Title: Is a past finite order the inner basis of space-time?

Date: Oct 29, 2004 11:30 AM

URL: http://pirsa.org/04100022

Abstract:

The poset as causet: deep structure of spacetime

causal set

Can illustrate the basic (kinematic) ideas with quotes

① ② ③

Continuum spacetime is already a poset

Robb axioms for $\mathbb{M}^4$ (but not only $\mathbb{M}^4$ satisfies them)

What is a spacetime in GR? A Lor. manifold: $ds^2 = g_{ab}\,dx^a\,dx^b$

$(-+++)$ ⇒ light cones ⟷ order (only for this signature)

Know cones ⇒ know $\Omega(x)^2 g_{ab}$ modulo $\Omega(x)$

so missing 1/0 of g_{ab}: volume info

Get it thru counting

Discreteness is the missing ingredient!

Unifies topology, diff. str., metric, causality in one simple mathematical structure (unifies "spacetime with its metric")

Geometry = Order + Number

Adjoin finitarity (discreteness) conditions:

Def causet = locally finite (resp. past finite) poset

(of course many practical reasons to want discreteness as well ....)

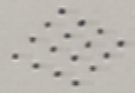
Kinematical Matters

(a) Sprinkling (and "Hawaiverification") Want $N = V$, $< \Leftrightarrow J^+$

When should we say that $M \approx C$?

if C "might have been made by Poisson process in M"?

NB: this is "locally Lorentz invariant" (exact in $\mathbb{M}^d$)

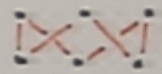
in contrast e.g.  is not invariant & not uniform

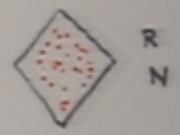
HENCE Lorentz invariance $\Rightarrow$ randomness!

(b) proper time = length of longest chain (proven for $\mathbb{M}^d$)

(spacelike distance, $d(x,y)$, is hard!)

(c) Dimensionality

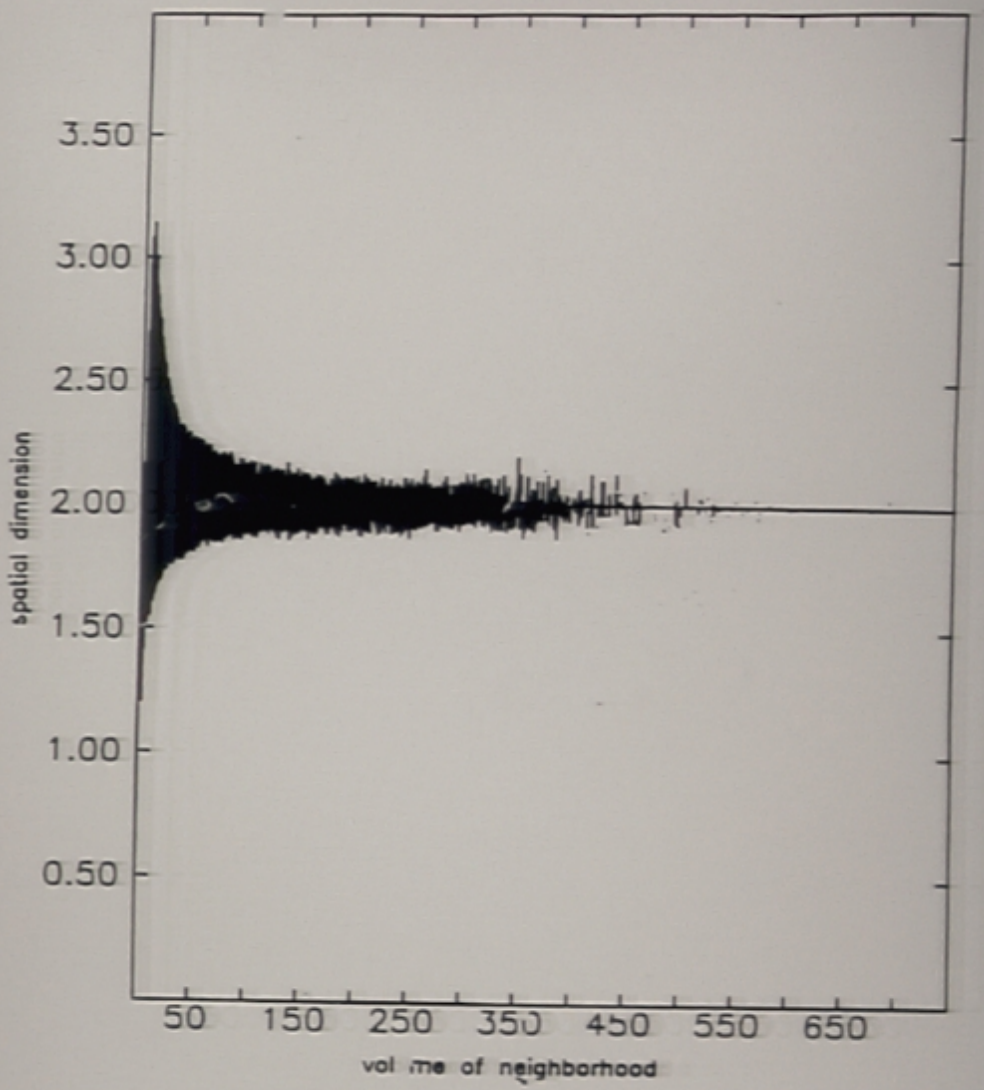
- "pixies" 
- MM-dim $\dfrac{\langle R \rangle}{\binom{\langle N \rangle}{2}} = \dfrac{3}{2}\binom{3d/2}{d}^{-1}$



for dim d need $\gg \left(\frac{27}{16}\right)^d$ points

- "midpoint scaling" dim

$$\frac{N}{N'} = 2^d$$


3.50
3.00
2.50
2.00
1.50
1.00
0.50
spatial dimension
50
150
250
350
450
550
650
volume of neighborhood

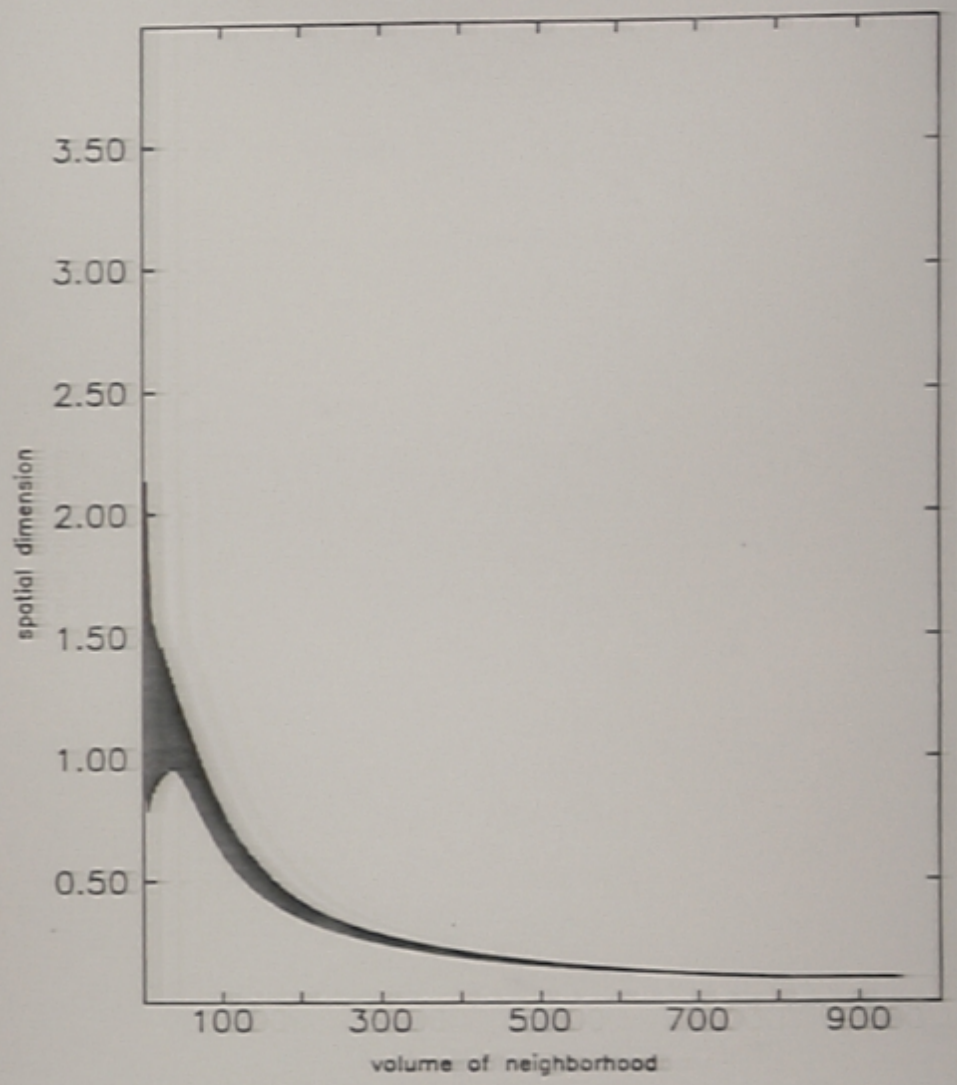
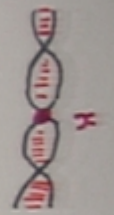


Fig. 10

(d) concept of post x

$\forall y \neq x,\ y \prec x$ or $y \succ x$



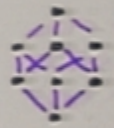
cycles of cosmic expansion + contraction

"percolation" ("random graph order") has ∞ posts

(e)–(g) coarse-graining, causal completion, Hauptvermutung

(h) Some important open questions

- Does "height = proper time" work in curved M too?
- Does MM-dim?
- Does $B_\ell \hookrightarrow M^\ell$?



B_3

- How estimate (e.g.) Ricci Scalar of sprinkled causet?

cf. $\frac{1}{2}\int R\,dV$

- How estimate spacelike distance in M^d?
- HOW RECOGNIZE A MANIFOLD?

Dynamical Matters I: Dynamics as Growth

(here consider causet as actor, later as background)

☺ CSG = classical sequential growth model: discrete cosmology

stepping stone (we hope) to QSG = QG

poscau

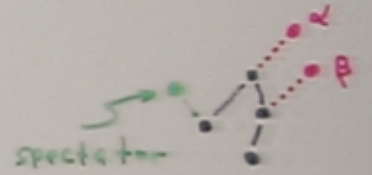
Assume

- sequential growth: elt's born (stochastically) one by one
- internal temporality: no elt born to past of any pre-existing element
- classical probabilities: no interference
- general covariance: probs at stage n are labeling independent

  ⇒ paths in "poscau" define a Markov process

- Bell causality:

  $\frac{\mathbb{P}(\alpha)}{\mathbb{P}(\beta)}$ indep. of spectator



- Markov Σ rule

(We do not require "locality")

These ⇒ almost uniquely a family of processes that turn out to be a kind of "generalized percolation"

Perc: pick a set S of ancestors independently with prob p, then close transitively

$$\Rightarrow \mathbb{P}(S) = p^{|S|}(1-p)^{n-|S|} \propto \left(\frac{p}{1-p}\right)^{|S|}$$

Gen. Perc: $\mathbb{P}(S)$ depends only on $k = |S|$, call it t_k

Dynamical Matters I: Dynamics as sequential Growth

(here consider causet as actor, later as background)

☺ CSG = classical sequential growth model: discrete cosmology

stepping stone (we have) to QSG = QG

poscau

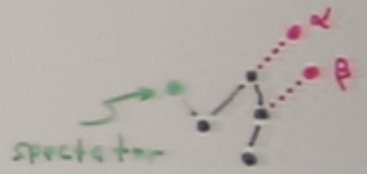
Assume

- sequential growth: elt's born (stochastically) one by one
- internal temporality: no elt born to past of any pre-existing element
- classical probabilities: no interference
- general covariance: probs at stage n are labeling independent

  ⇒ paths in "poscau" define a Markov process

- Bell causality:

  $\frac{\mathbb{P}(\alpha)}{\mathbb{P}(\beta)}$ indep. of spectator



- Markov Σ rule

(We do not require "locality")

These ⇒ almost uniquely a family of processes that turn out to be a kind of "generalized percolation"

Perc: pick a set S of ancestors independently with prob p, then close transitively

$$\Rightarrow \mathbb{P}(S) = p^{|S|}(1-p)^{n-|S|} \propto \left(\frac{p}{1-p}\right)^{|S|}$$

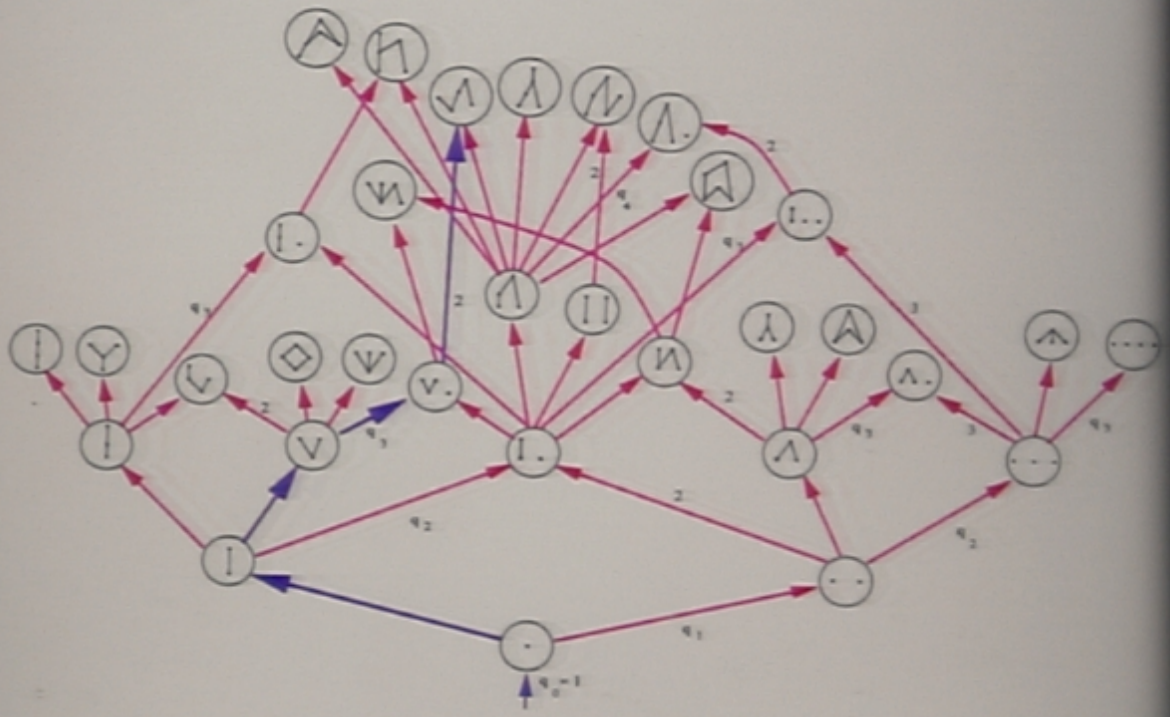


Figure 1: The poset of finite causets

Dynamical Matters I: Dynamics as (sequential) Growth

(here consider causet as actor, later as background)

☺ CSG = classical sequential growth model: discrete cosmology

stepping stone (we hope) to QSG = QG

poscau

Assume

- sequential growth: elt's born (stochastically) one by one
- internal temporality: no elt born to past of any pre-existing element
- classical probabilities: no interference
- general covariance: probs at stage n are labeling independent

⇒ paths in "poscau" define a Markov process

- Bell causality:

$\frac{\mathbb{P}(\alpha)}{\mathbb{P}(\beta)}$ indep. of spectator

- Markov Σ rule

(We do not require "locality")

These ⇒ almost uniquely a family of processes that turn out to be a kind of "generalized percolation"

Perc: pick a set S of ancestors independently with prob p, then close transitively

$$\Rightarrow \mathbb{P}(S) = p^{|S|}(1-p)^{n-|S|} \propto \left(\frac{p}{1-p}\right)^{|S|}$$

Gen. Perc: $\mathbb{P}(S)$ depends only on $k = |S|$, call it t_k

The parameters $t_0, t_1, t_2 \cdots$ define the stoch. proc.
Many results known for these CSG processes,
e.g. we can ask: "will posts occur?"
Claim critical falloff is $t_n = \left(\frac{t}{\log n}\right)^n$ with $t_c \stackrel{?}{=} \frac{\sqrt{m}}{3}$

$t > t_c \Rightarrow$ Tolman-Boltzmann cosmos, which naturally produces a homogeneous, isotropic, big early universe except it's probably not a manifold!

more generally, posts renormalize $t_n \to t_n + t_{n+1}$
stable set is percolation (+ convex combinations)

cosmic renormalization

To proceed to Quantum S.G. this way we need QBC!

omit?

(b) The other approach to causet dynamics

Could we base it on some analog of $\pm \int R\, dV$? (e.g. using $\Box\Box\sigma$?)

↑ Synge fcn.

Dynamical Matters II: "Matter" on a background causet

(a) scalar field ϕ

in $\mathbb{M}^n$ e.o.m. is $0 = \Box\phi = -\frac{\partial^2\phi}{\partial t^2} + \nabla^2\phi$

can we transcribe this to sprinklings of $\mathbb{M}^n$?

for starters can we deduce $\Box\phi$ itself?

Miraculous formula in $\mathbb{M}^2$

$$\Box\phi \propto -\tfrac{1}{2}\phi(x) + \left(\sum_I - 2\sum_{II} + \sum_{III}\right)\phi(x)$$

Fully Lor-invariant on test functions ϕ

Problem: only true on average, fluctuations huge, but $\exists$ variants to overcome this problem!

(relevant to "transplanckian puzzle" in: Hawking radiation; cosmology – e.g. $\Lambda < 1$ cm now $\Rightarrow$ once $\ll \ell_{planck}$ (worse with inflation.))

Needed: fuller analysis, 4D version of our formula

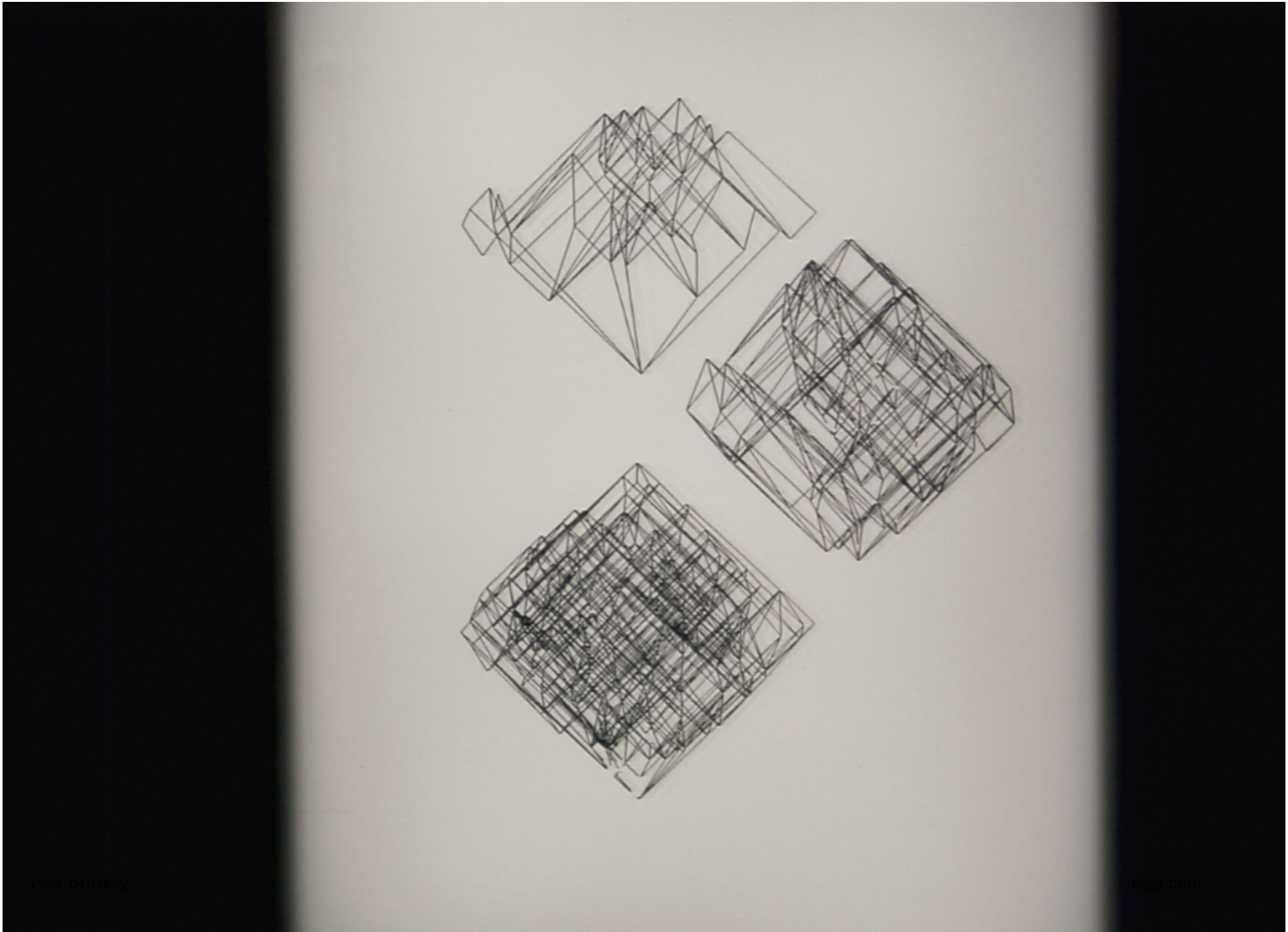
quantum version? other fields than scalars?

(b) particle on causet

exact "geodesic" cannot exist! what replaces it?

→ Lor invar diffusion in velocity space

Essentially unique!

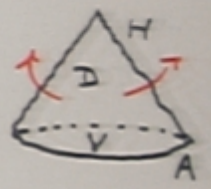


Phenomenology (just list)

- No time machines (purely kinematic reason)
- $\delta\Lambda \sim \frac{1}{\sqrt{N}}$ "confirmed" ($N \sim 10^{240}$) seee
- particle on causet ⇒ Lor. invar. diffusion
  (interesting problem in stochastic processes)
  relevant to cosmic rays?
- field on causet: light propagation from distant stars
  (using $\Box\phi$ — in progress)
  (relation to previous?)
- Black hole entropy seee
  can identify potential "horizon molecules" s.t.
  number ∝ area (works for non-equil. BH too)

omit?

related to "holography"?
(which claims that
"S crossing H" $\leq A$ (not V)
while can show
$|\text{maximal } D| \sim A$ (not V)



Some insights gained

- discreteness and Lorentz invariance can live together (just barely!)
  ⇒ kinematic randomness
  ⇒ UV-IR blurring

- locality must be given up and then $2^{n^2/4}$ is not a problem

- small Λ problem = why manifold problem
  also $\delta\Lambda$
  nonlocality the key?

- Solution of "problem of time" in terms of "stem predicates"

- toy cosmologies that explain the "cosmic big numbers" (without inflation)
  → new kind of "renormalization"

- dynamics = growth
  (covariance & becoming can coexist)

- And now we can ask new types of question:
  Why no holes or edges in spacetime?
  Why $d = 4$?
  Why are light cones round and not square?