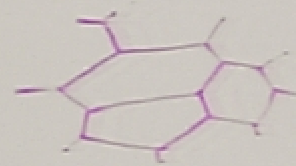


Given a specific (Ω, E, Φ) we can now compare the distribution of values of E^I_i with the one corresponding to a classical geometry.

$\rightarrow$ We need to show an example in detail.

2D Random Cell Complexes

- Consider a 2-sphere (S^2, s_{ab}). What are the distinguishing properties of a random graph on it?



The simplest way to get a graph of this type is from the Voronoi procedure.

- Can we tell just from the graph what metric it came from? (The metric must be slowly varying for this to make sense.)

- Each cell ω is characterized by

$$N_0(\omega), \quad N_1(\omega) \qquad (N_0(\omega) = N_1(\omega))$$

For the whole complex

$$N_0 - N_1 + N_2 = \chi(S^2) \quad \& \quad N_1 = \frac{3}{2} N_0$$

(voronoi)

so

$$\langle N_0(\omega) \rangle = \frac{3 N_0}{N_2} = 6\left(1 - \frac{R}{4\pi\rho}\right) = \overline{N_0(\omega)},$$

and

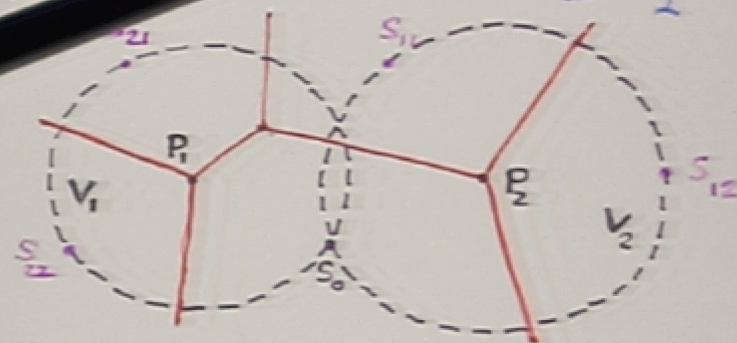
$$\omega \mapsto R_{est}(\omega) = 4\pi\rho\left(1 - \frac{1}{6} N_0(\omega)\right).$$

How reliable is this?

2D Curvature Uncertainties

- To know how reliable $R_{est}(\omega)$ is, we need to find out by how much N_o can vary, the value of
$$\sigma_o^2 = \overline{N_o^2} - \overline{N_o}^2 .$$

- To find [illegible] notice that $N_{oo'} = \frac{1}{2} N_o (N_o - 3)$



so that $\quad 2\overline{N_{oo'}} = \overline{N_o^2} - 3\overline{N_o}$

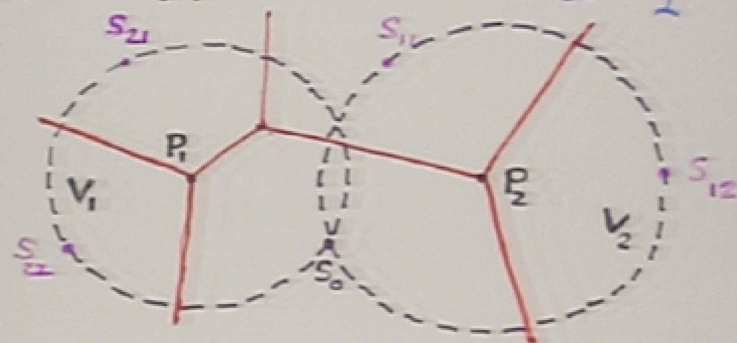
- To find $\overline{N_{oo'}}$, calculate
$$\overline{N_{o,o'}} = \int d\mu(P_1)\, d\mu(P_2)\, e^{-\rho A(V_1 \cup V_2)}$$
$\hookrightarrow P(V_1 \cup V_2 \text{ is empty})$

from which one gets an explicit integral.
So, finally
$$\sigma_o^2(R/\rho) = 2\overline{N_{oo'}} + 3\overline{N_o} - \overline{N_o}^2 .$$

2D Curvature Uncertainties.

- To know how reliable $R_{est}(\omega)$ is, we need to find out by how much N_0 can vary, the value of $\sigma_0^2 = \overline{N_0^2} - \overline{N_0}^2$.

- To find $\overline{N_0^2}$, notice that $N_{00'} = \frac{1}{2} N_0 (N_0 - 3)$



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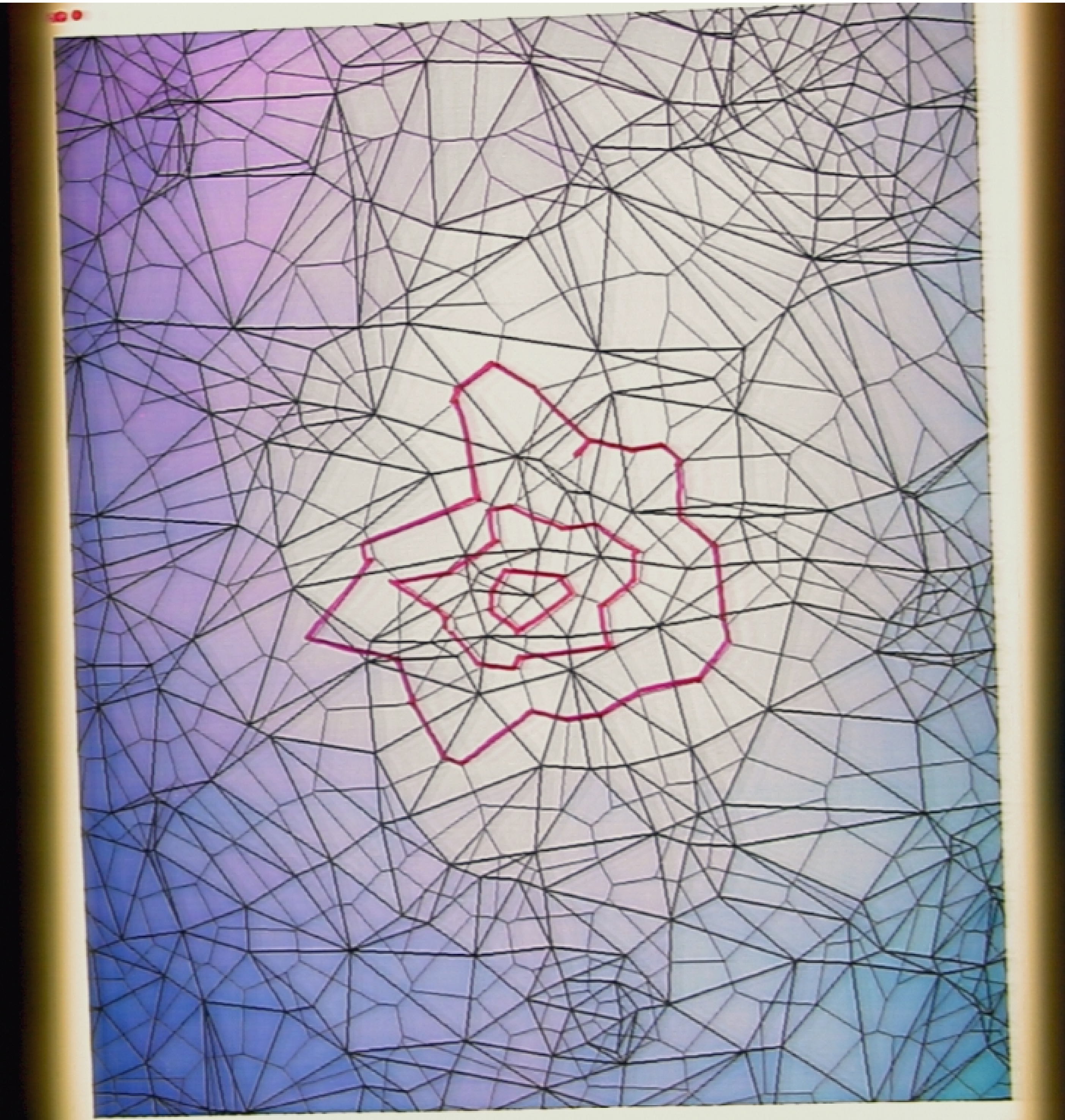

What to do now?

We need to think about

- The analogous 3D situation and calculations (estimate of curvature, and graph $\leftrightarrow$ cell complex correspondence).
- what to do when the smooth geometry tests fail, or the complex/graph is not Voronoi ("hard" coarse-graining?)
- Constructing fields and Lie-algebra valued quantities in the continuum

we have done these things to some extent.

For applications,

- (How) do these statistical uncertainties combine with the quantum ones?
- (How) does this affect predictions of quantum gravity phenomenology?

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