Title: Two Dimensional Strings, Matrix Models and the Rolling Tachyon

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Abstract:

2D Strings, Matrix Models and Rolling tachyon

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1. 2D Strings: tachyons, W_∞, discrete states, black hole
2. Matrix model: tachyon, W_∞ the NCFT of $u(p,q,t)$
3. Matrix model as open string: quantitative description of rolling tachyon using NCFT of $u(p,q,t)$
4. Discrete states and B.H. from matrix model ?

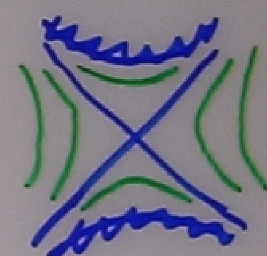
1·5

The linear, on-shell, gauge-inv. fluctuations $(\delta g_{\mu\nu}, \delta\Phi)$ are the limit $(m \to 0)$ of a full non-linear ~~gravit~~ metric-dilaton solution : The 2D B.H.

$$ds^2 = d\phi^2 - \tanh^2\left(2\phi + \tfrac{1}{2}\ln m\right)dt^2$$

$$\Phi = -\tfrac{1}{2}\ln\frac{m}{2} + \ln\cosh\left(2\phi + \tfrac{1}{2}\ln m\right)$$

$T = 0$



$\downarrow m \to 0$

$$ds^2 = -dt^2 + d\phi^2 + \underbrace{4me^{2\phi}}_{\delta g_{00}} dt^2 + \ldots$$

$O(m^2)$

$$\Phi = 2\phi + \underbrace{me^{2\phi}}_{} + \ldots$$

1.1

2D (BOSONIC) STRING THEORY

$$S = \frac{1}{4}\int dt\, d\phi \sqrt{-G}\, e^{-2\Phi}\,\mathcal{L}$$

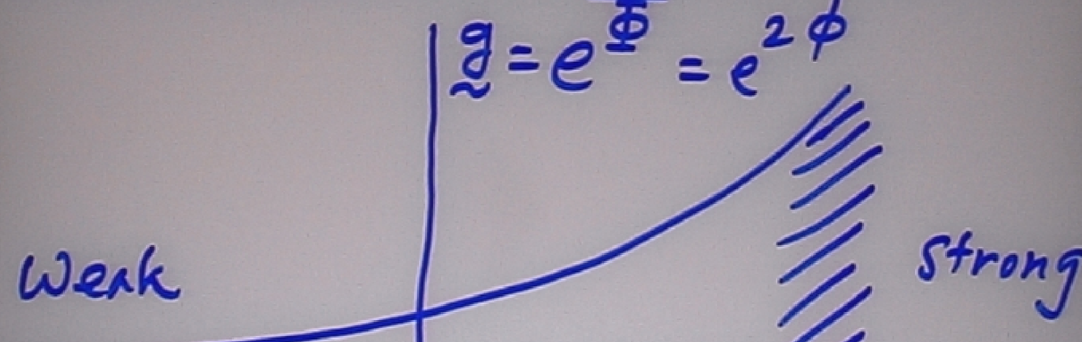
$$\mathcal{L} = R + 4(\nabla\Phi)^2 + \frac{2}{3}(26-D) - 2(\nabla T)^2 + 8T^2 - 2\tilde{V}(T)$$

Classical Solution #1

$$\overline{G}_{\mu\nu} = \eta_{\mu\nu} \qquad \overline{\Phi} = \frac{Q}{2}\phi \equiv 2\phi$$

$$\overline{T}(\phi) \underset{\sim}{\overset{\phi\to-\infty}{}} (b_1\phi + b_2)e^{2\phi}$$

$$g = e^{\overline{\Phi}} = e^{2\phi}$$



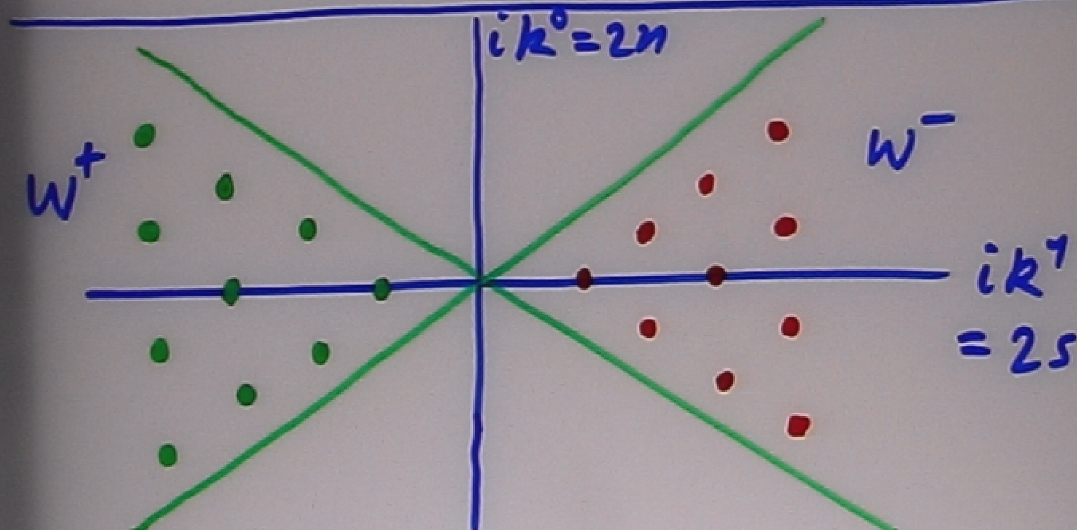
1·6

Other gauge-invariant on-shell fluctuations (world-sheet CFT analysis)

$$W^-_{s,n} \propto e^{2(1+s)\phi + 2nt}$$

$s = 1, \tfrac{3}{2}, 2, \cdots$

$n = -s+1, -s+2, \cdots, s-1$



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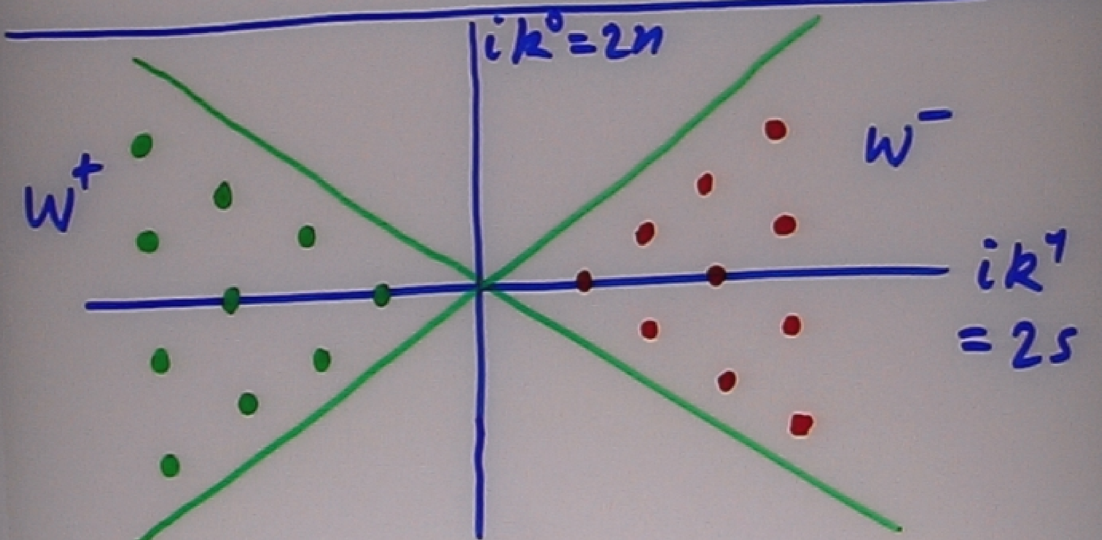

$(\delta g \ \ \delta \Phi) \Rightarrow W^-_{1,0}$

The W_∞ symmetry

$$W^+_{sn} \propto e^{2(1-s)\phi + 2nt}$$

$\Rightarrow$ ∞ no. of conserved currents (world sheet)

$\Rightarrow$ ∞-dim symmetry of string field theory

$$|\Psi\rangle \to |\Psi\rangle + Q\,|\Lambda_{s,n}\rangle$$

W_∞ algebra

W^+_{sn} : tachyons $\longrightarrow$ tachyons
discrete states $\to$ discrete states

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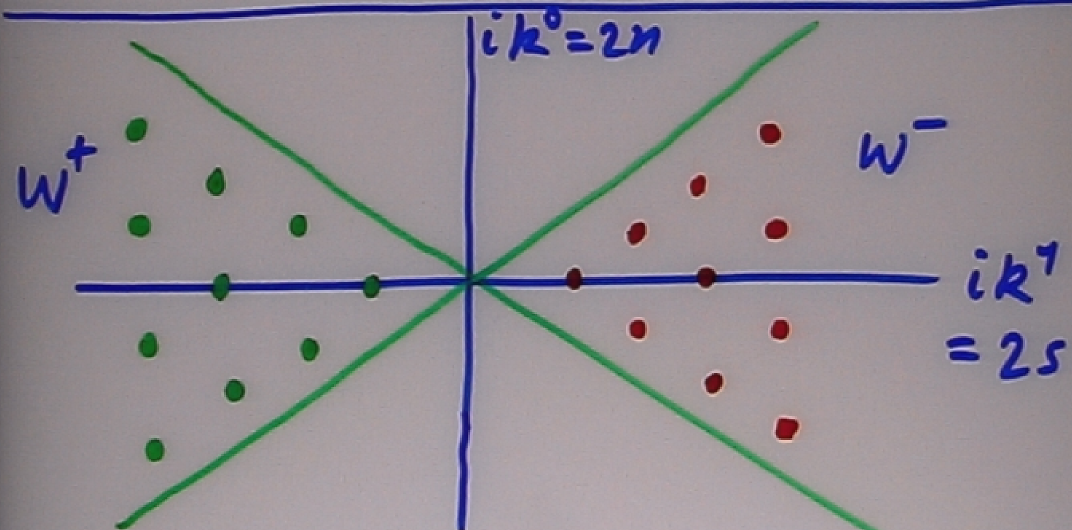

$$(\delta g, \delta \Phi) \Rightarrow W^-$$

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W_∞ algebra

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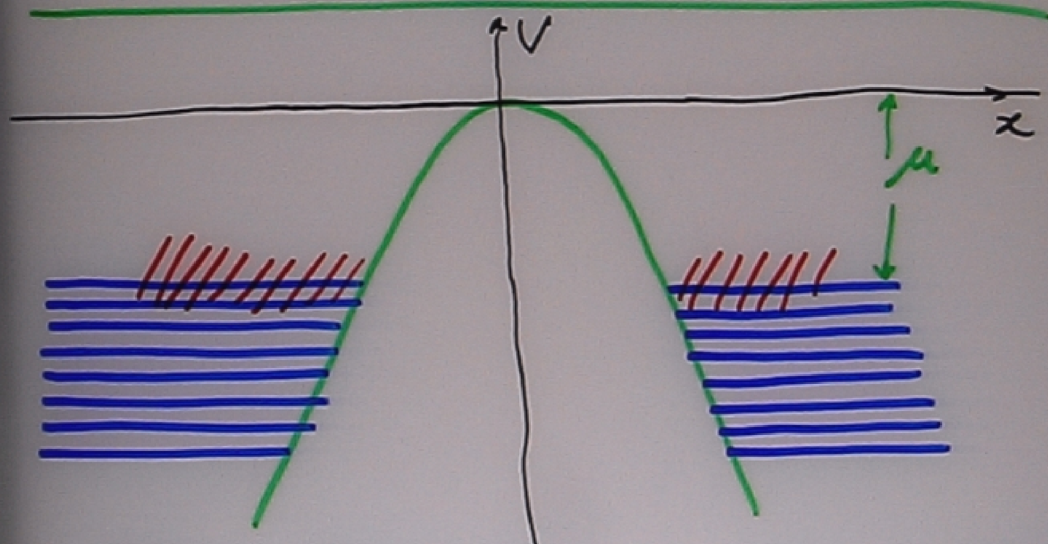
Matrix model

$$S = \int dt \; \mathrm{Tr}\left[\frac{\dot{M}^2}{2} + V(M)\right]$$

$$V(M) = -\frac{1}{2} M^2 \qquad [M_{ij}]_{N\times N}$$

N free fermions in a potential

$$h = -\frac{1}{2}\frac{\partial^2}{\partial x^2} - \frac{1}{2}x^2$$

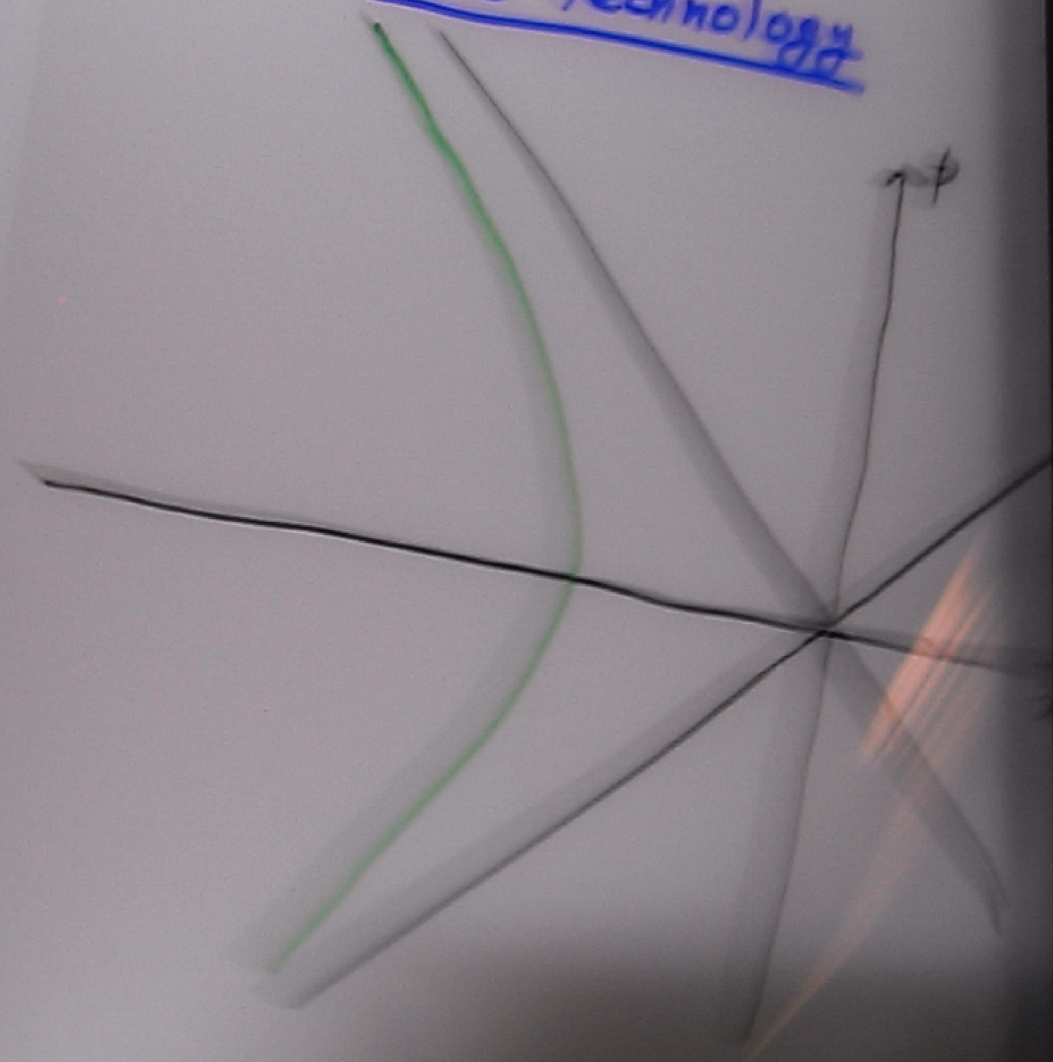


Near fermi level, $i(E-E_F)(\pm\tau - t)$

∴ Near fermi level,
matrix model fluctuations
= free massless fermions
in 1+1 dim
= free massless bosons
in 1+1 dim
($\Longrightarrow$ tachyons in 2D string theory)

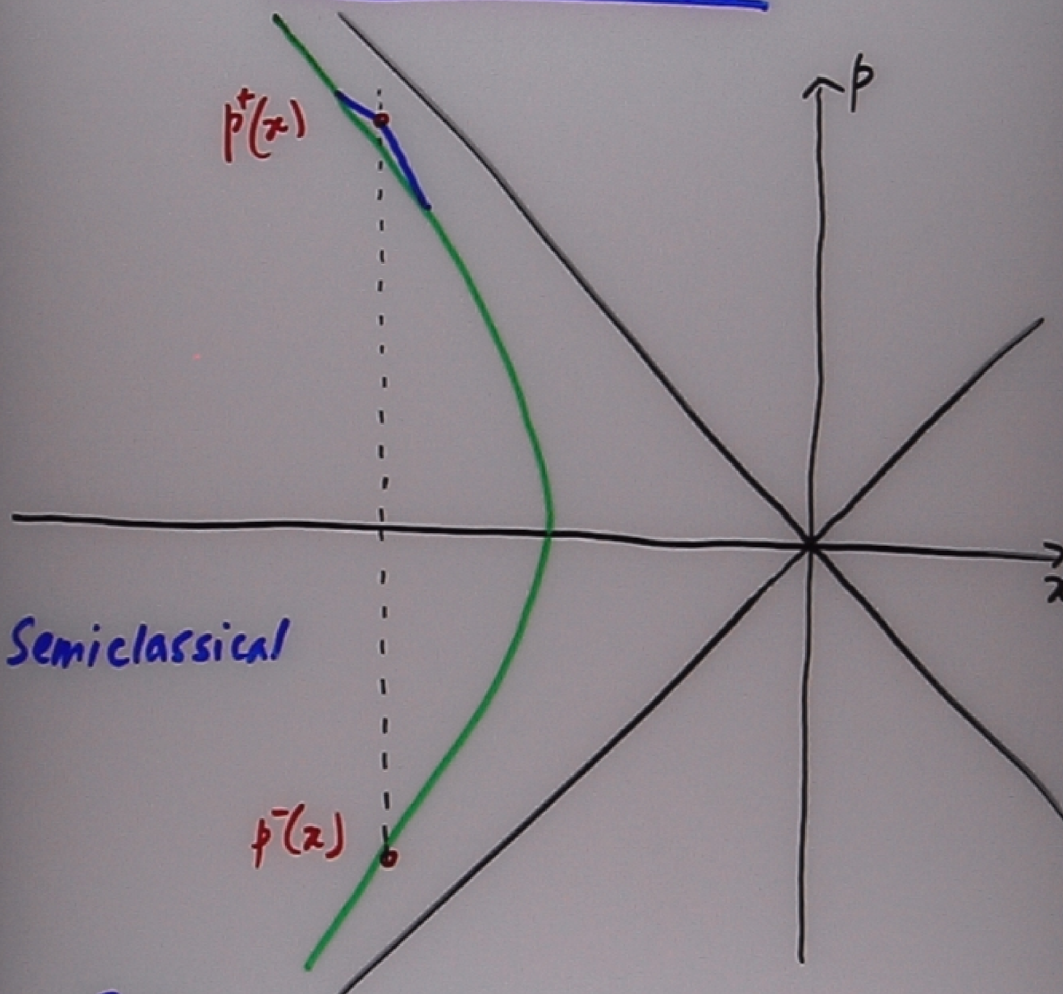
Non-relativistic corrections for fermions
$\Longrightarrow$ interaction for bosons

Some technology



2·3

Some technology



Semiclassical

$$\int \psi^*(x)\, \mathcal{O}\left(x, -i\frac{\partial}{\partial x}\right) \psi(x)\, dx$$

$$= \int dx \int^{p^+(x)} dp \quad \mathcal{O}(x,p)$$

2·4

Fermion no. density $\psi^\dagger\psi = p^+(x) - p^-(x) \equiv \rho(x)$

mom. density $\psi^\dagger\left(-i\frac{\partial}{\partial x}\right)\psi$

$$= \int_{p_-(x)}^{p_+(x)} p\,dp = \frac{(p^+(x))^2 - (p^-(x))^2}{2}$$

$$\equiv \frac{1}{2}\rho(x)\,\Pi(x)$$

Hamiltonian H

$$\int dx \int_{p_+(x)}^{p_-(x)} \left(\frac{p^2}{2} - \frac{x^2}{2}\right) dp = \int dx \left[\frac{p_+^3 - p_-^3}{6} - \frac{x^2}{2}(p_+ - p_-)\right]$$

$$p_+ = p_+^{(0)} + \delta p_+$$

$$p_- = p_-^{(0)} + \delta p_-$$

$\Rightarrow$ Interacting massless relativistic bosons: $\delta p_\pm = \left[(\partial_\tau \pm \partial_t)\,\bar{S}(\tau,t)\right]\frac{1}{p_+^{(0)}}$

Hamiltonian

$$H = \int d\tau \left[\bar{\pi}^2 + \bar{S}'^2 + e^{2\tau} O(\bar{S}^3) \right]$$

$$\bar{\pi} = \dot{\bar{S}}$$

Correspondence with 2D string tachyon

$$t \longleftrightarrow t$$

$$\tau \longleftrightarrow \phi$$

$$\bar{S}(t,\tau) \longleftrightarrow S(t,\phi)$$

$$\left.\begin{aligned} \bar{\alpha}_+(\omega) &= \frac{\Gamma(-i\omega)}{\Gamma(i\omega)}\,\alpha_+(\omega) \\ \bar{\alpha}_-(\omega) &= \frac{\Gamma(i\omega)}{\Gamma(-i\omega)}\,\alpha_-(\omega) \end{aligned}\right\} \text{leg pole}$$

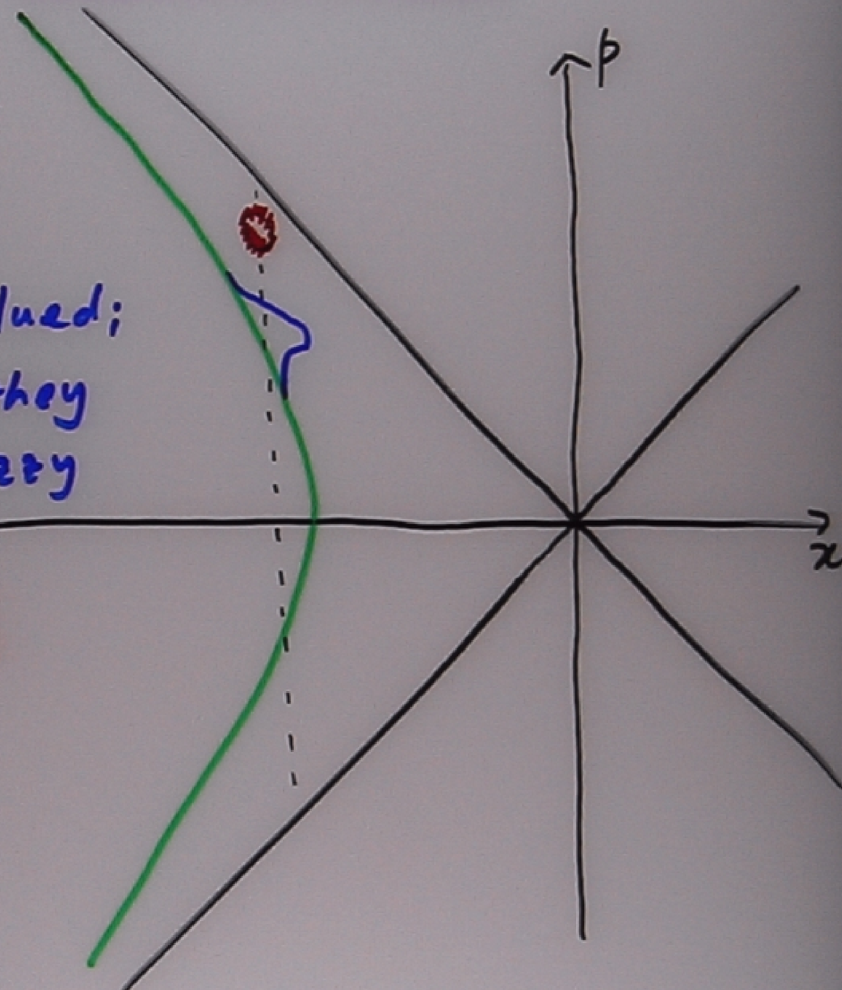
$$\bar{S}_+(t-\tau) = \int_{-\infty}^{+\infty} d\tilde{\tau}\, K(\tilde{\tau})\, S_+(t-\tau-\tilde{\tau})$$

$$S_-(t+\tau) = \int_{-\infty}^{+\infty} d\tilde{\tau}\, K(\tilde{\tau})\, \bar{S}_-(t+\tau-\tilde{\tau})$$

non

Some technology

$p^{\pm}(z)$
multi-valued;
worse, they
are fuzzy



$\hbar = g_s$

Correct bosonic description

$$u(p,q,t) = \int dx\,dy\; \psi^\dagger(x,t)\,\psi(y,t)\,\delta\!\left(q-\frac{x+y}{2}\right) \exp[i\,p(y-x)]$$

= Wigner phase space distribution

Fermion dynamics translates to

E.O.M $\quad i\,\partial_t u(p,q,t) = h * u - u * h$

$$h(p,q) \equiv \tfrac{1}{2}p^2 - \tfrac{1}{2}q^2$$

Constraint

$$u * u = u, \quad \int dp\,dq\; u(p,q) = N$$

Path integral $\int \mathcal{D}\mu(u)\, e^{iS[u(p,q,t)]}$

$$A * B = \exp\left[\tfrac{i}{2} g_s (\partial_p \partial_{q'} - \partial_q \partial_{p'})\right] [A(p,q)\,B(p',q')]_{p'=p,\,q'=q}$$

One can find a non-linear, non-local relation between $u(p,q,t)$ and the off-shell tachyon $S(x,t)$:

$$S(x,t) = \int dp\, dq\; K_1(p,q\,;x)\, \delta u(p,q,t) + \int dp\, dq \int dp'\, dq'\; K_2(p,q\,;p',q'\,;x)\, \delta u(p,q,t)\, \delta u(p',q',t) + \cdots$$

K_1, K_2 explicitly known up to 2nd order in pert. in e^{-2x} DMW '95

In case $u(p,q,t) = \Theta\left[\left(p^{+}(q,t) - p\right)\left(p - p^{-}(q,t)\right)\right]$ we get back leg poles

The above relation is more general. Works even when $u(p,q,t)$ describes "folds" or "fuzz".

W_∞ charges

2·7

Observables of free Fermi theory

$$\left[\int dx\, \Psi^\dagger(x)\, O_1(x, -i\partial_x)\, \Psi(x),\ \int dx'\, \Psi^\dagger(x')\, O_2(x', -i\partial_{x'})\, \Psi(x')\right]$$

$$= \int dx\, \Psi^\dagger(x)\, [O_1, O_2]\, \Psi(x)$$

isomorphic to algebra of hermitian operators in 1 particle Hilbert space $\equiv W_\infty$

Basis $\hat{W}_{sn} = \int dx\, \Psi^\dagger(x)\, (x+\hat{p})^{s+n} (x-\hat{p})^{s-n}\, \Psi(x)$

$$= \int dp\, dq\, U(p,q)\, (q+p)^{s+n} (q-p)^{s-n}$$

Semiclassical

$$[O_1, O_2] \xrightarrow{g_s \to 0} \{O_1, O_2\}_{PB}$$

PB ⟹ canonical transformation

$\xrightarrow{\text{in 2D}}$ area preserving diff

(shape of fermi fluid changes)

2·7

W_∞ charges

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$$\left[\int dx\, \Psi^\dagger(x)\, O_1(x, -i\partial_x)\, \Psi(x) , \int dx'\, \Psi^\dagger(x')\, O_2(x', -i\partial_{x'})\, \Psi(x')\right]$$

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PB $\Rightarrow$ canonical transformation

$\xrightarrow{\text{in 2D}}$ area preserving diff

(shape of fermi fluid changes)

3.1

Matrix model as Open String

D0-brane

The flat space lin. dilaton bgd. admits a D0-brane

Unstable, has open string tachyon

$$V(T) = -\frac{1}{2}T^2$$

For N D0-branes, $T \to M_{ij}$.

$$S_E = \int dt \, \mathrm{Tr}\left[\frac{1}{2}(D_t M)^2 - \frac{1}{2}M^2\right]$$

$$D_t M = \partial_t M + i[A_0, M]$$

Integrating out A_0 gives singlet sector of $c=1$ matrix model = fermion theory

KMS, MV '03

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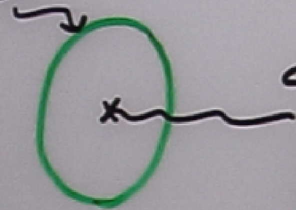
3.2

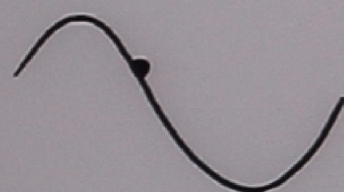
D0 B.C.



closed
string
tachyon

LLM '03

Rolling tachyon decay product
(world-sheet analysis)



The unstable D0 brane decays
into closed string tachyon.

Total energy of decay product

$$E_{tot} = \int_0^\infty dp\, p\, n(p) = \int_0^\infty dp = \infty$$

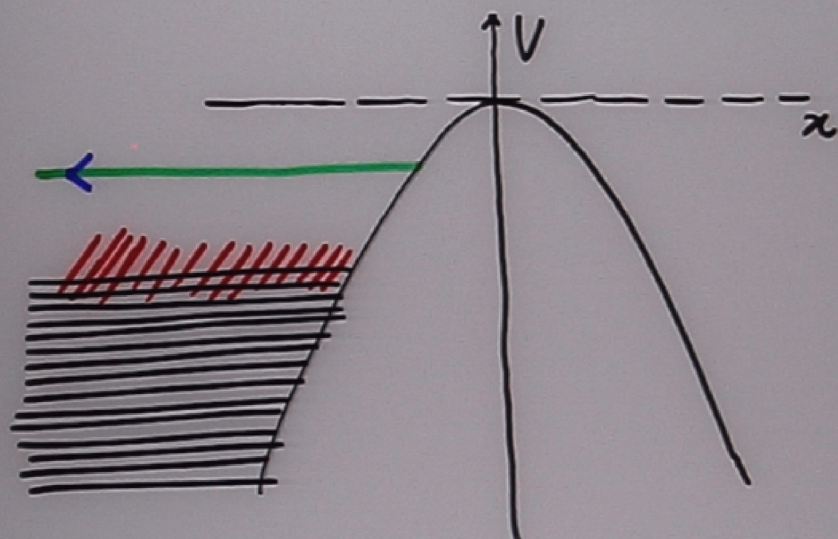
$$n(p) = p^{-1}$$

$\frac{1}{g_s}$?

Non-perturbative treatment required.

Proposal (KMS)

D0 brane = fermion of the matrix model



Puzzle:

Fermion at a given energy level close to the top will remain there. How does it enter the "relativistic band" near the Fermi level?

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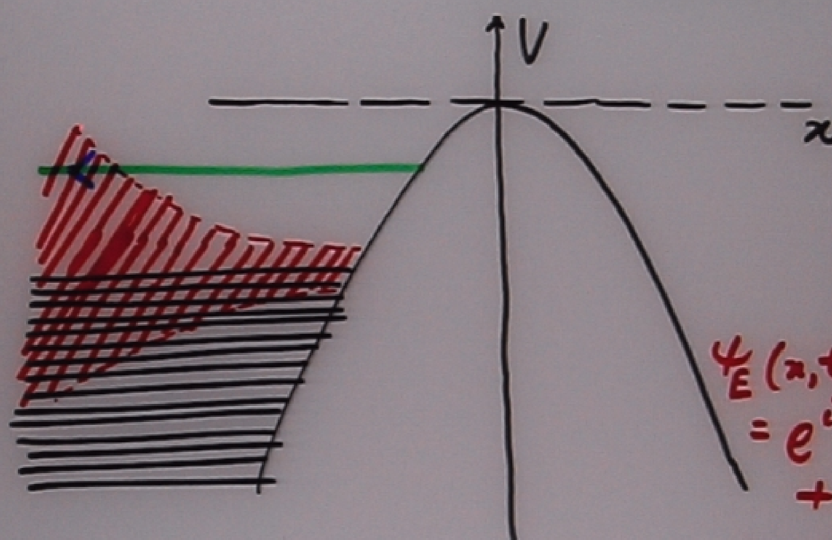

BMW '04

$\psi_E(x,t) = e^{i\epsilon(t+\tau)} + O\left(\frac{\epsilon}{x^2}\right)$

$\epsilon = E - E_F$

Puzzle:

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Quantitative description of rolling tachyon

At early times fermion is non-rel., even at late times $p^{\pm}$ description will not work.

Describe the motion of fermion in the $u(p,q,t)$ NCFT.

$$\left[\partial_t + p\partial_q + q\partial_p\right] u(p,q,t) = 0$$

$$u * u = u$$

$$\int dp\, dq\; u(p,q,t) = N$$

Solution :

$$u(p,q,t) = u_0\left(\bar{p}(t), \bar{q}(t)\right)$$

where

$$u_0 * u_0 = u_0$$

$$\int dp\, dq\; u_0(p,q) = N$$

$$\begin{pmatrix}\bar{p}(k)\\ \bar{q}(k)\end{pmatrix} = \begin{pmatrix}\cosh t & -\sinh t\\ -\sinh t & \cosh t\end{pmatrix}\begin{pmatrix}p\\ q\end{pmatrix}$$

3.4

Quantitative description of rolling tachyon

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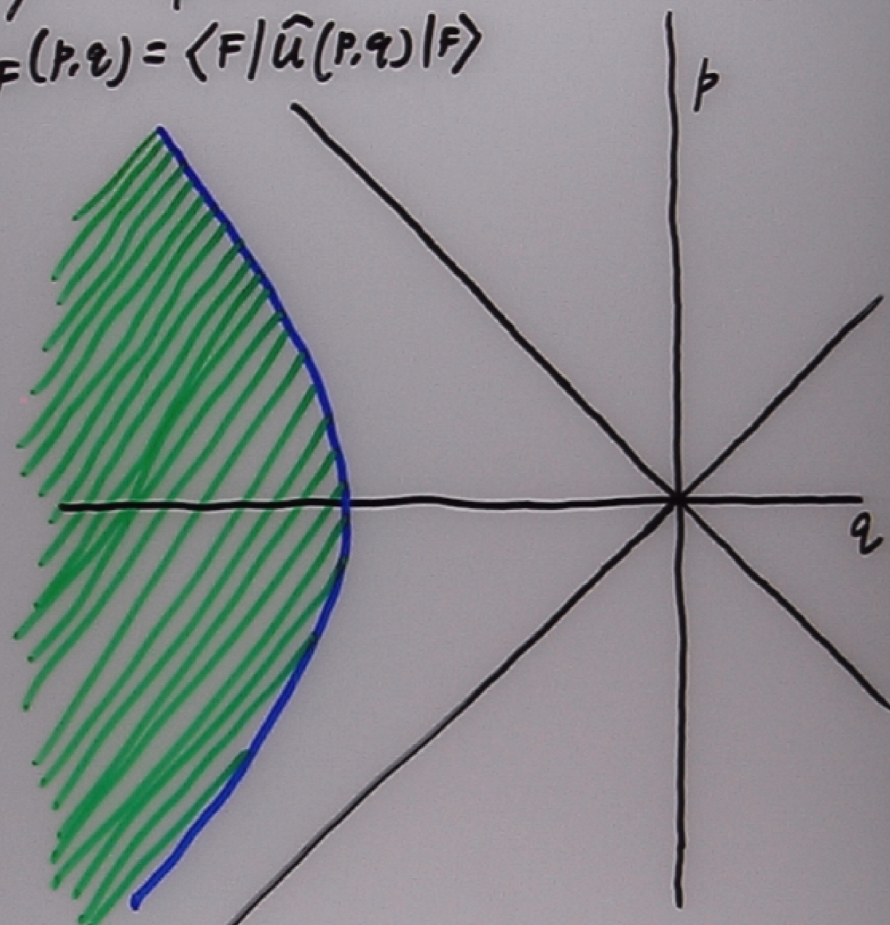
3.5

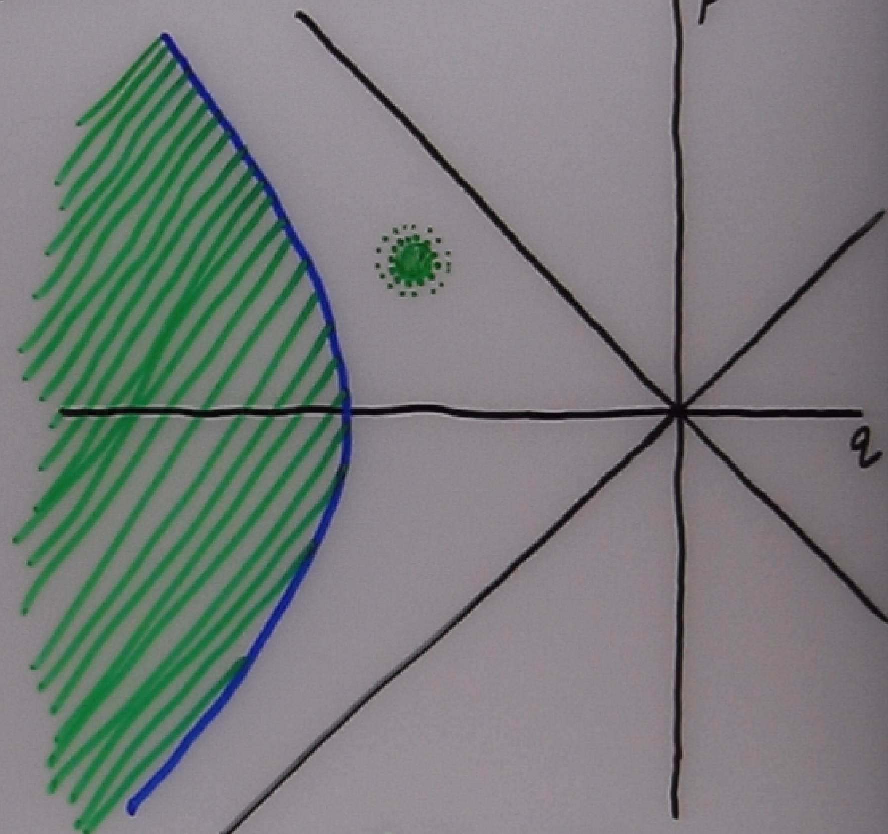
$$u(p,q) = u_F(p,q)$$

$$u_F(p,q) = \langle F | \hat{u}(p,q) | F \rangle$$



$$u_0(p,q) = u_F(p,q) + u_1(p,q) \qquad \underline{3.5}$$

$$u_F(p,q) = \langle F|\hat{u}(p,q)|F\rangle$$



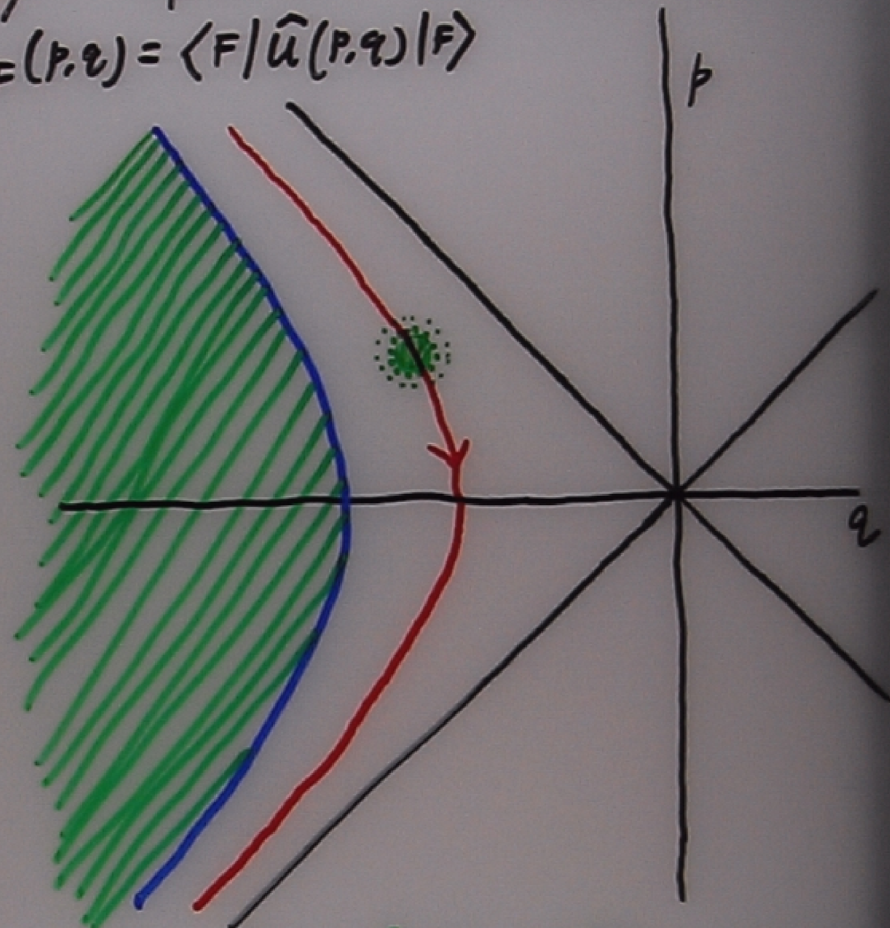
$$u_1(p,q) = \exp\left[-\frac{(p-p_0)^2+(q-q_0)^2}{g_s}\right] = \langle\psi_1|\hat{u}(p,q)|\psi_1\rangle$$

$$|\psi_1\rangle = \exp\left[-\frac{\frac{1}{2}(x-q_0)^2+ip_0x}{g_s}\right]$$

3.5

$$u(p,q,t) = u_F(p,q) + u_1(\bar{p}(t), \bar{q}(t))$$

$$u_F(p,q) = \langle F|\hat{u}(p,q)|F\rangle$$



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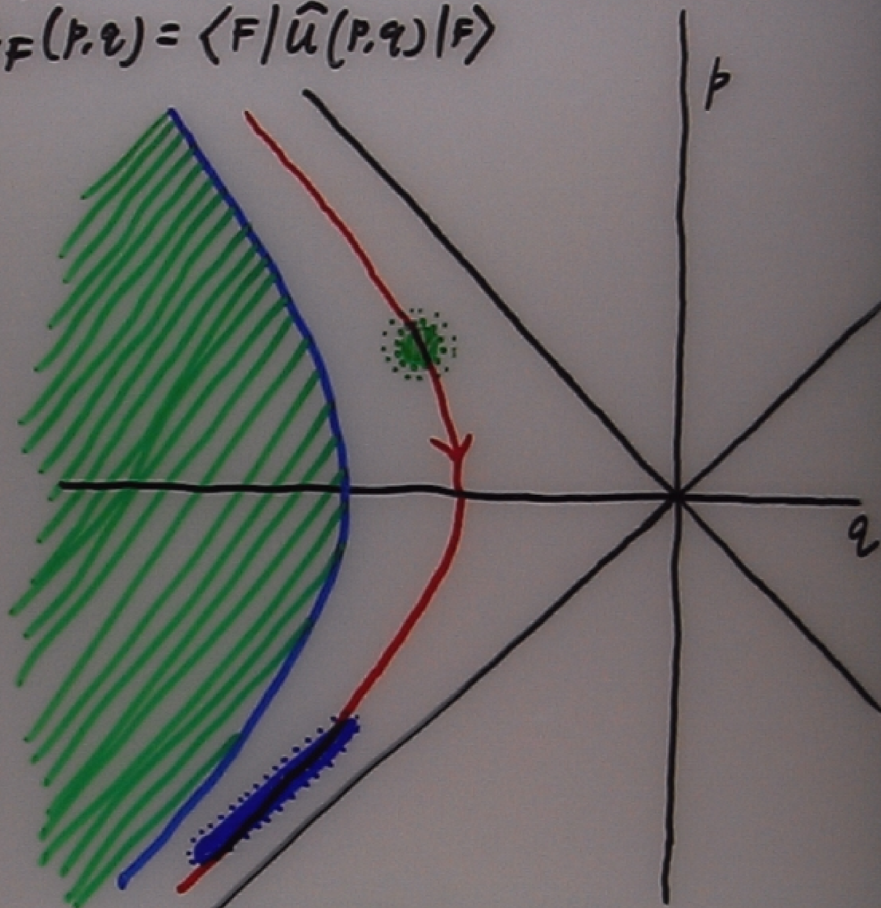
$$|\psi_1\rangle = \exp\left[-\frac{\frac{1}{2}(x-q_0)^2 + i p_0 x}{g_s}\right]$$

$$\begin{pmatrix} \bar{p}(t) \\ \bar{q}(t) \end{pmatrix} = \begin{pmatrix} \cosh t & -\sinh t \\ -\sinh t & \cosh t \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}$$

$$u(p,q,t) = u_F(p,q) + u_1(\bar{p}(t), \bar{q}(t))$$ 3.5

$$u_F(p,q) = \langle F|\hat{u}(p,q)|F\rangle$$



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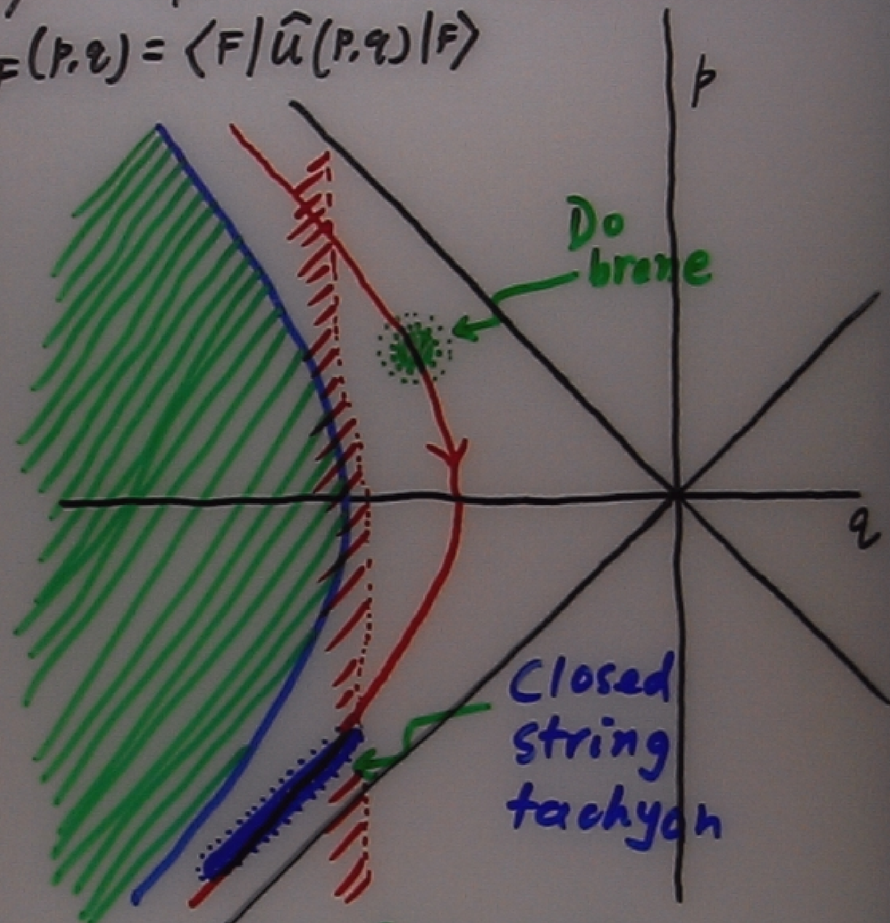
3.5

$$u(p,q,t) = u_F(p,q) + u_1(\bar{p}(t), \bar{q}(t))$$

$$u_F(p,q) = \langle F | \hat{u}(p,q) | F \rangle$$



$$u_1(p,q) = \exp\left[-\frac{(p-p_0)^2 + (q-q_0)^2}{g_s}\right] = \langle \psi_1 | \hat{u}(p,q) | \psi_1 \rangle$$

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$$S(x,t) = \int dp\,dq\; K_1(p,q;x)\, u_1(p,q,t)$$

reproduces tachyon distribution in the decay product.

Energy and all other W_∞ charges are finite and explicit:

$$W^+_{sn} = \frac{1}{g_s^s}(p_0+q_0)^{s+n}(p_0-q_0)^{s-n}(1+O(g_s))$$

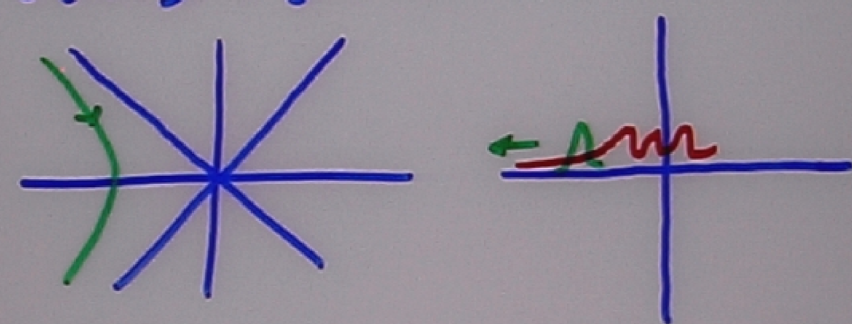
$$E = \frac{1}{2} W^+_{10} = \frac{p_0^2 - q_0^2}{2 g_s}$$

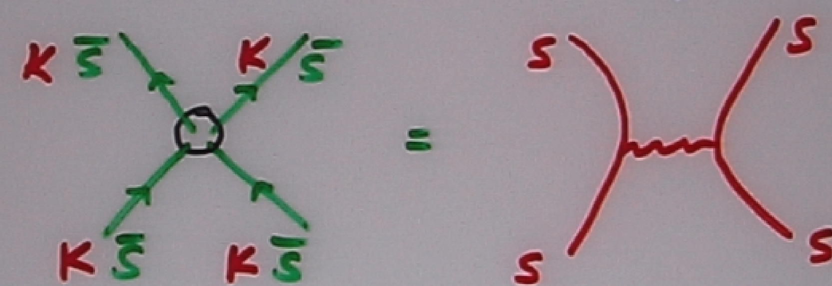
- A classical solution of $u(p,q,t)$ theory reproduces $\frac{1}{g_s}$ since it's NCFT with NC parameter g_s !

Discrete states and B.H.

- Gravitational interaction in the matrix model PN '94

$$S(x,t) = \int dx' K(x,x') \bar{S}(x',t)$$



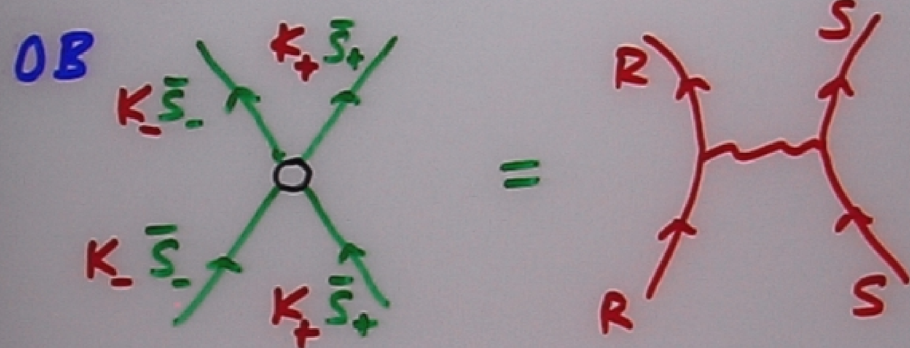


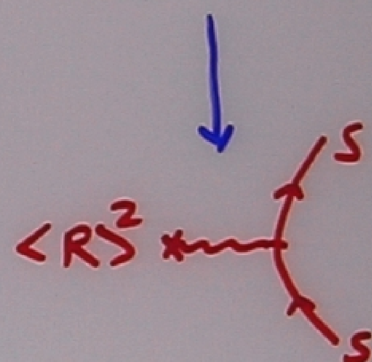
Tail of leg-pole transformation

⟶ long-range gravitational interaction

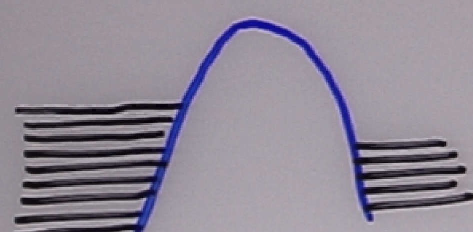
⟶ virtual discrete state

- Real gravity bgd.





If Ramond field has a vev



DMW '95 '04

4·3

• Discrete states from D0-brane

At early times

Sen '04
DMW '04

D0-branes $\neq$ tachyons

= discrete state ?

$$\langle D | \partial_p^m \partial_q^n \hat{U}(p,q) \Big|_{p=q=0} | D \rangle \xrightarrow{g_s \to 0} \underbrace{\partial_p^m \partial_q^n \delta(p)\,\delta(q)}_{\text{Witten's } W_{sn}^{-} \text{ module}}$$

(cf. $\langle \vec{n} | J_i | \vec{n} \rangle = -j\, n_i$.)

Rolling tachyon:

discrete states $\longrightarrow$ tachyons ?

(time reversed:

tachyons $\longrightarrow$ discrete states?)

- Discrete states from D0-brane

Sen '04
DMW '04

At early times
D0-branes $\neq$ tachyons
= discrete state ?

$$\langle D| \partial_p^m \partial_q^n \hat{U}(p,q)|_{p=q=0} |D\rangle \xrightarrow{g_s \to 0} \underbrace{\partial_p^m \partial_q^n \delta(p)\, \delta(q)}_{\substack{\text{Witten's} \\ W^-_{Sn}\ \text{module}}}$$

(cf. $\langle \vec{n}| J_i |\vec{n}\rangle = -j\, n_i$.)

Rolling tachyon:
discrete states $\longrightarrow$ tachyons ?

(time reversed:
tachyons $\longrightarrow$ discrete states?)

- **2D B.H.**

Temp $T = \text{const}$ (indep of mass)

$t_{decay} \propto \text{mass}$

Can a D0-brane or isolated fermion far above the fermi sea represent a B.H. ?

$\text{mass} \propto \frac{1}{g_s}$

$\therefore \; t_{decay} \propto \frac{1}{g_s}$

But rolling tachyon decay time $\propto O(1)$!

Could we slow the decay by considering a coherent superposition of M D0-branes

$\frac{M}{N} = \text{const}, \; M \to \infty, \; N \to \infty$?