Title: Supergravity Solutions of Intersecting Branes Based on Embedding Slef-Dual Geometries in M-theory

Date: Oct 26, 2004 11:05 AM

URL: http://pirsa.org/04100015

Abstract:

Supergravity Solutions of Intersecting Branes Based on Embedding Self-dual Geometries in M-theory

A.M. Ghezelbash
University of Waterloo

Based on papers

A.M.G. and R.B. Mann, JHEP 10 (2004) 012

R. Clarkson, A.M.G. and R.B. Mann, JHEP 08 (2004) 025
JHEP 04 (2004) 063

Motivations :

1) sugra solutions of D-systems are easy to find as long as they are smeared.

2) sugra solutions of the localized intersections are more difficult to find.

3) For brane intersecting systems with D6: the known methods are limited to the near core regions of the D6.

A. Hashimoto ; JHEP01 (1999) 018
S.A. Cherkis, hep-th/9906203
N. Itzhaki, A.A. Tseytlin, S. Yankielowicz, PLB432 (1998) 298

4) By embedding TN lifted to M-theory $\longrightarrow$ D2/D6 system

* without restriction to the near core region of the D6.
* instead of taking the near horizon limit of D6, one can consider the simpler near horizon limit of the D2.
* In decoupling limit $\Rightarrow$ dual gravity side to gauge theory on D2 is obtainable explicitly

Atiyah–Hitchin Space

↳ a part of moduli space of the set of two monopole solutions of Bogomolnyi eq.

$R^3 \otimes \frac{S^1 \otimes AH}{Z_2} \longrightarrow$ hyper-kähler $\Rightarrow$ Self-dual curvature

↳ full quantum moduli space of N=4 susy gauge theory in 3D

$$ds^2_{AH} = f^2(r)dr^2 + a^2(r)\sigma_1^2 + b^2(r)\sigma_2^2 + c^2(r)\sigma_3^2$$

σ_i : Maurer–Cartan one forms $\quad d\sigma_i = \frac{1}{2}\varepsilon_{ijk}\sigma_j \wedge \sigma_k$

Einstein's eqs (or Self-duality)

$$a' = f\,\frac{(b-c)^2 - a^2}{2bc}$$

$$b' = f\,\frac{(c-a)^2 - b^2}{2ca}$$

$$c' = f\,\frac{(a-b)^2 - c^2}{2ab}$$

$$a(r) = \sqrt{\frac{r S \sin\gamma \left\{ \frac{1-\cos\gamma}{2} r - \sin\gamma S \right\}}{S \sin\gamma + r\cos^2\left(\frac{\gamma}{2}\right)}}$$

$$b(r) = \sqrt{\frac{\left\{ S \sin\gamma - \frac{1-\cos\gamma}{2} r \right\} r \left\{ -S \sin\gamma - \frac{1+\cos\gamma}{2} r \right\}}{S \sin\gamma}}$$

$$c(r) = -\sqrt{\frac{r S \sin\gamma \left\{ \frac{1+\cos\gamma}{2} r + \sin\gamma S \right\}}{-S \sin\gamma + \frac{1-\cos\gamma}{2} r}}$$

where

$$S = \frac{2n E\left(\sin\frac{\gamma}{2}\right)}{\sin\gamma} - \frac{n K\left(\sin\frac{\gamma}{2}\right)\cos\frac{\gamma}{2}}{\sin\frac{\gamma}{2}}$$

$$K\left(\sin\frac{\gamma}{2}\right) = \frac{r}{2n} \qquad \begin{cases} n\pi \leq r < \infty \\ 0 \leq \gamma < \pi \end{cases}$$

Constant number of unit length

$$\begin{cases} E(k) = \int_0^{\pi/2} \sqrt{1-k^2\cos^2\theta}\, d\theta \\ K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k^2\cos^2\theta}} \end{cases}$$

Elliptic integrals

Asymptotic behaviours of AH metric functions:

$$a(r) = \sqrt{\frac{r\,\mathcal{S}\,\sin\gamma\left\{\frac{1-\cos\gamma}{2}r - \sin\gamma\,\mathcal{S}\right\}}{\mathcal{S}\sin\gamma + r\cos^2\left(\frac{\gamma}{2}\right)}}$$

$$b(r) = \sqrt{\frac{\left\{\mathcal{S}\sin\gamma - \frac{1-\cos\gamma}{2}r\right\} r \left\{-\mathcal{S}\sin\gamma - \frac{1+\cos\gamma}{2}r\right\}}{\mathcal{S}\,\sin\gamma}}$$

$$c(r) = -\sqrt{\frac{r\,\mathcal{S}\,\sin\gamma\left\{\frac{1+\cos\gamma}{2}r + \sin\gamma\,\mathcal{S}\right\}}{-\mathcal{S}\sin\gamma + \frac{1-\cos\gamma}{2}r}}$$

where

$$\mathcal{S} = \frac{2n\,E\left(\sin\frac{\gamma}{2}\right)}{\sin\gamma} - \frac{n\,K\left(\sin\frac{\gamma}{2}\right)\cos\frac{\gamma}{2}}{\sin\frac{\gamma}{2}}$$

$$K\left(\sin\frac{\gamma}{2}\right) = \frac{r}{2n} \qquad \left\{\begin{array}{l} n\pi \leq r < \infty \\ 0 \leq \gamma < \pi \end{array}\right.$$

Constant number of unit length

$$\left\{\begin{array}{l} E(k) = \int_0^{\pi/2} \sqrt{1-k^2\cos^2\theta}\,d\theta \\ K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k^2\cos^2\theta}} \end{array}\right.$$

Elliptic integrals

Asymptotic behaviours of AH metric functions:

$$a(r) \xrightarrow{r\to\infty} r\sqrt{1-\frac{2n}{r}} + \mathcal{O}(e^{-r/n})$$

$$b(r) \xrightarrow{r\to\infty} r\sqrt{1-\frac{2n}{r}} + \mathcal{O}(e^{-r/n})$$

$$a(r) = \sqrt{\frac{r\,\mathcal{S}\sin\gamma\left\{\frac{1-\cos\gamma}{2}r - \sin\gamma\,\mathcal{S}\right\}}{\mathcal{S}\sin\gamma + r\cos^2\left(\frac{\gamma}{2}\right)}}$$

$$b(r) = \sqrt{\frac{\left\{\mathcal{S}\sin\gamma - \frac{1-\cos\gamma}{2}r\right\} r \left\{-\mathcal{S}\sin\gamma - \frac{1+\cos\gamma}{2}r\right\}}{\mathcal{S}\sin\gamma}}$$

$$c(r) = -\sqrt{\frac{r\,\mathcal{S}\sin\gamma\left\{\frac{1+\cos\gamma}{2}r + \sin\gamma\,\mathcal{S}\right\}}{-\mathcal{S}\sin\gamma + \frac{1-\cos\gamma}{2}r}}$$

where

$$\mathcal{S} = \frac{2n\,E\left(\sin\frac{\gamma}{2}\right)}{\sin\gamma} - \frac{n\,K\left(\sin\frac{\gamma}{2}\right)\cos\frac{\gamma}{2}}{\sin\frac{\gamma}{2}}$$

$$K\left(\sin\frac{\gamma}{2}\right) = \frac{r}{2n} \qquad \begin{cases} n\pi \leq r < \infty \\ 0 \leq \gamma < \pi \end{cases}$$

Constant number of unit length

$$\begin{cases} E(k) = \int_0^{\pi/2} \sqrt{1-k^2\cos^2\theta}\,d\theta \\ K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k^2\cos^2\theta}} \end{cases}$$

Elliptic integrals

Asymptotic behaviours of AH metric functions:

$$a(r) \xrightarrow{r\to\infty} r\sqrt{1-\frac{2n}{r}} + \mathcal{O}(e^{-r/n})$$

$$b(r) \xrightarrow{r\to\infty} r\ldots + \mathcal{O}(e^{-r/n})$$

Atiyah–Hitchin Space

→ a part of moduli space of the set of two monopole solutions of Bogomolnyi eq.

$$R^3 \otimes \frac{S^1 \otimes AH}{Z_2} \longrightarrow \text{hyper-kähler} \Rightarrow \text{Self-dual curvature}$$

→ full quantum moduli space of $N=4$ susy gauge theory in 3D

$$ds^2_{AH} = f^2(r)\,dr^2 + a^2(r)\,\sigma_1^2 + b^2(r)\,\sigma_2^2 + c^2(r)\,\sigma_3^2$$

σ_i : Maurer–Cartan one forms $\quad d\sigma_i = \frac{1}{2}\varepsilon_{ijk}\,\sigma_j \wedge \sigma_k$

Einstein's eqs (or Self-duality)

$$a' = f\,\frac{(b-c)^2 - a^2}{2bc}$$

$$b' = f\,\frac{(c-a)^2 - b^2}{2ca}$$

$$c' = f\,\frac{(a-b)^2 - c^2}{2ab}$$

By choosing $f(r) = \frac{-b(r)}{r}$, we find

Interesting asymptotic metric!

$$ds^2_{AH} \xrightarrow{r\to\infty} \left(1-\frac{2n}{r}\right)\left(dr^2+r^2d\Omega^2\right)+4n^2\left(1-\frac{2n}{r}\right)^{-1}\left(d\psi+\cos\theta\, d\phi\right)^2$$

Euclidean Taub-NUT with a negative NUT charge $(-n)$

M2-brane Solution

$$ds_{11}^2 = \left\{H^{-1/3}\left(-dt^2+dx_1^2+dx_2^2\right)+H^{2/3}\left(dy^2+y^2d\Omega_3^2+ds^2_{AH}\right)\right\} \quad H^{-1/3}$$

$$H = H(y,r)$$

$$F_{tx_1x_2y} = \frac{-1}{2H^2}\frac{\partial H}{\partial y} \qquad F_{tx_1x_2r} = \frac{-1}{2H^2}\frac{\partial H}{\partial r}$$

Supergravity eqs: $\Rightarrow$

$$r^2\frac{\partial^2 H}{\partial r^2}+r\left\{r\frac{(ac)'}{ac}+1\right\}\frac{\partial H}{\partial r}+b^2\left\{\frac{\partial^2 H}{\partial y^2}+\frac{3}{y}\frac{\partial H}{\partial y}\right\}$$

Interesting asymptotic metric!

$$ds^2_{AH} \xrightarrow{r\to\infty} \left(1-\frac{2n}{r}\right)\left(dr^2+r^2 d\Omega^2\right)+4n^2\left(1-\frac{2n}{r}\right)^{-1}\left(d\psi+\cos\theta\, d\phi\right)^2$$

Euclidean Taub-NUT with a negative NUT charge $(-n)$

M2-brane Solution

$$ds^2_{11} = \left\{H^{-1/3}\left(-dt^2+dx_1^2+dx_2^2\right)+H^{2/3}\left(dy^2+y^2 d\Omega_3^2+ds^2_{AH}\right)\right\}$$

$H^{-1/3}$

$H = H(y, r)$

$$F_{t x_1 x_2 y} = \frac{-1}{2H^2}\frac{\partial H}{\partial y} \qquad F_{t x_1 x_2 r} = \frac{-1}{2H^2}\frac{\partial H}{\partial r}$$

Supergravity eqs: $\Rightarrow$

$$r^2\frac{\partial^2 H}{\partial r^2}+r\left\{r\frac{(ac)'}{ac}+1\right\}\frac{\partial H}{\partial r}+b^2\left\{\frac{\partial^2 H}{\partial y^2}+\frac{3}{y}\frac{\partial H}{\partial y}\right\}$$

Interesting asymptotic metric!

$$ds^2_{AH} \xrightarrow{r\to\infty} \left(1-\frac{2n}{r}\right)\left(dr^2+r^2 d\Omega^2\right)+4n^2\left(1-\frac{2n}{r}\right)^{-1}\left(d\psi+\cos\theta\, d\phi\right)^2$$

Euclidean Taub-NUT with a negative NUT charge $(-n)$

M2-brane Solution

$$ds_{11}^2 = H^{-1/3}\left(-dt^2+dx_1^2+dx_2^2\right)+H^{2/3}\left(dy^2+y^2 d\Omega_3^2+ds_{AH}^2\right)$$

$H^{-1/3}$

$H = H(y,r)$

$$F_{tx_1x_2y} = \frac{-1}{2H^2}\frac{\partial H}{\partial y} \qquad F_{tx_1x_2r} = \frac{-1}{2H^2}\frac{\partial H}{\partial r}$$

Supergravity eqs: $\Rightarrow$

$$r^2\frac{\partial^2 H}{\partial r^2}+r\left\{r\frac{(ac)'}{ac}+1\right\}\frac{\partial H}{\partial r}+b^2\left\{\frac{\partial^2 H}{\partial y^2}+\frac{3}{y}\frac{\partial H}{\partial y}\right\}=0$$

$$H(y,r) = 1+Q_2 R_k(r)\frac{J_1(ky)}{y}$$

$$I^{(\text{Bos. Sec.})}_{D=11} = \int d^{11}x \sqrt{-g} \left\{ R - \frac{1}{48} F^2_{[4]} + \frac{1}{6} F_{[4]} \wedge F_{[4]} \wedge A_{[3]} \right\}$$

$$F_{[4]} = dA_{[3]}$$

Eqs. of motion:

$$R_{MN} - \frac{1}{2} g_{MN} R = \frac{1}{3} \left\{ F_{MPQR} F_N{}^{PQR} - \frac{1}{8} g_{MN} F_{PQRS} F^{PQRS} \right\}$$

$$\nabla_M F^{MNPQ} = \frac{-1}{576} \varepsilon^{M_1 \cdots M_8 NPQ} F_{M_1 \cdots M_4} F_{M_5 \cdots M_8}$$

After reduction to $D=10$

$$F_{[4]} = \mathcal{F}_{[4]} + H_{[3]} \wedge dx_{10}$$

$$ds^2_{(1,10)} = e^{-2\phi/3} ds^2_{(1,9)} + e^{4/3\,\Phi} \left(dx_{10} + C_\alpha dx^\alpha \right)^2$$

$$I^{\text{string-frame}}_{IIA} = \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left[R + 4 \nabla_M \phi \nabla^M \phi - \frac{1}{12} e^{-\phi} H^2_{[3]} \right] \right.$$

Interesting asymptotic metric!

$$ds^2_{AH} \xrightarrow{r\to\infty} \left(1-\frac{2n}{r}\right)\left(dr^2+r^2 d\Omega^2\right) + 4n^2\left(1-\frac{2n}{r}\right)^{-1}\left(d\psi+\cos\theta\, d\phi\right)^2$$

Euclidean Taub-NUT with a negative NUT charge $(-n)$

M2-brane Solution

$$ds_{11}^2 = \left\{H^{-1/3}\left(-dt^2+dx_1^2+dx_2^2\right) + H^{2/3}\left(dy^2+y^2 d\Omega_3^2+ds_{AH}^2\right)\right\}$$

$H^{-1/3}$

$H = H(y,r)$

$$F_{t x_1 x_2 y} = \frac{-1}{2H^2}\frac{\partial H}{\partial y} \qquad F_{t x_1 x_2 r} = \frac{-1}{2H^2}\frac{\partial H}{\partial r}$$

Supergravity eqs: $\Rightarrow$

$$r^2\frac{\partial^2 H}{\partial r^2} + r\left\{r\frac{(ac)'}{ac}+1\right\}\frac{\partial H}{\partial r} + b^2\left\{\frac{\partial^2 H}{\partial y^2}+\frac{3}{y}\frac{\partial H}{\partial y}\right\} = 0$$

$$H(y,r) = 1 + Q_2 R_k(r)\frac{J_1(ky)}{y}$$

$$acr^2 R_k'' + \left\{acr - \tfrac{1}{2}r\left[(a-b)^2+(b-c)^2-a^2-c^2\right]\right\}R_k' - k^2 a b^2 c R_k = 0$$

Near $r \simeq n\pi$

$$R_k(r) \simeq K_0\big(k(r-n\pi)\big) = -\ln\left(\frac{k}{2}\right) - \gamma - \ln(r-n\pi) + O\left([r-n\pi]^2\right)$$

Modified Bessel function of the 2nd kind

Euler-Mascheroni Constant

Note:

$$ds^2_{AH} \xrightarrow{r \simeq n\pi} dr^2 + 4(r-n\pi)^2(d\tilde{\psi} + \cos\tilde{\theta}\, d\tilde{\varphi})^2 +$$
$$+ n^2\pi^2(d\tilde{\theta}^2 + \sin^2\tilde{\theta}\, d\tilde{\varphi}^2)$$

induced metric on the 2D Bolt located at $r = n\pi$

Near infinity:

$$R_k(r) \simeq \frac{e^{-k^2/|k|\, r}}{r}$$

So: $H(y, r) = 1 + Q_2 \int^{\infty} dk\, p(k)\, \frac{J_1(ky)}{\tilde{y}}\, R_k(r)$

Near $r \simeq n\pi$

$$R_\kappa(r) \simeq \underbrace{K_0}_{\text{Modified Bessel function of the 2nd kind}}(\kappa(r-n\pi)) = -\ln\left(\frac{\kappa}{2}\right) - \underbrace{\gamma}_{\text{Euler-Mascheroni Constant}} - \ln(r-n\pi) + O([r-n\pi]^2)$$

Note:

$$ds^2_{AH} \xrightarrow{r \simeq n\pi} dr^2 + 4(r-n\pi)^2 (d\tilde{\psi} + \cos\tilde{\theta}\, d\tilde{\varphi})^2 + \underbrace{+ n^2\pi^2 (d\tilde{\theta}^2 + \sin^2\tilde{\theta}\, d\tilde{\varphi}^2)}_{\text{induced metric on the 2D Bolt located at } r=n\pi}$$

Near infinity:

$$R_\kappa(r) \simeq \frac{e^{-\kappa^2/|\kappa|\, r}}{r}$$

So:
$$H(y,r) = 1 + Q_2 \int_0^\infty d\kappa \, \underbrace{p(\kappa)}_{\text{measure function}} \frac{J_1(\kappa y)}{y} R_\kappa(r)$$

To obtain $p(\kappa)$; we are looking at near Bolt limit.

Transverse space to M2-brane at the near Bolt limit is : $R^4 \otimes R^2 \otimes S^2$

$$\Downarrow$$

$$H_0 \sim 1 + \frac{Q_2}{\mathcal{R}^6}$$

$$\mathcal{R} = \sqrt{y^2 + \tilde{r}^2}$$

$$\tilde{r} = r - n\pi$$

Comparing $H(y,r)$ with H_0

$\Rightarrow p(k) \sim k^4$

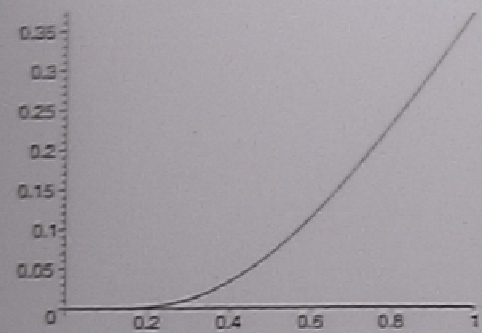
$$\boxed{H_{AH}(y,r) = 1 + Q_2 \int_0^\infty dk\, k^4 \frac{J_1(ky)}{y} R_k(r)}$$

Second Metric function for AH M2

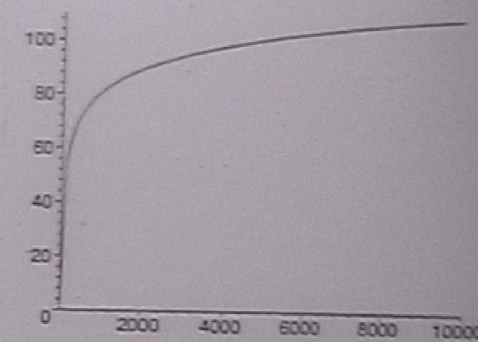
$$\tilde{H}_{AH}(y,r) = 1 + Q_2 \int_0^\infty d\tilde{k}\, \tilde{k}^4 \frac{K_1(\tilde{k}y)}{y} \tilde{R}_{\tilde{k}}(r)$$

Diff. eq. for $\tilde{R}_{\tilde{k}}(r)$ = Diff eq. for $R_k(r)$ with $k \to i\tilde{k}$

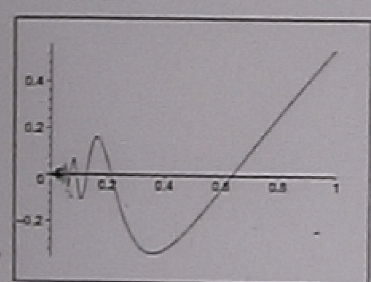
Near Bolt limit



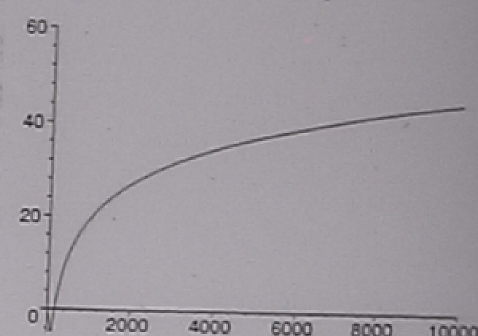
Radial function as a function of $\frac{1}{r}$, near infinity



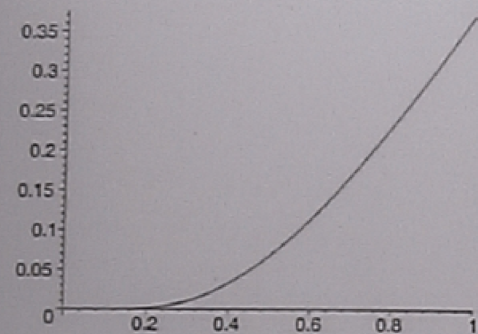
Radial function as a function of $\frac{1}{r-\pi}$.
For $r \simeq \pi$, R diverges logarithmically as $\ln\left(\frac{1}{r-\pi}\right)$



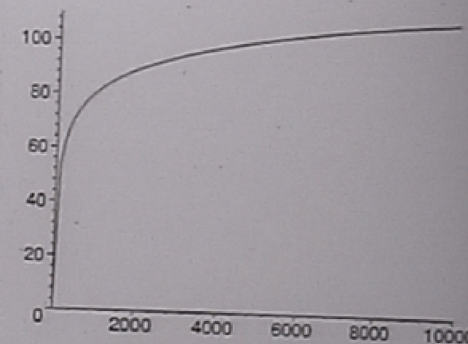
Second radial function as a function of $\frac{1}{r}$,



Second radial function as a function of $\frac{1}{r-\pi}$.

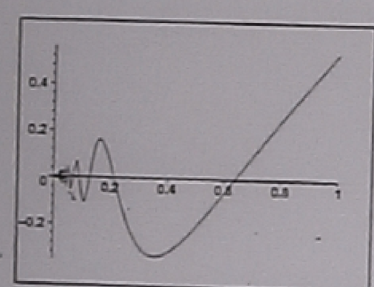


Radial function as a function of $\frac{1}{r}$, near infinity

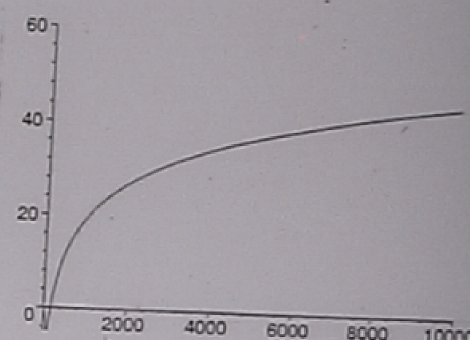


Radial function as a function of $\frac{1}{r-\pi}$.

For $r \simeq \pi$, R diverges logarithmically as $\ln\left(\frac{1}{r-\pi}\right)$



Second radial function as a function of $\frac{1}{r}$,



Second radial function as a function of $\frac{1}{r-\pi}$.

g $H(y,r)$ with r_0

$(k) \sim k^4$

$R = \sqrt{y^2 + \tilde{r}^2}$

$\tilde{r} = r - n\pi$

$$\boxed{_{AH}(y,r) = 1 + Q_2 \int_0^\infty dk\, k^4 \frac{J_1(ky)}{y} R_k(r)}$$

ond Metric function for AH N2

$$\tilde{H}_{AH}(y,r) = 1 + Q_2 \int_0^\infty d\tilde{k}\, \tilde{k}^4 \frac{K_1(\tilde{k}y)}{y} \tilde{R}_{\tilde{k}}(r)$$

f. eq. for $\tilde{R}_{\tilde{k}}(r)$ = Diff eq. for $R_k(r)$

with $k \to i\tilde{k}$

olt limit $n\pi$: $\tilde{R}_{\tilde{k}}(r) \simeq -Y_0(\tilde{k}(r-n\pi)) = \frac{-2}{\pi}\left\{\ln\frac{\tilde{k}}{2} + \gamma + \ln(r-n\pi)\right\}$

Bessel fun. of the 2nd kind.

Logarithmic divergence

er ∞ : $\tilde{R}_{\tilde{k}}(r) \simeq \frac{\cos(\tilde{k}r)}{r}$

Transverse space to M2-brane at the near Bolt limit is : $R^4 \otimes R^2 \otimes S^2$

$$\Downarrow$$

Comparing $H(y,r)$ with H_0

$\Rightarrow \rho(k) \sim k^4$

$H_0 \sim 1 + \frac{Q_2}{\mathcal{R}^6}$

$\mathcal{R} = \sqrt{y^2 + \tilde{r}^2}$

$\tilde{r} = r - n\pi$

$$\boxed{H_{AH}(y,r) = 1 + Q_2 \int_0^\infty dk\, k^4 \frac{J_1(ky)}{y} R_k(r)}$$

Second Metric function for AH M2

$$\tilde{H}_{AH}(y,r) = 1 + Q_2 \int_0^\infty d\tilde{k}\, \tilde{k}^4 \frac{K_1(\tilde{k}y)}{y} \tilde{R}_{\tilde{k}}(r)$$

Diff. eq. for $\tilde{R}_{\tilde{k}}(r)$ = Diff eq. for $R_k(r)$ with $k \to i\tilde{k}$

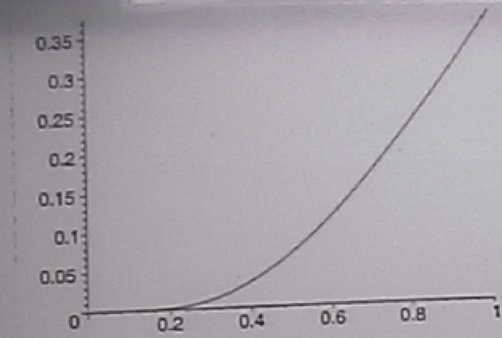
Near Bolt limit

$r \simeq n\pi$: $\tilde{R}_{\tilde{k}}(r) \simeq -Y_0(\tilde{k}(r-n\pi)) = \frac{-2}{\pi}\left\{\ln\frac{\tilde{k}}{2} + \gamma + \ln(r-n\pi)\right\}$

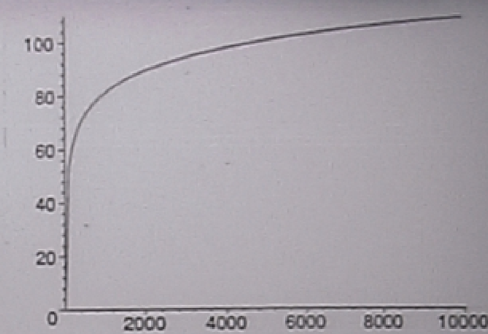
Bessel fun. of the 2nd Kind.

Logarithmic divergence

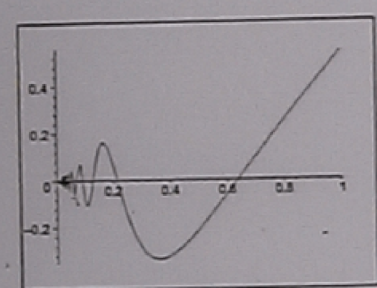
larger $r \to \infty$: $\tilde{R}_{\tilde{k}}(r) \simeq \frac{\cos(\tilde{k}r)}{r}$



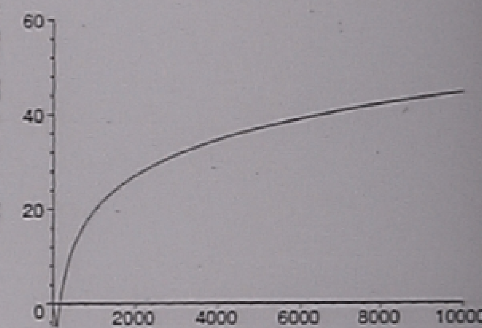
Radial function as a function of $\frac{1}{r}$, near infinity



Radial function as a function of $\frac{1}{r-\pi}$.
For $r \simeq \pi$, R diverges logarithmically as $\ln\left(\frac{1}{r-\pi}\right)$



Second radial function as a function of $\frac{1}{r}$, near infinity



Second radial function as a function of $\frac{1}{r-\pi}$.

M5 - brane Solutions

$$ds_{11}^2 = H^{-1/3}(-dt^2+dx_1^2+dx_2^2+\cdots+dx_5^2) + H^{2/3}(dy^2+ds_{AH}^2)$$

$$F_{\tau\theta\varphi y} = \frac{\alpha}{2}\sin\theta\, r\, a(r)\, c(r)\, \frac{\partial H}{\partial r} = \frac{\alpha}{2}\varepsilon_{\tau\theta\varphi y\mu}\partial^\mu H$$

$$F_{\tau\theta\varphi r} = \frac{-\alpha}{2r}\sin\theta\, a(r)\, b^2(r)\, c(r)\frac{\partial H}{\partial y} = \frac{\alpha}{2}\varepsilon_{\tau\theta\varphi r\mu}\partial^\mu H$$

Sugra eqs : $r^2\frac{\partial^2 H}{\partial r^2} + r\left\{1+r\left(\frac{a'}{a}+\frac{c'}{c}\right)\right\}\frac{\partial H}{\partial r} + b^2\frac{\partial^2 H}{\partial y^2} = 0$

After fixing the measure functions:

$$\boxed{H_{AH}(y,r) = 1+Q_5\int_0^\infty dk\, k^2 \cos(ky) R_k(r)}$$

Second solution

$$\boxed{\tilde{H}_{AH}(y,r) = 1+Q_5\int_0^\infty d\tilde{k}\, \tilde{k}^2 e^{-\tilde{k}y}\, \tilde{R}_{\tilde{k}}(r)}$$

Compactification over τ direction

M2 | M5

M5-brane Solutions

$$ds_{11}^2 = H^{-1/3}(-dt^2 + dx_1^2 + dx_2^2 + \cdots + dx_5^2) + H^{2/3}(dy^2 + ds_{AH}^2)$$

$$F_{\tau\theta\varphi y} = \frac{\alpha}{2}\sin\theta\, r\, a(r)\, c(r)\, \frac{\partial H}{\partial r} = \frac{\alpha}{2}\, \varepsilon_{\tau\theta\varphi y\mu}\, \partial^{\mu} H$$

$$F_{\tau\theta\varphi r} = \frac{-\alpha}{2r}\sin\theta\, a(r)\, b^2(r)\, c(r)\, \frac{\partial H}{\partial y} = \frac{\alpha}{2}\, \varepsilon_{\tau\theta\varphi r\mu}\, \partial^{\mu} H$$

Sugra eqs : $r^2 \frac{\partial^2 H}{\partial r^2} + r\left\{1 + r\left(\frac{a'}{a} + \frac{c'}{c}\right)\right\}\frac{\partial H}{\partial r} + b^2 \frac{\partial^2 H}{\partial y^2} = 0$

After fixing the measure functions:

$$\boxed{H_{AH}(y,r) = 1 + Q_5 \int_0^\infty dk\, k^2 \cos(ky)\, R_k(r)}$$

Second solution

$$\boxed{\tilde{H}_{AH}(y,r) = 1 + Q_5 \int_0^\infty d\tilde{k}\, \tilde{k}^2\, e^{-\tilde{k}y}\, \tilde{R}_{\tilde{k}}(r)}$$

Compactification over τ direction

	M2	M5
large r :	$D2 \perp D6(2)$ system with 8 susy's.	$NS5 \perp D6(5)$ with 8 susy's

M2-brane solutions based on TN_4 and TB_4

$$ds_{11}^2 = H^{-2/3}(-dt^2 + dx_1^2 + dx_2^2) + H^{1/3}(dy^2 + y^2 d\Omega_3^2 + ds_4^2)$$

$$F_{tx_1x_2y} = -1/2H\ \partial H/\partial y \quad , \quad F_{tx_1x_2r} = -1/2H\ \partial H/\partial r$$

$$ds^2_{4(TN)} = \tilde{f}_4(r)(dr^2 + r^2 d\Omega^2) + \frac{16n^2}{\tilde{f}_4(r)}\left(d\psi + \frac{1}{2}\cos\theta\, d\varphi\right)^2$$

$$\tilde{f}_4(r) = 1 + 2n/r$$

$r \geq 0$
$0 \leq \theta \leq \pi$
$0 \leq \varphi, \psi \leq 2\pi$

Sugra eqs : $$r\frac{\partial^2 H}{\partial r^2} + 2\frac{\partial H}{\partial r} + \frac{3(r+2n)}{y}\frac{\partial H}{\partial y} + (r+2n)\frac{\partial^2 H}{\partial y^2} = 0$$

$$H_{TN}(y,r) = 1 + Q_2\int_0^\infty dp\, p^3\, \frac{J_1(py)}{y}\, \Gamma(pn)\, \frac{W_W(-pn, \frac{1}{2}, 2pr)}{r}$$

Whittaker-W function

$$W_W(x_0, y_0, z) = e^{-\frac{1}{2}z} z^{\frac{1}{2}+y_0}\, \mathcal{U}\left(\tfrac{1}{2}+y_0-x_0, 1+2y_0, z\right)$$

Confluent hypergeometric function (Kummer function)

Second Solution

$$\tilde{H} \ldots = 1 + Q \int \ldots \frac{-iW_W(-i\tilde{p}n, \frac{1}{2}, 2i\tilde{p}r)}{\ldots}\, K_1(\tilde{p}y)$$

M2-brane solutions based on TN_4 and TB_4

$$ds_{11}^2 = H^{-2/3}\left(-dt^2 + dx_1^2 + dx_2^2\right) + H^{1/3}\left(dy^2 + y^2 d\Omega_3^2 + ds_4^2\right)$$

$$F_{tx_1x_2y} = -1/2H \,\partial H/\partial y \quad , \quad F_{tx_1x_2r} = -1/2H\,\partial H/\partial r$$

$$ds^2_{4(TN)} = \tilde{f}_4(r)\left(dr^2 + r^2 d\Omega^2\right) + \frac{16n^2}{\tilde{f}_4(r)}\left(d\psi + \frac{1}{2}\cos\theta\, d\varphi\right)^2$$

$r \geqslant 0$

$0 \leqslant \theta \leqslant \pi$

$0 \leqslant \varphi, \psi \leqslant 2\pi$

$$\tilde{f}_4(r) = 1 + 2n/r$$

Sugra eqs : $$r\frac{\partial^2 H}{\partial r^2} + 2\frac{\partial H}{\partial r} + \frac{3(r+2n)}{y}\frac{\partial H}{\partial y} + (r+2n)\frac{\partial^2 H}{\partial y^2} = 0$$

$$H_{TN}(y,r) = 1 + Q_2\int_0^\infty dp\, p^3\, \frac{J_1(py)}{y}\,\Gamma(pn)\,\frac{\underbrace{W_W(-pn, 1/2, 2pr)}_{\text{Whittaker-W function}}}{r}$$

$$W_W(x_0, y_0, z) = e^{-\frac{1}{2}z} z^{\frac{1}{2}+y_0}\,\underbrace{\mathcal{U}\left(\tfrac{1}{2}+y_0-x_0, 1+2y_0, z\right)}_{\substack{\text{Confluent hypergeometric function} \\ \text{(Kummer function)}}}$$

Second Solution

$$\tilde{H}_{TN}(y,r) = 1 + Q_2\int d\tilde{p}\; \frac{-iW_M(-i\tilde{p}n, \frac{1}{2}, 2i\tilde{p}r)}{\ldots}\;\frac{K_1(\tilde{p}y)}{y}$$

M2-brane solutions based on TN_4 and TB_4

$$ds_{11}^2 = H^{-2/3}(-dt^2 + dx_1^2 + dx_2^2) + H^{1/3}(dy^2 + y^2 d\Omega_3^2 + dS_4^2)$$

$$F_{tx_1x_2y} = -1/2H\, \partial H/\partial y \quad , \quad F_{tx_1x_2r} = -1/2H\, \partial H/\partial r$$

$$dS^2_{4(TN)} = \tilde{f}_4(r)(dr^2 + r^2 d\Omega^2) + \frac{16n^2}{\tilde{f}_4(r)}\left(d\psi + \tfrac{1}{2}\cos\theta\, d\varphi\right)^2$$

$$r \geq 0$$
$$0 \leq \theta \leq \pi$$
$$0 \leq \varphi, \psi \leq 2\pi$$

$$\tilde{f}_4(r) = 1 + 2n/r$$

Sugra eqs: $r\dfrac{\partial^2 H}{\partial r^2} + 2\dfrac{\partial H}{\partial r} + \dfrac{3(r+2n)}{y}\dfrac{\partial H}{\partial y} + (r+2n)\dfrac{\partial^2 H}{\partial y^2} = 0$

$$H_{TN}(y,r) = 1 + Q_2 \int_0^\infty dp\, p^3\, \frac{J_1(py)}{y}\, \Gamma(pn)\, \underbrace{\frac{W_W(-pn, \tfrac{1}{2}, 2pr)}{r}}_{\text{Whittaker-W function}}$$

$$W_W(x_0, y_0, z) = e^{-\frac{1}{2}z}\, z^{\frac{1}{2}+y_0}\, \underbrace{U(\tfrac{1}{2}+y_0-x_0, 1+2y_0, z)}_{\text{Confluent hypergeometric function (Kummer function)}}$$

Second Solution

$$\tilde{H}_{TN}(y,r) = 1 + Q_2 \int d\tilde{p}\, \frac{-iW_M(-i\tilde{p}n, \tfrac{1}{2}, 2i\tilde{p}r)}{r}\, \frac{K_1(\tilde{p}y)}{y}$$

M2-brane solutions based on TN_4 and TB_4

$$ds_{11}^2 = H^{-2/3}(-dt^2+dx_1^2+dx_2^2)+H^{1/3}(dy^2+y^2 d\Omega_3^2+ds_4^2)$$

$$F_{tx_1x_2y} = -1/2H\ \partial H/\partial y \quad , \quad F_{tx_1x_2r} = -1/2H\ \partial H/\partial r$$

$$ds^2_{4(TN)} = \tilde{f}_4(r)(dr^2+r^2 d\Omega^2) + \frac{16n^2}{\tilde{f}_4(r)}\left(d\psi+\frac{1}{2}\cos\theta\, d\varphi\right)^2$$

$$\tilde{f}_4(r) = 1+2n/r$$

$r \geq 0$

$0 \leq \theta \leq \pi$

$0 \leq \varphi, \psi \leq 2\pi$

Sugra eqs: $$r\frac{\partial^2 H}{\partial r^2} + 2\frac{\partial H}{\partial r} + \frac{3(r+2n)}{y}\frac{\partial H}{\partial y} + (r+2n)\frac{\partial^2 H}{\partial y^2} = 0$$

$$H_{TN}(y,r) = 1+Q_2\int_0^\infty dp\, p^3\, \frac{J_1(py)}{y}\, \Gamma(pn)\, \frac{W_W(-pn, \frac{1}{2}, 2pr)}{r}$$

Whittaker-W function

$$W_W(x_0, y_0, z) = e^{-\frac{1}{2}z} z^{\frac{1}{2}+y_0}\, U\left(\tfrac{1}{2}+y_0-x_0, 1+2y_0, z\right)$$

Confluent hypergeometric function (Kummer function)

Second Solution

$$\tilde{H}_{TN}(y,r) = 1+Q_2\int d\tilde{p}\ \frac{-iW_M(-i\tilde{p}n, \frac{1}{2}, 2i\tilde{p}r)}{r}\ \frac{K_1(\tilde{p}y)}{y}$$

$$W_M(x_0, y_0, z) = e^{-\frac{1}{2}z} z^{\frac{1}{2}+y_0}\, F\left(\tfrac{1}{2}+y_0-x_0; 1+2y_0; z\right)$$

The measure function in $\tilde{H}_{TN}$ is fixed by Consideration of the small r limit : $r \ll n$

$$ds^2_{TN} \rightarrow dz^2 + z^2 d\Omega_3^2 \quad ; \quad z = 2\sqrt{2nr}$$

So, transverse geometry is $R^4 \otimes R^4$ with the metric

$$dy^2 + dz^2 + y^2 d\Omega_3^2 + z^2 d\Omega_3'^2$$

and so $\tilde{H}_{TN} \rightarrow 1 + \frac{Q_2}{\mathcal{R}^6}$ $\qquad \mathcal{R}^2 = y^2 + z^2$

$$\tilde{H}_{TN} = 1 + Q_2 \int d\tilde{p}\, \tilde{f}(\tilde{p})\, e^{-i\tilde{p}r} \mathcal{F}(1 + i\tilde{p}n, 2, 2i\tilde{p}r) \frac{K_1(\tilde{p}y)}{y}$$

$$\xRightarrow{r \ll n} 1 + Q_2 \int d\tilde{p}\, \tilde{f}(\tilde{p}) \frac{I_1(2i\tilde{p}\sqrt{2nr})}{i\tilde{p}\sqrt{2nr}} \frac{K_1(\tilde{p}y)}{y}$$

$$= 1 + \frac{Q_2}{(y^2 + 8nr)^3}$$

$$\Downarrow$$

$$\tilde{f}(\tilde{p}) = \frac{\tilde{p}^4}{16}$$

$$\Downarrow$$

$$\tilde{H}_{\ldots}(y, r) = 1 + Q \int^{\infty} d\tilde{p}\, \tilde{p}^4 e^{-i\tilde{p}r} \mathcal{F}(1 + i\tilde{p}n, 2, 2i\tilde{p}r) \times$$

The measure function in $\tilde{H}_{TN}$ is fixed by Consideration of the small r limit : $r \ll n$

$$ds^2_{TN} \to dz^2 + z^2 d\Omega_3^2 \quad ; \quad z = 2\sqrt{2nr}$$

So, transverse geometry is $R^4 \otimes R^4$ with the metric

$$dy^2 + dz^2 + y^2 d\Omega_3^2 + z^2 d\Omega_3'^2$$

and so $\tilde{H}_{TN} \to 1 + \frac{Q_2}{\mathcal{R}^6} \qquad \mathcal{R}^2 = y^2 + z^2$

$$\tilde{H}_{TN} = 1 + Q_2 \int d\tilde{p}\, \tilde{f}(\tilde{p}) e^{-i\tilde{p}r} \mathcal{F}(1+i\tilde{p}n, 2, 2i\tilde{p}r) \frac{K_1(\tilde{p}y)}{y}$$

$$\xRightarrow{r \ll n} 1 + Q_2 \int d\tilde{p}\, \tilde{f}(\tilde{p}) \frac{I_1(2i\tilde{p}\sqrt{2nr})}{i\tilde{p}\sqrt{2nr}} \frac{K_1(\tilde{p}y)}{y}$$

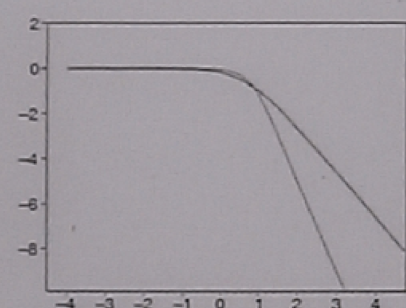
$$= 1 + \frac{Q_2}{(y^2 + 8nr)^3}$$

$$\Downarrow$$

$$\tilde{f}(\tilde{p}) = \frac{\tilde{p}^4}{16}$$

$$\Downarrow$$

$$\tilde{H}_{TN}(y, r) = 1 + Q_2 \int^{\infty} d\tilde{p}\, \tilde{p}^4 e^{-i\tilde{p}r} \mathcal{F}(1+i\tilde{p}n, 2, 2i\tilde{p}r) \times$$



Log-Log plots of

— $h(r) \simeq H_{TN}(y=0, r) - 1$ versus $\frac{r}{n}$

..... $\tilde{h}(y) \simeq \tilde{H}_{TN}(y, r=0) - 1$ versus $\frac{y}{n}$

Normalization is chosen such that
$h, \tilde{h} \to 1$ at $r, y \to 0$

KK reduction of the 11D sugra to 10D

$$ds^2_{(1,10)} = e^{-2\Phi/3} ds^2_{(1,9)} + e^{4\Phi/3}(dx_{10} + C_\alpha dx^\alpha)^2$$

$$F_{[4]} = \mathcal{F}_{[4]} + H_{[3]} \wedge dx_{10}$$

$\mathcal{F}_{[4]}$: RR 4-form field strength

$H_{[3]}$: NSNS 3-form field strength

Conventions

$$de^a = g^a_{bc} e^b \wedge e^c$$

$$\omega^a_{bc} = \frac{1}{2}(g^a_{bc} + g^b_{ca} - g^c_{ab})$$

$$\omega_{dbM} = \omega^a_{bc} \eta_{ad} e^c_M$$

$$e^a = e^a_M dx^M \qquad g_{MN} = \eta_{ab} e^a e^b$$

$$\Gamma^{A_1 \cdots A_p} = \Gamma^{[A_1} \cdots \Gamma^{A_p]}$$

$$\{\Gamma^a, \Gamma^b\} = -2\eta^{ab}$$

Rep of clifford alg.: $\Gamma_{\hat{i}} = \gamma_{\hat{i}} \otimes \mathbb{1}_8$ $\quad \hat{i} = 0,1,2,3 \; : \; SO(1,3)$

Dimensional reduction along the Coordinate ψ of the Taub-NUT

$$ds_{10}^2 = H^{-1/2}\tilde{f}_4^{-1/2}(-dt^2+dx_1^2+dx_2^2) + H^{1/2}\tilde{f}_4^{-1/2}(dy^2+y^2 d\Omega_3^2)$$
$$+H^{1/2}\tilde{f}_4^{1/2}(dr^2+r^2 d\Omega_2^2)$$

$$\Phi = \frac{3}{4}\ln(H^{1/3}/\tilde{f}_4) \quad B_{\mu\nu}=0$$
$$C_\phi = 2n\cos\theta \quad A_{tx_1x_2} = H^{-1}$$

D2 ⊥ D6 (2)

preservation of SUSY

Since we consider only the bosonic sector of 11D sugra;

$$\underset{\text{gravitino}}{\delta_\varepsilon \psi_a}\Big|_{\text{fermion}=0} = 0 \qquad \psi_a = e_a^M \psi_M$$

$$\Rightarrow \partial_M \varepsilon + \frac{1}{4}\omega_{Mab}\Gamma^{ab}\varepsilon + \frac{1}{144}\Gamma_M^{NPQR}F_{NPQR}\varepsilon - \frac{1}{18}\Gamma^{PQR} \times$$
$$F_{MPQR}\,\varepsilon = 0$$

Dimensional reduction along the Coordinate ψ of the Taub-NUT

$$ds^2_{10} = H^{-1/2}\tilde{f}_4^{-1/2}(-dt^2+dx_1^2+dx_2^2) + H^{1/2}\tilde{f}_4^{-1/2}(dy^2+y^2 d\Omega_3^2) + H^{1/2}\tilde{f}_4^{1/2}(dr^2+r^2 d\Omega_2^2)$$

$$\Phi = \frac{3}{4}\ln(H^{1/3}/\tilde{f}_4) \quad B_{\mu\nu}=0$$

$$C_\phi = 2n\cos\theta \quad A_{tx_1x_2} = H^{-1}$$

D2 ⊥ D6 (2)

preservation of SUSY

Since we consider only the bosonic sector of 11D sugra;

$$\delta_\varepsilon \psi_a \Big|_{\text{fermion}=0} = 0 \qquad \psi_a = e_a^M \psi_M$$

graritino

$$\Rightarrow \underbrace{\partial_M}_{\text{d=11 world indices}} \varepsilon + \frac{1}{4}\underbrace{\omega_{Mab}}_{\text{tangent space indices}}\Gamma^{ab}\varepsilon + \frac{1}{144}\Gamma_M{}^{NPQR}F_{NPQR}\varepsilon - \frac{1}{18}\Gamma^{PQR} \times F_{MPQR}\varepsilon = 0$$

M2-brane based on Taub-Bolt

$$ds^2_{TB} = f_B(r)\,dr^2 + \frac{1}{f_B(r)}\left(d\psi + 2n\cos\theta\, d\varphi\right)^2 + (r^2 - n^2)\,d\Omega^2$$

$$f_B(r) = \frac{2(r^2-n^2)}{(r-2n)(2r-n)} \quad ; \quad r \geq 2n \; ; \; 0 \leq \psi \leq 8\pi n$$

Difference between TB and TN

general metric function of the TN

$$f(r) = \frac{r^2 - n^2}{r^2 + n^2 - 2mr}$$

Fixed point set of the Killing vector $\partial/\partial\psi$ is :

$$\begin{cases} \text{zero-dim. if } m = n \Rightarrow \left.\frac{1}{f}\right|_{r=n} = 0 \quad ; \text{ NUT} \\ \text{two-dim. if } m = \frac{5}{4}n \Rightarrow \left.\frac{1}{f}\right|_{r=2n>n} = 0 \quad ; \text{ Bolt} \end{cases}$$

Equivalent form of TB

M2-brane based on Taub-Bolt

$$ds^2_{TB} = f_B(r)\,dr^2 + \frac{1}{f_B(r)}\left(d\psi + 2n\cos\theta\, d\varphi\right)^2 + (r^2 - n^2)\,d\Omega^2$$

$$f_B(r) = \frac{2(r^2-n^2)}{(r-2n)(2r-n)} \quad ; \quad r \geqslant 2n \quad ; \quad 0 \leqslant \psi \leqslant 8\pi n$$

Difference between TB and TN

general metric function of the TN

$$f(r) = \frac{r^2-n^2}{r^2+n^2-2mr}$$

Fixed point set of the Killing vector $\partial/\partial\psi$ is :

$$\begin{cases} \text{zero-dim. if } m=n \Rightarrow \left.\frac{1}{f}\right|_{r=n} = 0 \quad ; \text{ NUT} \\ \text{two-dim. if } m=\frac{5}{4}n \Rightarrow \left.\frac{1}{f}\right|_{r=2n>n} = 0 \quad ; \text{ Bolt} \end{cases}$$

Equivalent form of TB

$$ds^2_{TB} = \tilde{f}_B(r)\,dr^2 + 16n^2/\tilde{f}_B(r)\left(d\Psi + \frac{1}{2}\cos\theta\, d\varphi\right)^2 + r(r+2n)\,d\Omega^2$$

$$H_{TB}(y,r) = 1 + Q_2 \int_0^\infty \frac{J_1(cy)}{y} R_c(r)\, c^4\, dc$$

$$(2r^2 - n^2 - rn) R_c'' + (4r - n) R_c' - 2rc^2 (r + 2n) R_c = 0$$

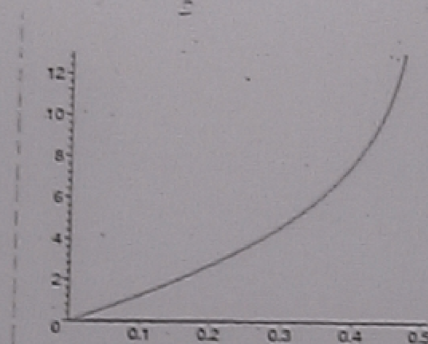
* Both radial functions for TN and TB diverge at the brane location and vanish at ∞.

<u>Other solution</u>

$$\tilde{H}_{TB}(y,r) = 1 + Q_2 \int_0^\infty \frac{K_1(cy)}{y} \tilde{R}_c(r)\, c^4\, dc$$

$$H_{TB}(y,r) = 1 + Q_2 \int_0^\infty \frac{J_1(cy)}{y} R_c(r)\, c^4\, dc$$

$$(2r^2 - n^2 - rn) R_c'' + (4r - n) R_c' - 2rc^2 (r + 2n) R_c = 0$$



Radial function versus $n/2r$ for TB case. Bolt is at $r=n$

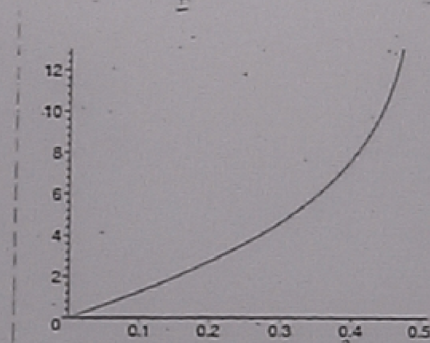
* Both radial functions for TN and TB diverge at the brane location and vanish at ∞.

Other solution

$$\tilde{H}_{TB}(y,r) = 1 + Q_2 \int_0^\infty \frac{K_1(cy)}{y} \tilde{R}_c(r)\, c^4\, dc$$

$$H_{TB}(y,r) = 1 + Q_2 \int_0^\infty \frac{J_1(cy)}{y} R_c(r)\, c^4\, dc$$

$$(2r^2 - n^2 - rn) R_c'' + (4r - n) R_c' - 2rc^2 (r + 2n) R_c = 0$$

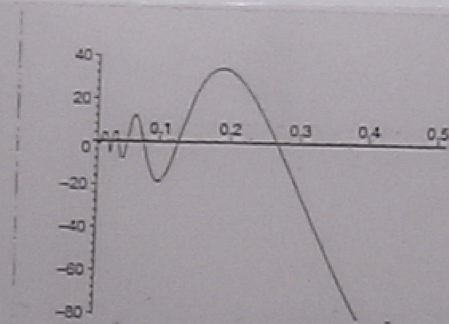


Radial function versus $n/2r$ for TB Case .. Bolt is at $r=n$

* Both radial functions for TN and TB diverge at the brane location and vanish at ∞.

Other solution

$$\tilde{H}_{TB}(y,r) = 1 + Q_2 \int_0^\infty \frac{K_1(cy)}{y} \tilde{R}_c(r)\, c^4\, dc$$



Damped Oscillating second radial function in TB as a function of $n/2r$ • Bolt: $r=n$

After reduction to 10D

$D2 \perp D6(2)$

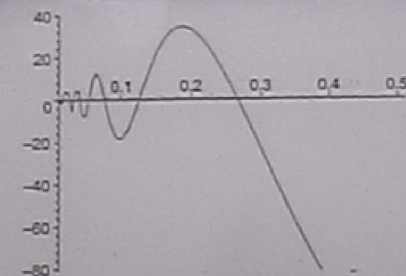
No susy is preserved

$$ds_{10}^2 = \frac{H^{-1/2}}{\sqrt{F_{Bolt}}}\left(-dt^2+dx_1^2+dx_2^2\right) + \frac{H^{1/2}}{\sqrt{F_{Bolt}}}\left(dy^2+y^2 d\Omega_3^2\right)$$

$$+ H^{1/2}\sqrt{F_{Bolt}}\left(dr'^2 + r'\left(r'+\frac{3}{2}n\right)d\Omega^2\right)$$

$\Phi = \frac{3}{4}\ln\left(\frac{H^{1/3}}{F_{Bolt}}\right)$ $B_{\mu\nu}=0$

distortion with



Damped Oscillating second radial function in TB as a function of $n/2r$. Bolt: $r=n$

After reduction to 10D

$D2 \perp D6(2)$

No susy is preserved

$$ds_{10}^2 = \frac{H^{-1/2}}{\sqrt{F_{Bolt}}}\left(-dt^2+dx_1^2+dx_2^2\right) + \frac{H^{1/2}}{\sqrt{F_{Bolt}}}\left(dy^2+y^2 d\Omega_3^2\right)$$

$$+ H^{1/2}\sqrt{F_{Bolt}}\left(dr'^2 + r'\left(r'+\frac{3}{2}n\right)d\Omega^2\right)$$

distortion with respect to TN Case in normal space to D2/D6 system

$$\Phi = \frac{3}{4}\ln\left(\frac{H^{1/3}}{F_{Bolt}}\right) \quad B_{\mu\nu}=0$$

$$C_\phi = 2n\cos\theta \quad A_{t x_1 x_2} = 1/H$$

* properties of solution are qualitatively similar to TN Case; but no susy is preserved. (TB is not self-dual)

Embedding EH in M5

$$ds_{11}^2 = H^{-1/3}(-dt^2 + dx_1^2 + \cdots + dx_5^2) + H^{2/3}(dy^2 + ds_{EH}^2)$$

$$F_{m_1 \cdots m_4} = \frac{\alpha}{2}\varepsilon_{m_1 \cdots m_5}\partial^{m_5} H \Rightarrow F_{t\theta\varphi y} = \frac{\sin\theta(r^4 - a^4)}{16 r}\frac{\partial H}{\partial r}$$

$$F_{t\psi\varphi r} = -\frac{r^3 \sin\theta}{16}\frac{\partial H}{\partial y}$$

$$ds_{EH}^2 = \frac{r^2}{4g(r)}\{d\psi + \cos\theta\, d\varphi\}^2 + g(r)dr^2 + \frac{r^2}{4}d\Omega^2$$

$$g^{-1}(r) = 1 - \frac{a^4}{r^4}$$

$\left\{ \begin{array}{l} \text{asymptotically flat} \\ \text{with self-dual} \\ \text{curvature} \end{array} \right.$ $\left\{ \begin{array}{l} \text{Near } r=a \Rightarrow R^2 \otimes S^2 \\ ds^2_{r\sim a} \simeq dz^2 + z^2 d\psi^2 + \frac{a^2}{4}d\Omega^2 \end{array} \right.$

$$H_{EH}(y,r) = 1 + Q_5\int_0^\infty dc\{g_1(c)\cos(cy) + g_2(c)\sin(cy)\}R_c(r)$$

where

$$r(r^4 - a^4)R_c'' + (3r^4 + a^4)R_c' - c^2 r^5 R_c = 0$$

To fix $g_1(c)$ and $g_2(c)$;

Embedding EH in M5

$$ds_{11}^2 = H^{-1/3}(-dt^2 + dx_1^2 + \cdots + dx_5^2) + H^{2/3}(dy^2 + ds_{EH}^2)$$

$$F_{m_1 \cdots m_4} = \frac{\alpha}{2}\, \varepsilon_{m_1 \cdots m_5} \partial^{m_5} H \Rightarrow F_{\psi\theta\varphi y} = \frac{\sin\theta\,(r^4 - a^4)}{16 r} \frac{\partial H}{\partial r}$$

$$F_{\psi\theta\varphi r} = -\frac{r^3 \sin\theta}{16} \frac{\partial H}{\partial y}$$

$$ds_{EH}^2 = \frac{r^2}{4 g(r)}\{d\psi + \cos\theta\, d\varphi\}^2 + g(r)\, dr^2 + \frac{r^2}{4} d\Omega^2$$

$$g^{-1}(r) = 1 - \frac{a^4}{r^4}$$

asymptotically flat
with self-dual
curvature

Near $r = a \Rightarrow R^2 \otimes S^2$

$$ds_{r\sim a}^2 \simeq dz^2 + z^2 d\psi^2 + \frac{a^2}{4} d\Omega^2$$

$$H_{EH}(y, r) = 1 + Q_5 \int_0^\infty dc \{ g_1(c) \cos(cy) + g_2(c) \sin(cy) \} R_c(r)$$

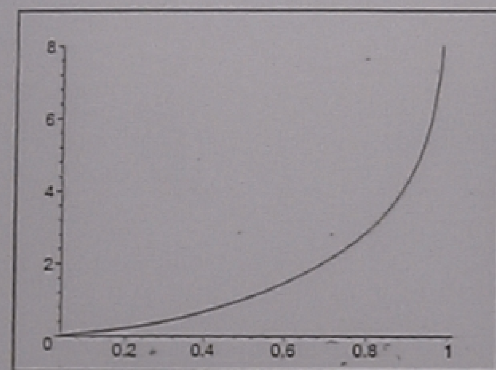
where

$$r(r^4 - a^4) R_c'' + (3r^4 + a^4) R_c' - c^2 r^5 R_c = 0$$

To fix $g_1(c)$ and $g_2(c)$;

So $H_{EH} \xrightarrow{r \sim a} 1 + \frac{Q_5}{R^3} \Rightarrow$

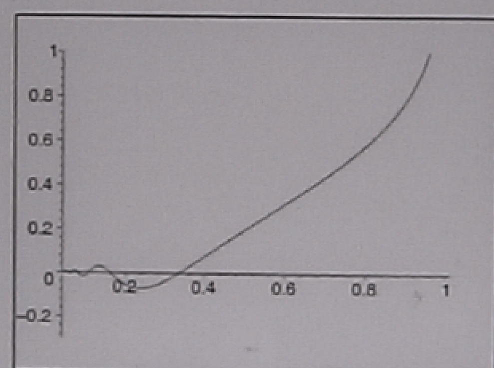
$$H_{EH}(y,r) = 1 + Q_5 \int_0^\infty dc\, c^2 \cos(cy) R_c(r)$$



Radial function in EH Case
as a function of a/r

Second Solution

$$\tilde{H}_{EH}(y,r) = 1 + Q_5 \int_0^\infty d\tilde{c}\, \tilde{c}^2 e^{-\tilde{c}y} \tilde{R}_{\tilde{c}}(r)$$



Reduction along ψ Coord:

$$ds_{10}^2 = \frac{\omega}{2}\left\{\frac{1}{\sqrt{g}}\left(-dt^2+dx_1^2+\dots+dx_5^2\right) + H g^{-1/2} dy^2 + H g^{1/2} a^2\left(d\omega^2 + \frac{\omega^2}{4g} d\Omega_2^2\right)\right\}$$

asymptotically flat

NS5 $\perp$ D6(5) system $\quad \omega = \frac{r}{a}$

$\to$ 8 susy's

$$\text{NS NS}\begin{cases} \Phi = \frac{3}{4}\ln\left\{\frac{\omega^2 H^{2/3}}{4g}\right\} \\ H_{[3]} = \frac{F_{\theta\varphi y\psi}}{a}\, d\theta\wedge d\varphi\wedge dy + \frac{F_{\theta\varphi r\psi}}{a}\, d\theta\wedge d\varphi\wedge dr \end{cases}$$

$$\text{RR}\begin{cases} C_{[1]} = a\cos\theta\, d\varphi \\ \mathcal{A}_{[3]} = 0 \end{cases}$$

T-duality along x_1

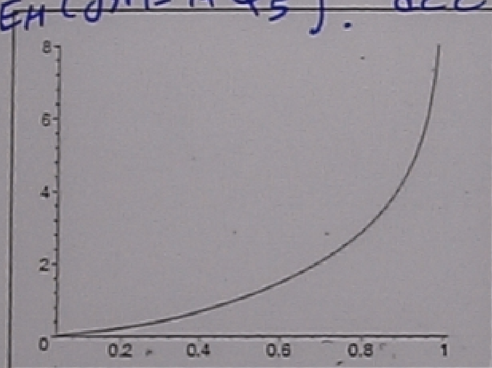
$$\text{NS NS}\left\{ \tilde{\Phi} = \frac{1}{2}\ln\left(\frac{\omega^2 H}{4g}\right) \quad \tilde{B}_{r\phi} = -\frac{r^3\cos\theta}{8a} H_y \quad \tilde{B}_{y\phi} = \frac{(r^4-a^4)\cos\theta\, H_r}{8ar} \right.$$

$$\tilde{H}_{[3]} = d\tilde{B}_{[2]}$$

$$\text{RR}\left\{ \left(\tilde{F}_{[2]}\right)_{\phi x_1} = a\cos\theta \right.$$

So $H_{EH} \xrightarrow{r\sim a} 1+\frac{Q_5}{R^3} \Rightarrow$

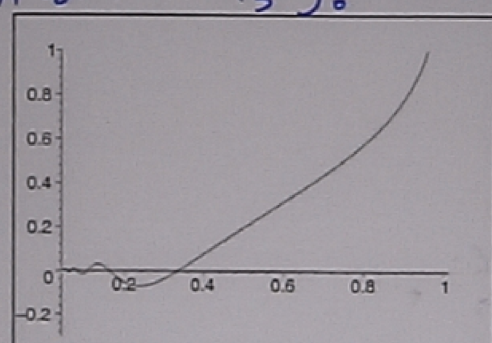
$$H_{EH}(y,r) = 1 + Q_5 \int_0^\infty dc\, c^2 \cos(cy)\, R_c(r)$$



Radial function in EH Case
as a function of a/r

Second Solution

$$\tilde{H}_{EH}(y,r) = 1 + Q_5 \int_0^\infty d\tilde{c}\, \tilde{c}^2 e^{-\tilde{c}y}\, \tilde{R}_{\tilde{c}}(r)$$



M2-brane solution with AH and TN

$$ds_{11}^2 = H^{-1/3}\left(-dt^2 + dx_1^2 + dx_2^2\right) + H^{2/3}\left(ds_{TN}^2 + ds_{AH}^2\right)$$

$$ds_{TN}^2 = f(y)\left(dy^2 + y^2\left(d\alpha^2 + \sin^2\alpha\, d\beta^2\right)\right) + \frac{16n^2}{f(y)}\left(d\sigma + \frac{1}{2}\cos\alpha\, d\beta\right)^2$$

$$f(y) = 1 + 2n/y$$

$$H_{TN\otimes AH}(y,r) = 1 - iQ_2 \int_0^\infty dk\, \frac{W_M\left(-ikn, \frac{1}{2}, 2iky\right)}{y}\, k^4 R_k(r)$$

Whittaker function

Compactification over the circle σ

$$ds_{10}^2 = H_{TN\otimes AH}^{-1/2} f^{-1/2}\left(-dt^2 + dx_1^2 + dx_2^2\right) +$$
$$+ H_{TN\otimes AH}^{+1/2} f^{-1/2} ds_{AH}^2 +$$
$$+ H_{TN\otimes AH}^{1/2} f^{1/2}\left(dy^2 + y^2 d\Omega^2\right)$$

NSNS fields $\Phi = \frac{3}{4}\ln\left(\frac{H^{1/3}}{f}\right)$, $B_{\mu\nu} = 0$

M2-brane solution with AH and TN

$$ds_{11}^2 = H^{-1/3}(-dt^2 + dx_1^2 + dx_2^2) + H^{2/3}(ds_{TN}^2 + ds_{AH}^2)$$

$$ds_{TN}^2 = f(y)\left(dy^2 + y^2(d\alpha^2 + \sin^2\alpha\, d\beta^2)\right) + \frac{16n^2}{f(y)}\left(d\sigma + \frac{1}{2}\cos\alpha\, d\beta\right)^2$$

$$f(y) = 1 + 2n/y$$

$$H_{TN\otimes AH}(y,r) = 1 - iQ_2 \int_0^\infty dk\, \frac{W_M(-ikn, \frac{1}{2}, 2iky)}{y} k^4 R_k(r)$$

Whittaker function

Compactification over the circle σ

$$ds_{10}^2 = H_{TN\otimes AH}^{-1/2} f^{-1/2}(-dt^2 + dx_1^2 + dx_2^2) +$$
$$+ H_{TN\otimes AH}^{+1/2} f^{-1/2} ds_{AH}^2 +$$
$$+ H_{TN\otimes AH}^{1/2} f^{1/2}(dy^2 + y^2 d\Omega^2)$$

NSNS fields $\Phi = \frac{3}{4}\ln\left(\frac{H^{1/3}}{f}\right)$, $B_{\mu\nu} = 0$

Summary

M2 susy	M2 non-susy	M5 susy	M5 non-susy
TN	TB	TN	TB
EH	D2⊥D6(2)	EH	NS5⊥D6(5)
AH	$\begin{cases} TN_6 \\ TB_6 \end{cases}$	AH	
TN⊗TN	D2⊥D4(2)	NS5⊥ D6(5)	
TN⊗EH	$\begin{cases} TN_8 \\ TB_8 \end{cases}$	$\updownarrow T$	
TN⊗AH		NS5⊥ D5(4)	
EH⊗EH	D(2)		
EH⊗AH			
AH⊗AH			
$\underbrace{}_{D2\perp D6(2)}$			

- new M-theory solutions as realization of fully localized D/D and NS/D systems.
- holographic dual of the theories on D2 and NS5 branes
- New LST's on the worldvolumes of NS5 branes
- Similar behaviour of M-brane metric functions H of non-susy Cases with that of susy cases.

Decoupling limits of D/D systems

$$g_{YM2} = \sqrt{\frac{g_s}{\ell_s}}$$

$$g_{YM6} = (2\pi)^2 \ell_s \sqrt{g_s \ell_s} = (2\pi)^2 \ell_s^2 \, g_{YM2}$$

$\ell_s \to 0$ with $g_{YM2} = \text{fixed} \Rightarrow g_{YM6} \to 0$

Dynamics on D6 decouples.

Then by rescaling the coordinates r, y, NUT charge, Eguchi–Hanson parameter a and integration parameter c, and relations $R_\infty = g_s \ell_s$, $Q_2 = 32\pi^2 N_2 \ell_p^6 = 32\pi^2 N_2 g_{YM2}^2 \ell_s^8$

$$\Rightarrow H(y,r) \to H(Y,U) = \frac{1}{\ell_s^4} h(Y,U)$$

No dependence on ℓ_s

$$h(Y,U) = \pi^2 g_{YM2}^4 N_2 \int dp \, \frac{p^4}{Y} J_1(pY) \, \Gamma\!\left(\frac{g_{YM}^2 N_6 p}{4}\right) e^{-pU} \, \mathcal{U}\!\left(1 + \frac{g_{YM2}^2 N_6 p}{4}, 2, 2pU\right)$$

$$\Rightarrow ds_{10}^2 = \ell_s^2 \left\{ h^{-1/2} f^{-1/2} (-dt^2 + dx_1^2 + dx_2^2) + h^{1/2} f^{-1/2} (dY^2 + Y^2 d\Omega_3^2) + h^{1/2} f^{1/2} (dU^2 + U^2 d\Omega_2^2) \right\}$$

10D metric of D2/D6 system

$$g_{YM2} = \sqrt{\frac{g_s}{\ell_s}}$$

$$g_{YM6} = (2\pi)^2 \ell_s^2 \sqrt{g_s \ell_s} = (2\pi)^2 \ell_s^2 g_{YM2}$$

$\ell_s \to 0$ with $g_{YM2} = \text{fixed} \Rightarrow g_{YM6} \to 0$

Dynamics on D6 decouples.

Then by rescaling the coordinates r, y, NUT charge, Eguchi–Hanson parameter a and integration parameter c, and relations $R_\infty = g_s \ell_s$, $Q_2 = 32\pi^2 N_2 \ell_p^6 = 32\pi^2 N_2 g_{YM2}^2 \ell_s^8$

$$\Rightarrow \quad H(y,r) \to H(Y,U) = \frac{1}{\ell_s^4} h(Y,U)$$

No dependence on ℓ_s

$$h(Y,U) = \pi^2 g_{YM2}^4 N_2 \int dp \frac{p^4}{Y} J_1(pY) \Gamma\left(\frac{g_{YM}^2 N_6 p}{4}\right) e^{-pU} \mathcal{U}\left(1 + \frac{g_{YM2}^2 N_6 p}{4}, 2, 2pU\right)$$

$$\to ds_{10}^2 = \ell_s^2 \Big\{ h^{-1/2} f^{-1/2} (-dt^2 + dx_1^2 + dx_2^2) + h^{1/2} f^{-1/2} (dY^2 + Y^2 d\Omega_3^2) + h^{1/2} f^{1/2} (dU^2 + U^2 d\Omega_2^2) \Big\}$$

10D metric of D2/D6 system obtained for TN M2-brane

sugra solution for decoupled D2/D6

Then by rescaling the ...

Eguchi–Hanson parameter a and integration parameter c,

and relations $R_\infty = g_s l_s$, $Q_2 = 32\pi^2 N_2 l_p^6 = 32\pi^2 N_2 g_{YM2}^2 l_s^8$

$$\Rightarrow \quad H(y,r) \rightarrow H(Y,U) = \frac{1}{l_s^4}\, h(Y,U)$$

No dependence on l_s

$$h(Y,U) = \pi^2 g_{YM2}^4 N_2 \int dp\, \frac{p^4}{Y} J_1(pY)\, \Gamma\!\left(\frac{g_{YM}^2 N_6 p}{4}\right) e^{-pU}\, \mathcal{U}\!\left(1+\frac{g_{YM2}^2 N_6 p}{4}, 2, 2pU\right)$$

$$\rightarrow \quad ds_{10}^2 = l_s^2 \Big\{ h^{-1/2} f^{-1/2}(-dt^2 + dx_1^2 + dx_2^2) + h^{1/2} f^{-1/2}(dY^2 + Y^2 d\Omega_3^2) + h^{1/2} f^{1/2}(dU^2 + U^2 d\Omega_2^2) \Big\}$$

10D metric of D2/D6 system obtained for TN M2-brane

sugra solution for decoupled D2/D6

So, we have an explicit rep. of dual $\overbrace{\text{theory}}^{\text{sugra}}$ ~~to~~

Superrenormalizable super YM in 2+1 dimension

(Gauge group $SU(N_2)$)

For TB_4

$$h_{TB}(Y,U) = 32\pi^2 N_2\, g_{YM2}^2 \int dp\, p^4 \frac{J_1(pY)}{Y} R_c(U)$$

$$\ldots \sqrt{h_{TB}}\,(dY^2 + Y^2 d\Omega_3^2)$$

Then by rescaling the [illegible]
Eguchi–Hanson parameter a and integration parameter c,
and relations $R_\infty = g_s l_s$, $Q_2 = 32\pi^2 N_2 l_p^6 = 32\pi^2 N_2 g_{YM2}^2 l_s^8$

$$\Rightarrow \quad H(y,r) \rightarrow H(Y,U) = \frac{1}{l_s^4} h(Y,U)$$

No dependence on l_s

$$h(Y,U) = \pi^2 g_{YM2}^4 N_2 \int dp \frac{p^4}{Y} J_1(pY) \Gamma\left(\frac{g_{YM}^2 N_6 p}{4}\right) e^{-pU} \mathcal{U}\left(1 + \frac{g_{YM2}^2 N_6 p}{4}, 2, 2pU\right)$$

$$\rightarrow ds_{10}^2 = l_s^2 \left\{ h^{-1/2} f^{-1/2}(-dt^2 + dx_1^2 + dx_2^2) + h^{1/2} f^{-1/2}(dY^2 + Y^2 d\Omega_3^2) + h^{1/2} f^{1/2}(dU^2 + U^2 d\Omega_2^2) \right\}$$

10D metric of D2/D6 system obtained for TN M2-brane

sugra solution for decoupled D2/D6

So, we have an explicit rep. of dual sugra theory ~~of~~ to
Superrenormalizable super YM in 2+1 dimension
(Gauge group $SU(N_2)$)

For TB4

$$h_{TB}(Y,U) = 32\pi^2 N_2 g_{YM2}^2 \int dp\, p^4 \frac{J_1(pY)}{Y} R_c(U)$$

$$\ldots (\ldots dx^2 \ldots dx^2) + \sqrt{h_{TB}}\,(dY^2 + Y^2 d\Omega_3^2)$$

So, we have an explicit rep. of dual $\overset{\text{sugra}}{\vee}$ theory ~~to~~ of

Superrenormalizable super YM in 2+1 dimension

(Gauge group $SU(N_2)$)

For TB4

$$h_{TB}(Y,U) = 32\pi^2 N_2 g_{YM2}^2 \int dp\, p^4 \frac{J_1(pY)}{Y} R_c(U)$$

$$ds^2 = l_s^2 \left\{ \frac{1}{\sqrt{F_4(U) h_{TB}}}(-dt^2 + dx_1^2 + dx_2^2) + \frac{\sqrt{h_{TB}}}{\sqrt{F_4}}(dY^2 + Y^2 d\Omega_3^2) + \sqrt{F_4 h_{TB}}\left(dU^2 + \left(U^2 + \frac{3}{8} g_{YM2}^2 U\right) d\Omega_2^2\right)\right\}$$

and for TN6/TB6

$$ds_{10}^2 = l_s^2 \left\{ \frac{1}{\sqrt{h_6 g_6(U)}}(-dt^2 + dx_1^2 + dx_2^2) + \sqrt{\frac{h_6}{g_6}}(dY^2 + Y^2 d\alpha^2) + \sqrt{h_6 g_6}\left(dU^2 + \frac{U^2(\sqrt{3}U \pm 2g_{YM}^2)}{3(\sqrt{3}U \pm g_{YM}^2)}(d\Omega^2 + d\Omega'^2)\right)\right\}$$

So, we have an explicit rep. of dual $\overset{\text{sugra}}{\vee}$ theory ~~to~~ of

Superrenormalizable super YM in 2+1 dimension

(Gauge group $SU(N_2)$)

For TB4

$$h_{TB}(Y,U) = 32\pi^2 N_2\, g_{YM2}^2 \int dp\, p^4 \frac{J_1(pY)}{Y} R_c(U)$$

$$ds^2 = l_s^2 \left\{ \frac{1}{\sqrt{F_4(U) h_{TB}}}\left(-dt^2 + dx_1^2 + dx_2^2\right) + \frac{\sqrt{h_{TB}}}{\sqrt{F_4}}\left(dY^2 + Y^2 d\Omega_3^2\right) + \sqrt{F_4 h_{TB}}\left(dU^2 + \left(U^2 + \frac{3}{8} g_{YM2}^2 U\right) d\Omega_2^2\right)\right\}$$

and for TN6/TB6

$$ds_{10}^2 = l_s^2 \left\{ \frac{1}{\sqrt{h_6\, g_6(U)}}\left(-dt^2 + dx_1^2 + dx_2^2\right) + \sqrt{\frac{h_6}{g_6}}\left(dY^2 + Y^2 d\alpha^2\right) + \sqrt{h_6\, g_6}\left(dU^2 + \frac{U^2(\sqrt{3}U \pm 2g_{YM}^2)}{3(\sqrt{3}U \pm g_{YM}^2)}(d\Omega^2 + d\Omega'^2)\right)\right\}$$

Decoupling limits of NS/D systems

At low energies, the dynamics of IIA NS5 decouple from the bulk.

In the limit of $g_s \to 0$ and $\ell_s =$ fixed (near NS5 horizon)

we can rescale $r, y \to \frac{r}{g_s \ell_s^2} = U$, $\frac{y}{g_s \ell_s^2} = Y$

$\Longrightarrow$ All Harmonic functions for NS5 brane rescale as

$$H(r,y) \to H(U,Y) = g_s^{-2} h(U,Y)$$

Ex.

$$H_{TN_4}(U,Y) = \frac{1}{g_s^2}\left\{ \frac{\pi N_5}{\ell_s^3} \int dp\, p^2\, \Gamma\!\left(\frac{p}{4\ell_s}\right) \cos(pY)\, e^{-pU} \times \mathcal{U}\!\left(1+\frac{p}{4\ell_s}, 2, 2pU\right) \right\}$$

[After rescaling integration variable and using $Q_5 = \pi N_5 g_s \ell_s^3 = \pi N_5 \ell_p^3$]

$$ds^2_{10,IIA} = f^{-1/2}\left(-dt^2 + dx_1^2 + \cdots + dx_5^2\right) + \ell_s^4 \left\{ h f^{-1/2} dY^2 + h f^{1/2}\left(dU^2 + U^2 d\Omega^2\right) \right\}$$

↳ sugra dual to decoupled free theory on NS5

Decoupling limits of NS/D systems

At low energies, the dynamics of IIA NS5 decouple from the bulk.

In the limit of $g_s \to 0$ and $\ell_s = $ fixed (near NS5 horizon)
we can rescale $r, y \to \frac{r}{g_s \ell_s^2} = U, \ \frac{y}{g_s \ell_s^2} = Y$

$\Rightarrow$ All Harmonic functions for NS5 brane rescale as

$$H(r,y) \to H(U,Y) = g_s^{-2} h(U,Y)$$

Ex.

$$H_{TN_4}(U,Y) = \frac{1}{g_s^2}\left\{ \frac{\pi N_5}{\ell_s^3} \int dp\, p^2\, \Gamma\left(\frac{p}{4\ell_s}\right) \cos(pY)\, e^{-pU} \times \mathcal{U}\left(1+\frac{p}{4\ell_s}, 2, 2pU\right) \right\}$$

[After rescaling integration variable and using $Q_5 = \pi N_5 g_s \ell_s^3 = \pi N_5 \ell_p^3$]

$$ds^2_{10,IIA} = f^{-1/2}(-dt^2 + dx_1^2 + \cdots + dx_5^2) + \ell_s^4 \left\{ h f^{-1/2} dY^2 + h f^{1/2} (dU^2 + U^2 d\Omega^2) \right\},$$

$\hookrightarrow$ sugra dual to decoupled free theory on NS5 (LST_{IIA}) in low energy.

For IIB : at low energies, the dynamics of IIB NS5 decouples from the bulk.

After rescaling the radial coordinates $\Longrightarrow$

$$H(u,Y) = \frac{1}{g_s^2}\tilde{h}(u,Y)$$

$$ds^2_{10,IIB} = \tilde{f}^{-1/2}\left(-dt^2 + \tilde{f}\,dx_1^2 + dx_2^2 + \cdots + dx_5^2\right) + g^4_{YM5}\left(\tilde{f}^{-1/2}dY^2 + \tilde{f}^{1/2}(du^2 + u^2 d\Omega_2^2)\right)$$

Sugra dual of SYM with $g_{YM} = l_s$

low energy limit of LST_{IIB}

We found our solutions preserves 1/4 of the SUSY; in good agreement with the number of susy for LST's.

LST_{IIA} : $(2,0)$ susy $\xrightarrow[\text{of D brane}]{\text{in the presence}}$ $(1,0)$ susy $\equiv 8$ super symmetries

16 susy's

For IIB : at low energies, the dynamics of IIB NS5 decouples from the bulk.

After rescaling the radial coordinates $\Longrightarrow$

$$H(u,Y) = \frac{1}{g_s^2}\tilde{h}(u,Y)$$

$$ds^2_{10,IIB} = \tilde{f}^{-1/2}\left(-dt^2 + \tilde{f}\,dx_1^2 + dx_2^2 + \cdots + dx_5^2\right) + g^4_{YM5}\left(\tilde{f}^{-1/2}dY^2 + \tilde{f}^{1/2}(du^2 + u^2 d\Omega_2^2)\right)$$

Sugra dual of SYM with $g_{YM} = l_s$

low energy limit of LST_{IIB}

We found our solutions preserves 1/4 of the susy; in good agreement with the number of susy for LST's.

LST_{IIA} : $(2,0)$ susy $\xrightarrow[\text{presence of D brane}]{\text{in the}}$ $(1,0)$ susy $\equiv$ 8 super symmetries

16 susy's

LST_{IIB} : $(1,1)$ susy $\longrightarrow$ 8 susy's.

At low energies, the dynamics of IIA NS5 decouple from the bulk.

In the limit of $g_s \to 0$ and ℓ_s = fixed (near NS5 horizon)

we can rescale $r, y \to \frac{r}{g_s \ell_s^2} = U$, $\frac{y}{g_s \ell_s^2} = Y$

$\Longrightarrow$ All Harmonic functions for NS5 brane rescale as

$$H(r,y) \to H(U,Y) = g_s^{-2} h(U,Y)$$

Ex.

$$H_{TN_4}(U,Y) = \frac{1}{g_s^2}\left\{ \frac{\pi N_5}{\ell_s^3} \int dp\, p^2\, \Gamma\!\left(\frac{p}{4\ell_s}\right) \cos(pY)\, e^{-pU} \times \mathcal{U}\!\left(1+\frac{p}{4\ell_s}, 2, 2pU\right) \right\}$$

[After rescaling integration variable and using $Q_5 = \pi N_5 g_s \ell_s^3 = \pi N_5 \ell_P^3$]

$$ds^2_{10,\mathrm{IIA}} = f^{-1/2}(-dt^2 + dx_1^2 + \cdots + dx_5^2) + \ell_s^4 \left\{ h f^{-1/2} dY^2 + h f^{1/2} (dU^2 + U^2 d\Omega^2) \right\}$$

$\hookrightarrow$ sugra dual to decoupled free theory on NS5 (LST$_{\mathrm{IIA}}$) in low energy.

Conclusions and future works

* 11D sugra solutions of fully localized D2/D6, [illegible], NS5/D6, NS5/D5 systems

* Explicit rep. of dual gravity theory for gauge theory on D2 (and NS5).

* Applications of our solutions:

  1) Calculation of $\langle TT \rangle_{LST}$ from classical action of gravity (dual theory)

  2) Possible computation of spectrum of fields in LST

  3) Computation of some of the states in LST

Conclusions and future works

* 11D sugra solutions of fully localized D2/D6, D2⊥D4, NS5⊥D6, NS5/D5 systems

* Explicit rep. of dual gravity theory for gaug theory on D2 (and NS5).

* Applications of our solutions:

  1) Calculation of $\langle TT \rangle_{LST}$ from classical action of gravity (dual theory)

  2) possible Computation of spectrum of fields in LST

  3) Computation of some of the states in LST

Conclusions and future works

* 11D sugra solutions of fully localized D2/D6, D2⊥D4, NS5⊥D6, NS5/D5 systems

* Explicit rep. of dual gravity theory for gaug theory on D2 (and NS5).

* Applications of our solutions:

  1) Calculation of $\langle TT \rangle_{LST}$ from classical action of gravity (dual theory)

  2) possible Computation of spectrum of fields in LST

  3) Computation of some of the states in LST

$$H_{TB}(y,r) = 1 + Q_2 \int_0^{\infty} \frac{J_1(cy)}{y} R_c(r)\, c^4\, dc$$

$$(2r^2 - n^2 - rn) R_c'' + (4r - n) R_c' - 2rc^2 (r + 2n) R_c = 0$$

* Both radial functions for TN and TB diverge at the brane location and vanish at ∞.

<u>Other solution</u>

$$\int^{\infty} K_1(cy)\, \tilde{R}(r)\, c^4\, dc$$

$$H_{TB}(y,r) = 1 + Q_2 \int_0^\infty \frac{J_1(cy)}{y} R_c(r)\, c^4\, dc$$

$$(2r^2 - n^2 - rn) R_c'' + (4r - n) R_c' - 2rc^2 (r + 2n) R_c = 0$$

* Both radial functions for TN and TB diverge at the brane location and vanish at ∞.

Other solution

$$\tilde{H}_{TB}(y,r) = 1 + Q_2 \int_0^\infty \frac{K_1(cy)}{y} \tilde{R}_c(r)\, c^4\, dc$$

$\tilde{R}_c$ is a damped oscillating function.