

Title: Emergent physics a condensed matter primer.

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Abstract:



Emergent physics: a condensed matter primer

G. Volovik

Low Temperature Lab, Helsinki & Landau Institute, Moscow



- * Emergent relativity from many *Theories of Everything*:
universality classes of *Theories of Everything*
from **p-space topology** of *quantum vacua*

* Fermi point in p-space and emergent physical laws

emergence of chiral particles (quarks and leptons)

as fermion zero modes of quantum vacuum

emergence of relativistic spin

and effective Lorentz invariance

effective gauge fields

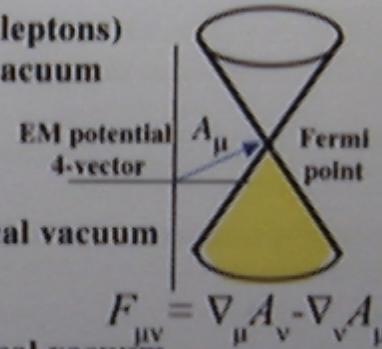
as bosonic collective modes of physical vacuum

effective gauge invariance

effective metric and gravity

as bosonic collective modes of physical vacuum

quantization of physical parameters, ...



$$(\sqrt{-g}/12\pi^2) \ln(\Lambda/T) g^{\alpha\beta} g^{\mu\nu} F_{\alpha\mu} F_{\beta\nu}$$

* Application to yet unknown physics of quantum vacuum

dark matter and dark energy

cosmological constant problems

vacuum instability, ergoregion instability

chiral anomaly and baryogenesis, ...

$$\Lambda\sqrt{-g} \quad \Lambda=0 \text{ in equilibrium}$$

baryoproduction

$$\dot{B} = (1/4\pi^2) (\mathbf{B} \cdot \mathbf{E}) [B_R e_R^2 + B_L e_L^2]$$

momentoproduction

$$\dot{P} = (1/4\pi^2) (\mathbf{B} \cdot \mathbf{E}) [P_R - P_L]$$

a condensed matter primer

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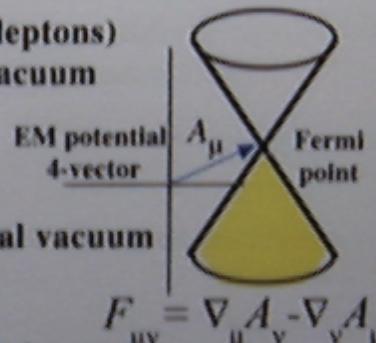
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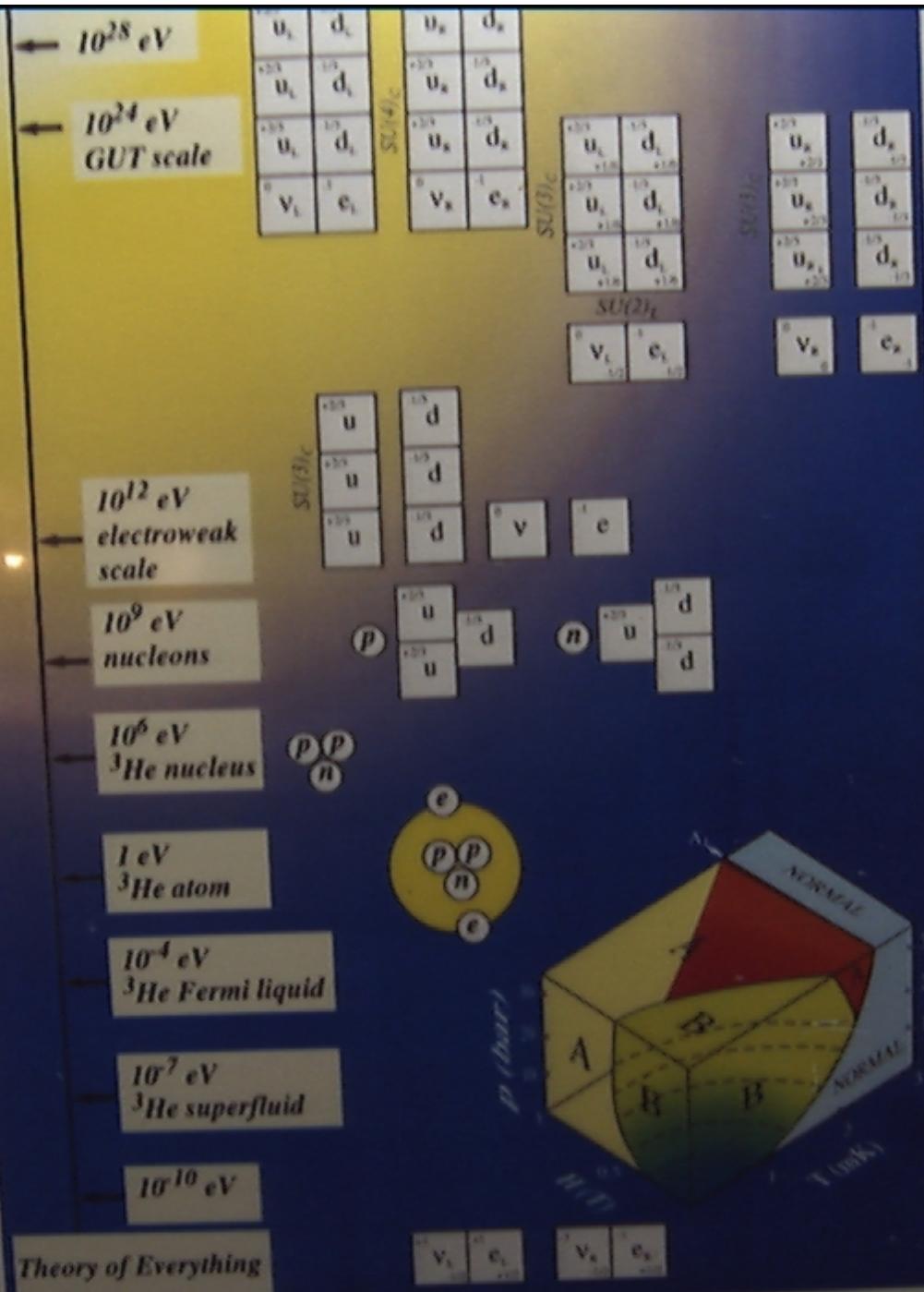
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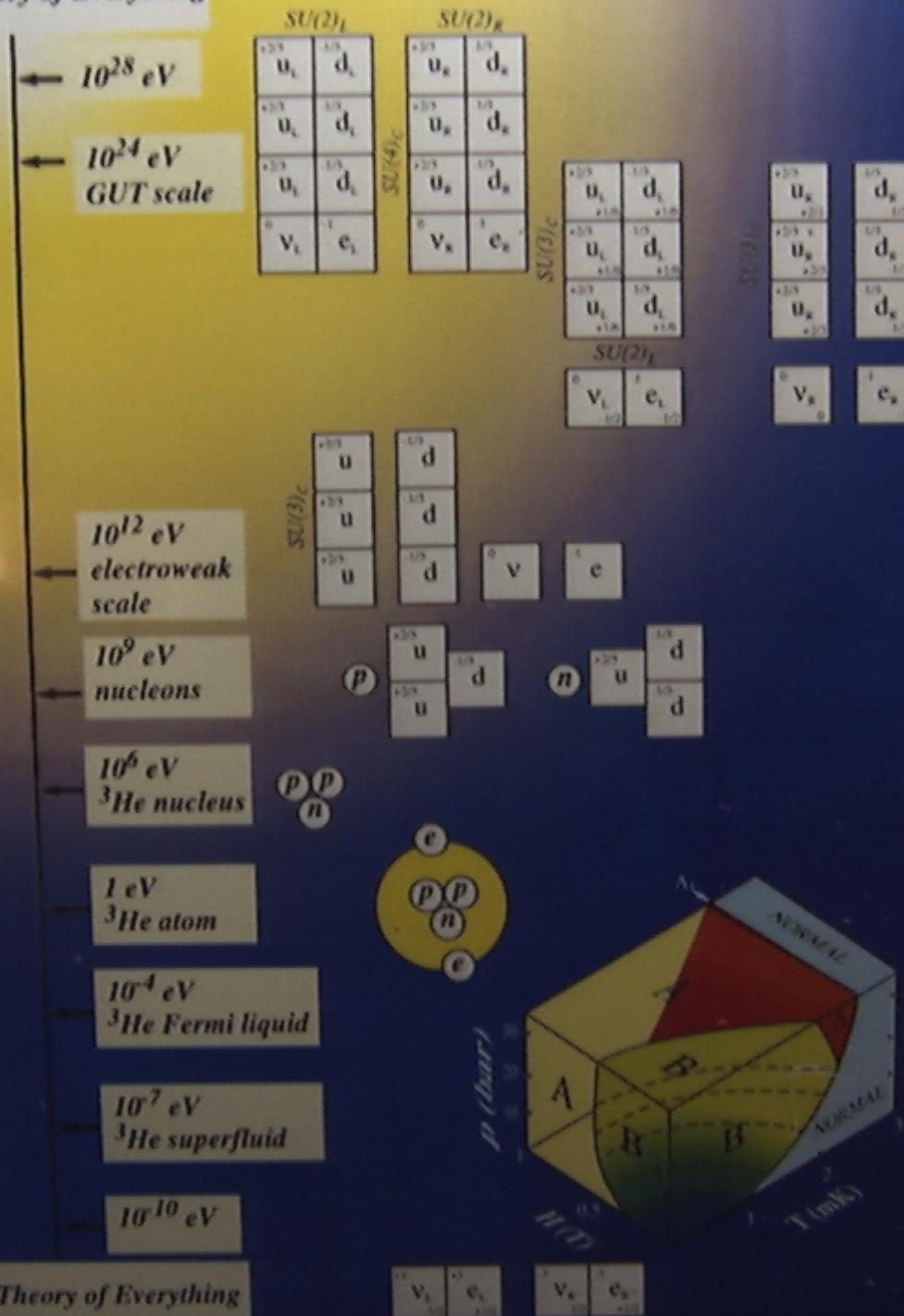
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Theory of Everything

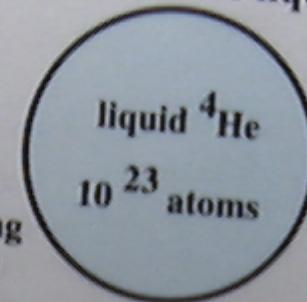




Bangalore
18.02.2004

Landau concept of quasiparticles

Quantum Bose liquid



complicated
many-body system
of strongly interacting
strongly correlated
non-relativistic atoms



India-Finland
Workshop

Theory of Everything:
huge amount of
degrees of freedom

*analog of
Planck scale*

↓
in low temperature limit $T \ll 1^0 \text{ K}$

$$\boxed{\text{Effective theory}} + \boxed{\text{Quasiparticles (phonons)}} + \boxed{\text{Collective hydrodynamic modes}} = \boxed{\text{QFT}}$$

dilute system of elementary particles of effective theory

bosonic

analog of gravity

velocity and density fields provide effective acoustic metric for phonons

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$H_{\text{eff}} = E_{\text{vac}} + \sum_{\mathbf{p}} c_{\mathbf{p}} a_{\mathbf{p}}^+ a_{\mathbf{p}}$$

$$E_{\text{vac}} = 0$$

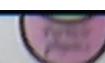
speed of sound

hint

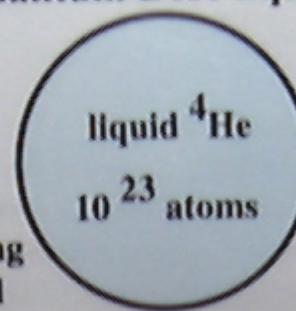
for solution of cosmological problem

$g_{\mu\nu}$ obey hydrodynamic equations rather than Einstein equations

Quantum Bose liquid



complicated
many-body system
of strongly interacting
strongly correlated
non-relativistic atoms



Theory of Everything:
huge amount of
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analog of
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Effective theory ↓ in low temperature limit $T \ll 1^0 \text{K}$

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hint
for solution of
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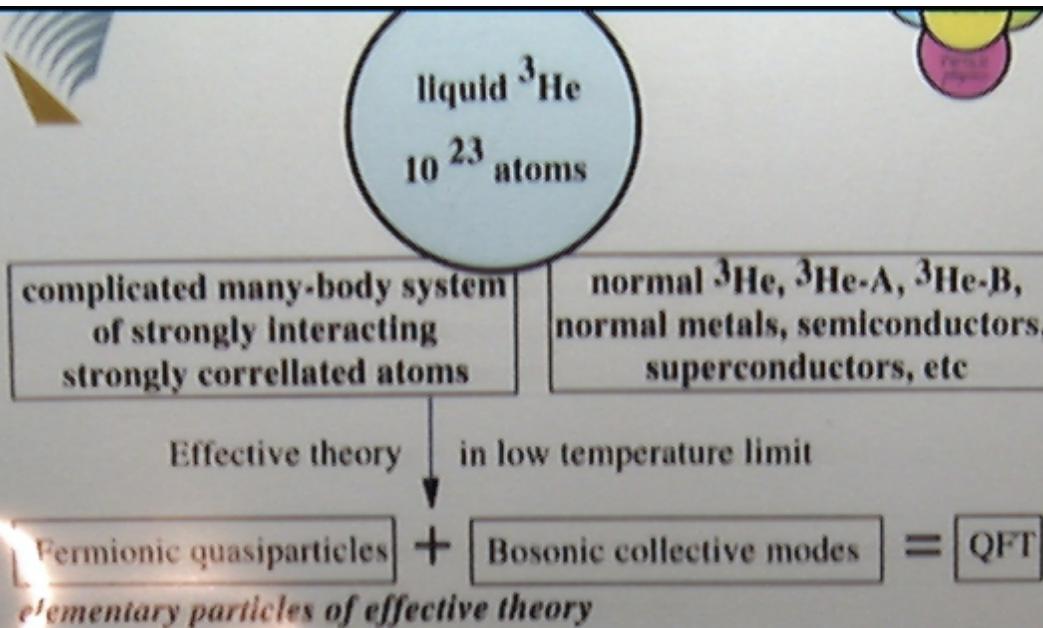
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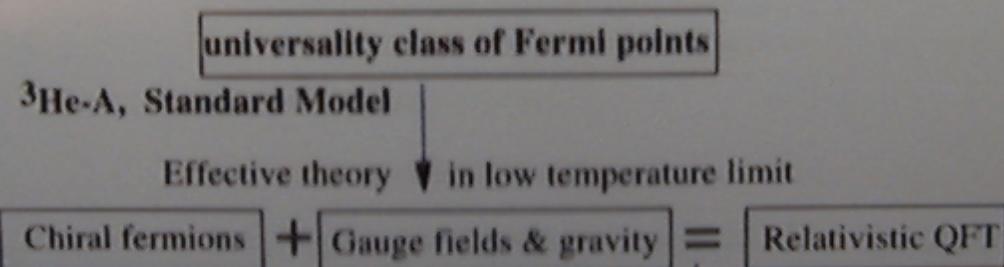


Scan ©American Institute of Physics

Description:
The Theoretical Division at the Institute of Physical Problems, 1956. Standing, l-R: Ginzburg, L.R.; Gershtein, Prilepskii, Arkhipov, Dzhobava; Seated, l-R: Prozorov, Aleksei Adrikov, Khaznikov, Lev Davidovich Landau, Evgenii Konstantinovich Lifshitz.



Type of Quantum Field Theory depends on universality class



left-handed
fermions live here

right-handed
fermions live here

emergent phenomena:
Gauge invariance
Lorentz invariance
General covariance (partly)
chiral fermions
gauge fields
gravity
spin

and emergent effective QFT

quasiparticles: propagating particle-like excitations
above the ground state (vacuum) of system

- * quasiparticle does not scatter on atoms of system,
if system is in the ground state

for quasiparticle:
ground state = vacuum

- * quasiparticles obey effective Quantum Field Theory

physical laws in effective QFT are more symmetric
than physical laws in the underlying
microscopic 'Planck-scale' system

- * type of effective QFT depends on Universality Class

details of underlying 'Planck-scale' physics are irrelevant:
they are lost in coarse-grained description

many 'Theories of Everything'
give the same Effective QFT

acoustic phonons in crystals
do not depend on details of Atomic Structure of Solid;
electronic excitations near Fermi surface
do not depend on details of Electronic Structure of Solid

- * cond-mat provides us with a broad class of Hamiltonians for fermions,
(in RQFT they are very restricted due to Lorentz invariance)
General consideration of the Hamiltonians
(actually, of Green's functions)
revealed that properties of a given QFT are determined
by momentum-space topology

- * p-space topology determines Universality Class of QFT

Universality classes of fermionic vacua



Systems with Fermi surface

Normal metal

Normal ${}^3\text{He}$

$$E = \frac{p^2 - p_F^2}{2m}$$

Fermi surface $E=0$

$E > 0$

$E < 0$
occupied
levels:
Fermi sea

Quark matter

Vacuum within
black hole

Fully gapped Fermi systems

Ground state of superconductor

quasiparticles

$E(p)$

gap Δ

$p=p_F$

Vacuum of Dirac fermions

quarks
electrons

$E(p)$

conduction
bands

electrons

quarks

$p=0$

p

occupied
valence
bands:

Dirac sea

$$E^2 = v_F^{-2} (p \cdot p_F)^2 + \Delta^2$$

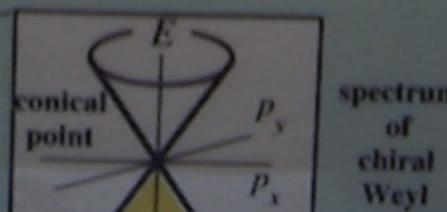
$$E^2 = p^2 c^2 + M^2$$

Systems with Fermi points

Ground state of ${}^3\text{He-A}$

Fermi point

Vacuum of Standard Model



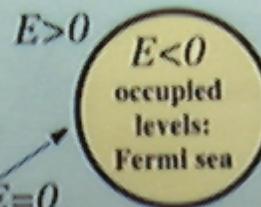
spectrum
of
chiral
Weyl

Systems with Fermi surface

Normal metal
Normal ^3He

$$E = \frac{p^2 + p_F^2}{2m}$$

Fermi surface $E=0$

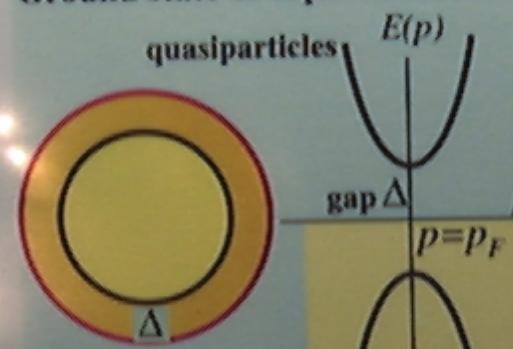


Quark matter

Vacuum within
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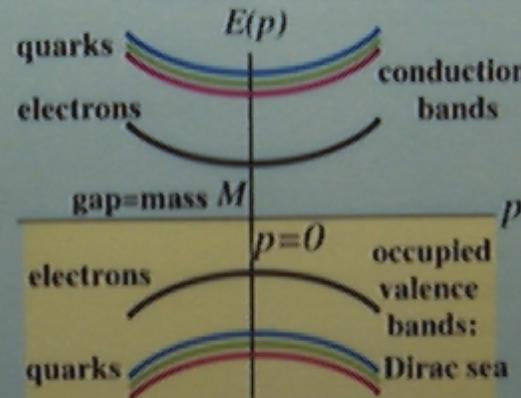
Fully gapped Fermi systems

Ground state of superconductor



$$E^2 = v_F^2(p-p_F)^2 + \Delta^2$$

Vacuum of Dirac fermions



$$E^2 = p^2 c^2 + M^2$$

Systems with Fermi points

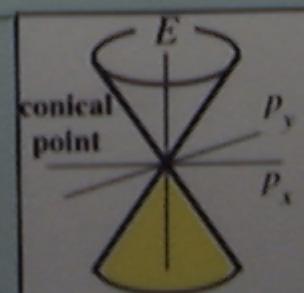
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Fermi point



$$E^2 = p^2 c^2$$

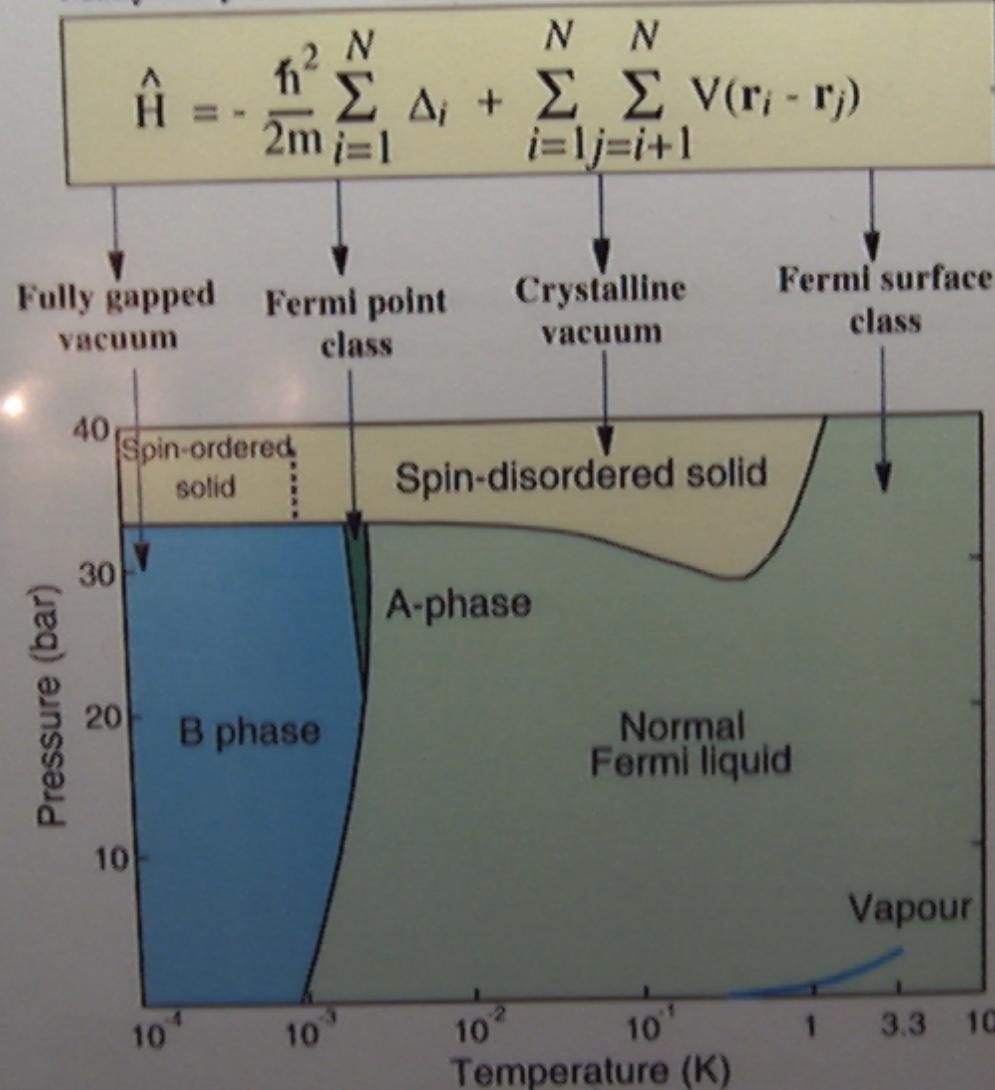
Vacuum of Standard Model



spectrum
of
chiral
Weyl
fermion

Theory of Everything (in system of ^3He atoms)

Many-body Schrödinger quantum mechanics for N atoms



the only role of microscopic parameters $V(\mathbf{r}_i - \mathbf{r}_j)$:
to choose between 4 classes of quantum vacua

* Fermi gas

$$E = \frac{p^2}{2m} - \mu$$

$$E = \frac{p^2 + p_F^2}{2m}$$

Fermi surface
 $E=0$

$E>0$

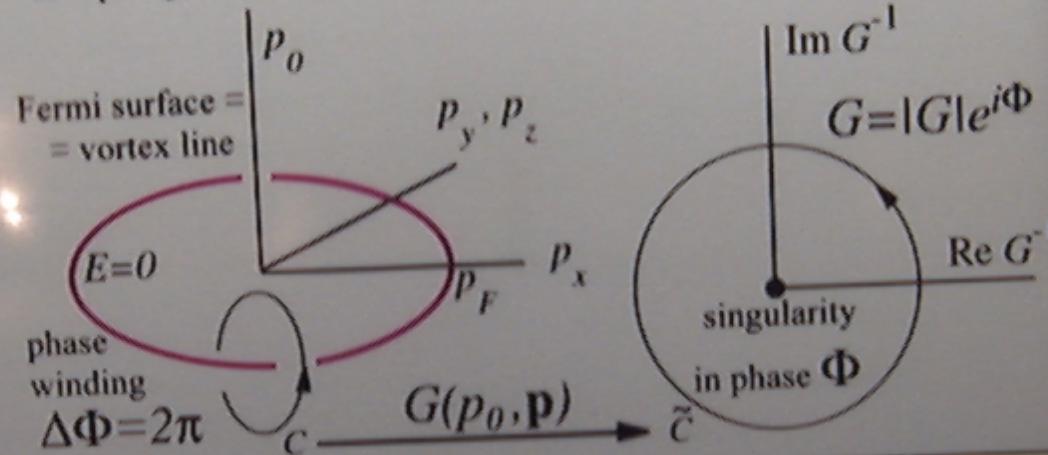
$E<0$
occupied
levels:
Fermi sea

$$p=p_F$$

* Vortex in 4-momentum space

Fermi surface is robust to perturbations and interactions
as topologically stable singularity of Green function:

$$G^{-1} = i p_0 - E$$

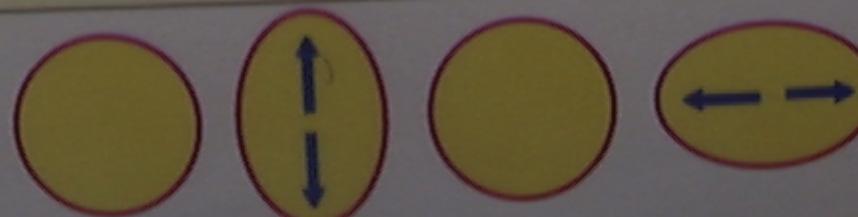


* Interacting Fermi system:
general topological invariant

$$N_1 = \frac{1}{2\pi i} \text{tr} \oint_{\text{around Fermi surface}} d\mu \mathbf{G} \partial_\mu \mathbf{G}^{-1}$$

* Collective bosonic modes of fermionic vacua:
how quasiparticles view collective motion of vacuum

Oscillations of shape of Fermi surface (zero sounds)



Fermi surface Universality class

* Fermi gas

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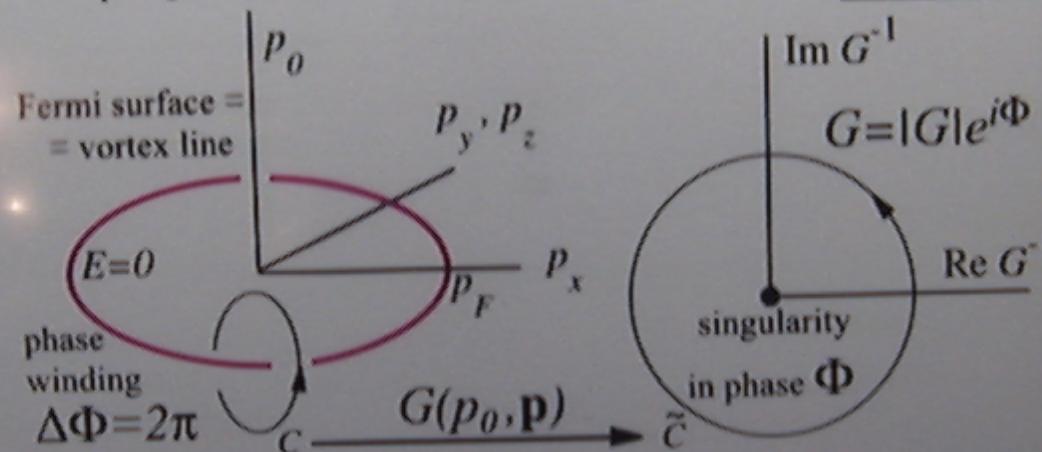
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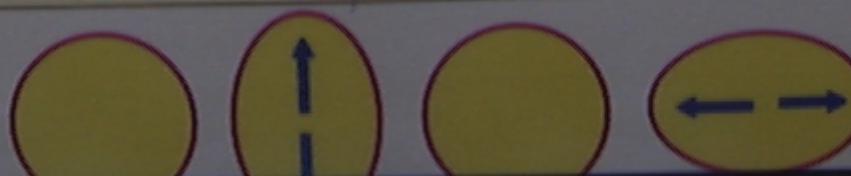
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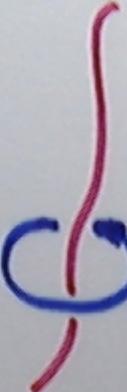
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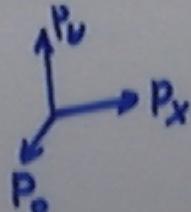
$$\Psi = |\Psi| e^{i\Phi}$$

$\Phi(\vec{r})$ - phase of condensate

$$\Delta\Phi = 2\pi N$$

↓
integer

Fermi surface - vortex in momentum space

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↓
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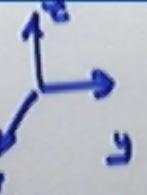
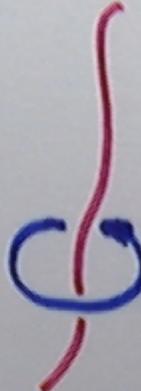
$$\Phi(\vec{p}, p_0)$$

Vortex in \vec{r} -space

$$\Psi = |\Psi| e^{i\Phi}$$

$\Delta\Phi = 2\pi N$
↓
integer

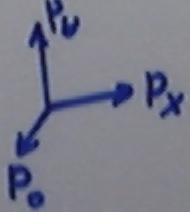
$\Phi(\vec{r})$ - phase of condensate



Fermi surface - vortex in momentum space

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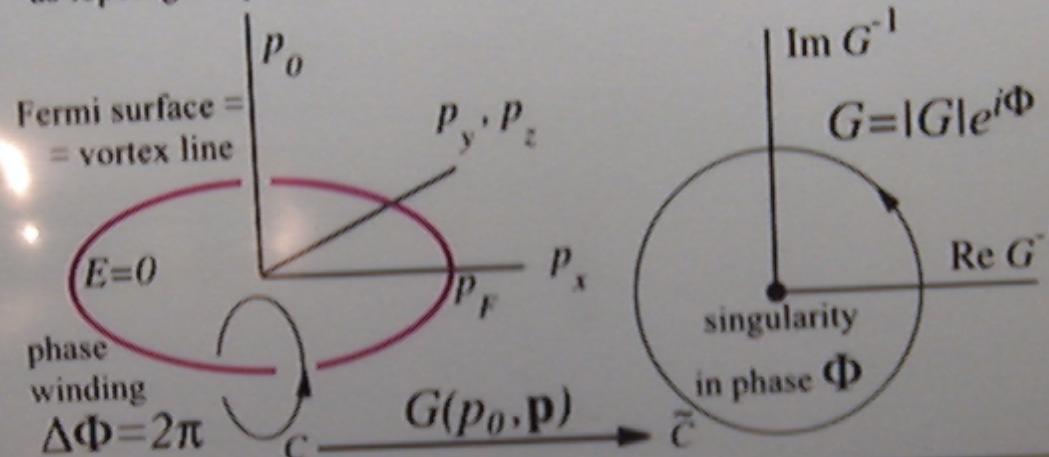
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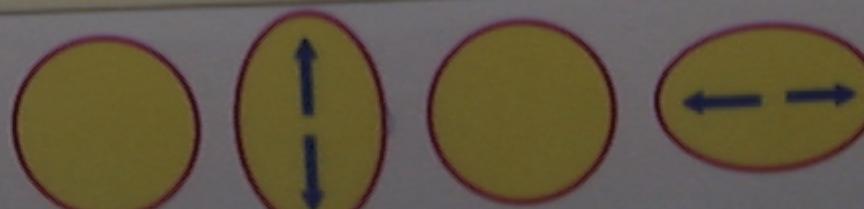


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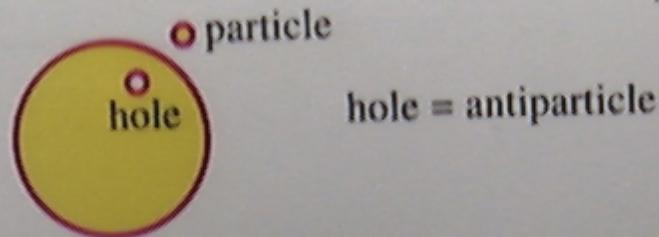
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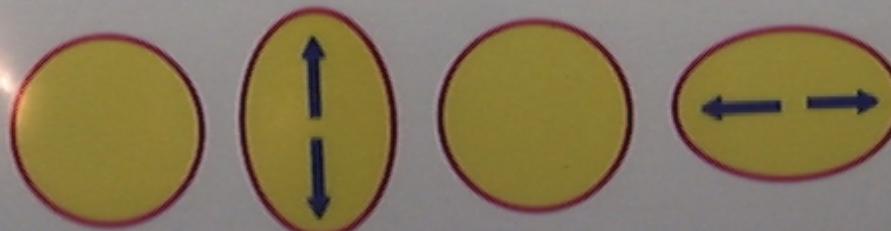


**Effective nonrelativistic Quantum Field Theory:
interacting fermions and bosons**

* fermions



bosons



bosons are not fundamental.
they are 'composite' or effective:
collective modes of fermionic vacuum emerging
in the low-energy corner

**Can Relativistic Quantum Field Theory
naturally emerge in fermionic vacuum ?**

Compare these Hamiltonians

Bogoliubov-Nambu
quasiparticle in QFT for
 $^3\text{He-A}$ & chiral superconductors

left-handed neutrino
 $H = -c \sigma \cdot \mathbf{p}$

$$H = \begin{pmatrix} \frac{p^2 + p_F^2}{2m} & c_\perp(p_x + ip_y) \\ c_\perp(p_x - ip_y) & -\frac{p^2 + p_F^2}{2m} \end{pmatrix}$$

right-handed neutrino
 $H = +c \sigma \cdot \mathbf{p}$

What is common for them?

$$H(\mathbf{p}) = \sigma \cdot \mathbf{m}(\mathbf{p}) \quad E^2(\mathbf{p}) = \mathbf{m}^2(\mathbf{p})$$

1. $E(\mathbf{p}) = 0$ at points (Fermi points)

$$\mathbf{p} = +p_F \mathbf{e}_z \quad N_3 = +1$$

$$N_3 = +1$$

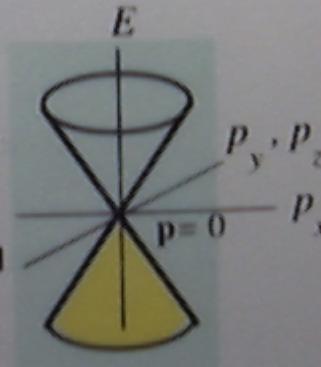
right-handed neutrino

$$\text{left-handed neutrino}$$

$$N_3 = -1$$



$$\mathbf{p} = -p_F \mathbf{e}_z \quad N_3 = -1$$



2. Fermi points are topologically stable
& described by topological invariant in momentum space

$$N_3 = \frac{1}{8\pi} e_{ijk} \int_{\text{over 2D surface around Fermi point}} dS^i \hat{\mathbf{m}} \cdot (\partial^j \hat{\mathbf{m}} \times \partial^k \hat{\mathbf{m}})$$

3. Close to Fermi points (quasi)particles are
relativistic left or right-handed chiral Weyl fermions

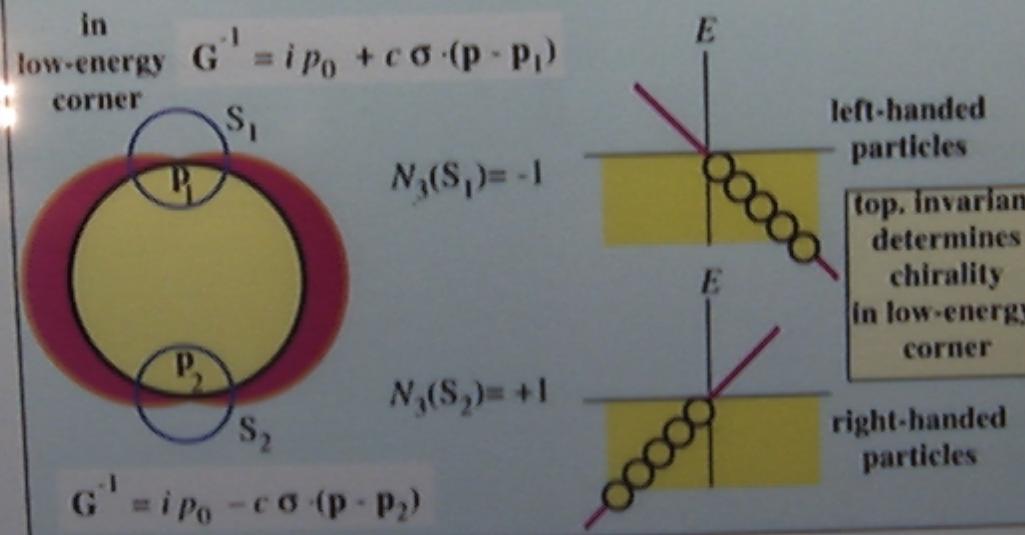
$$L = e^\mu_a \sigma^a (p_\mu \cdot e A_\mu)$$

Topological stability of Fermi point (general case)

Topological invariant in 4D momentum space (\mathbf{p}, p_0)
in terms of fermionic propagator:
matrix Green's function $\mathbf{G}(\mathbf{p}, p_0)$

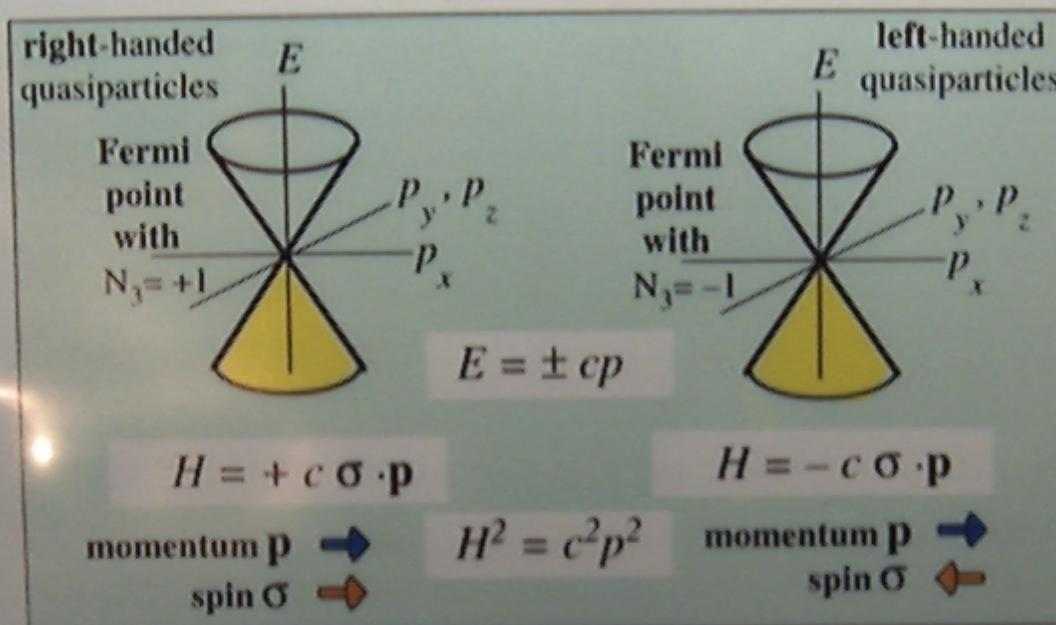
$$N_3 = \frac{1}{24\pi^2} e_{\mu\nu\lambda\gamma} \text{tr} \int dS^\gamma \mathbf{G} \partial^\mu \mathbf{G}^{-1} \mathbf{G} \partial^\nu \mathbf{G}^{-1} \mathbf{G} \partial^\lambda \mathbf{G}^{-1}$$

over 3D surface S in 4D momentum space

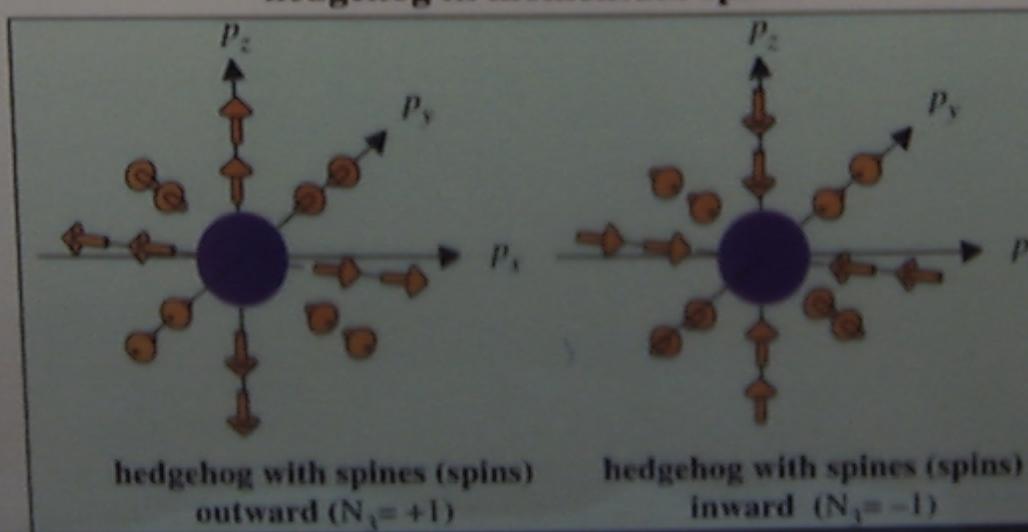


Chiral particles

Quasiparticles near Fermi points are relativistic:
left or right-handed chiral Weyl fermions



Topological stability of Fermi point:
hedgehog in momentum space





Classes of quantum field theories

Compare these Hamiltonians

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left-handed
neutrino

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right-handed
neutrino

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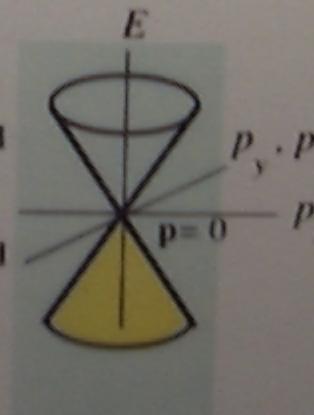
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Classes of quantum field theories



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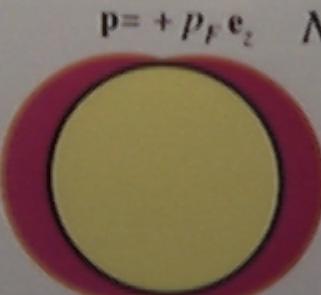
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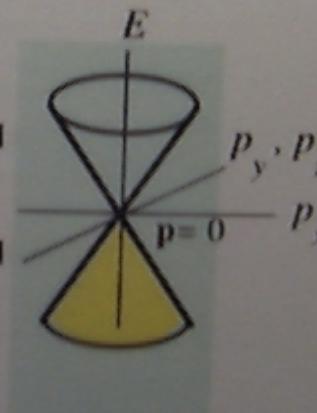
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right-handed
neutrino



left-handed
neutrino
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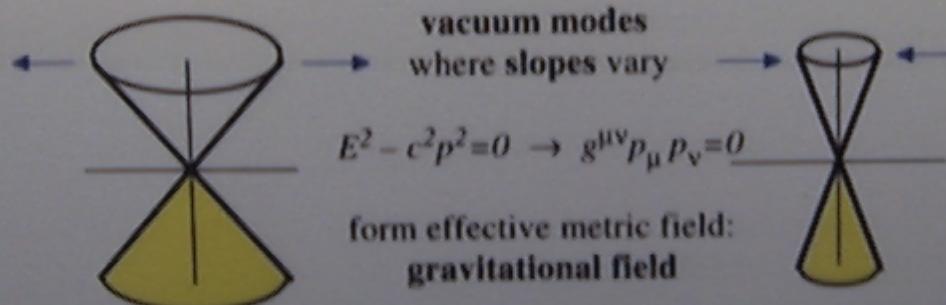
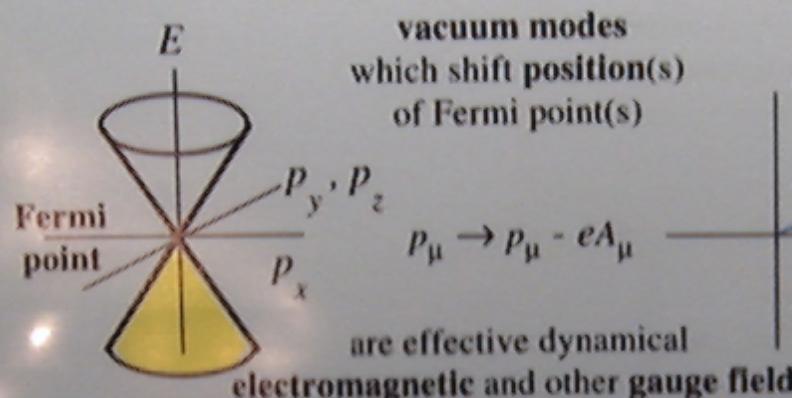
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relativistic left or right-handed chiral Weyl fermions

of Fermi-point Universality Class: gauge fields & gravity

Vacuum low-energy dynamics cannot destroy the Fermi point.
Shifts A_μ and slopes $g^{\mu\nu}$ are propagating collective modes:



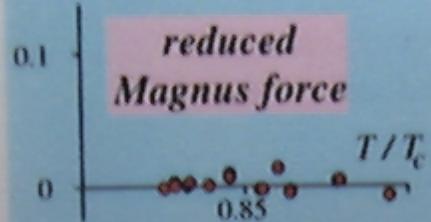
Quasiparticle near Fermi point is
left or right particle moving in effective
gravitational, electromagnetic, weak fields

$$g^{\mu\nu} (p_\mu - eA_\mu - eVW_\mu)(p_\nu - eA_\nu - eVW_\nu) = 0$$

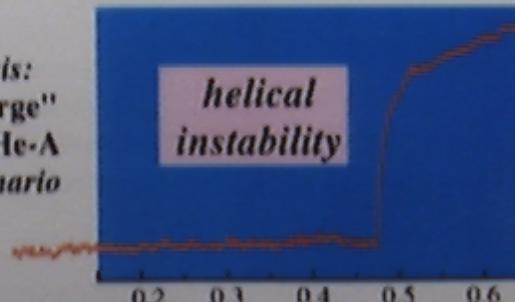
chiral fermions,
gauge fields and gravity
appear
in low-energy corner
together with spin and
physical laws:
Lorentz and gauge
invariance,
and general covariance

properties of quantum vacuum

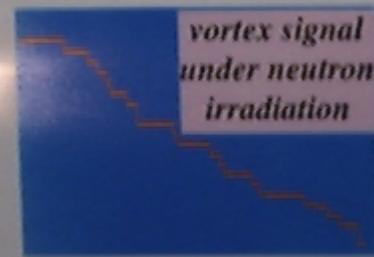
With superfluid ^3He we simulated:



(Manchester) Axial anomaly:
creation of charge from vacuum
was demonstrated in $^3\text{He}\text{-A}$

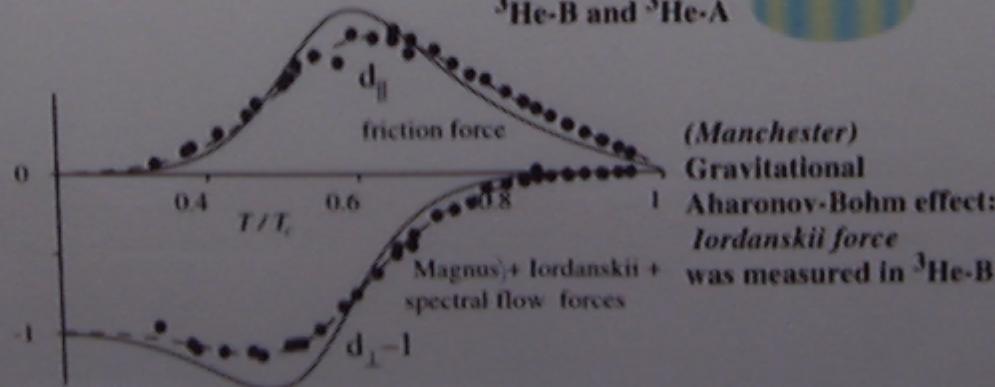


(Helsinki) Magnetogenesis:
Transformation of "charge"
to "magnetic field" in $^3\text{He}\text{-A}$
Joyce-Shaposhnikov scenario



vortex signal
under neutron (Helsinki, Grenoble, Lancaster)
irradiation Kibble-Zurek scenario
of defect formation
was tested in $^3\text{He}\text{-B}$

(Helsinki)
ergoregion instability
at the brane
between 2 vacua
 $^3\text{He}\text{-B}$ and $^3\text{He}\text{-A}$



(Manchester)
Gravitational
Aharonov-Bohm effect:
Iordanskii force
was measured in $^3\text{He}\text{-B}$

In ${}^3\text{He-A}$

Bevan, et al. Nature 386, 689 (1997)

$$\dot{\mathbf{P}} = (1/4\pi^2) \mathbf{B} \cdot \mathbf{E} \sum_a P_a C_a e_a^2$$

Momentogenesis

$$\dot{\mathbf{B}} = (1/4\pi^2) \mathbf{B}_Y \cdot \mathbf{E}_Y \sum_a B_a C_a Y_a^2$$

baryogenesis

P_a -- momentum

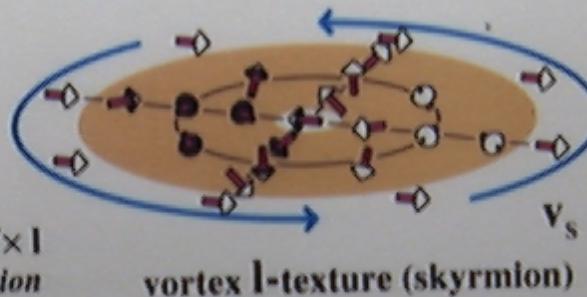
e_a -- effective charge

$C_a = +1$ for right

-1 for left

Effective magnetic field $\mathbf{B} = p_F \nabla \times \mathbf{l}$
is produced by vortex skyrmion

Effective electric field $\mathbf{E} = p_F d\mathbf{l}/dt$
is produced by motion of skyrmion



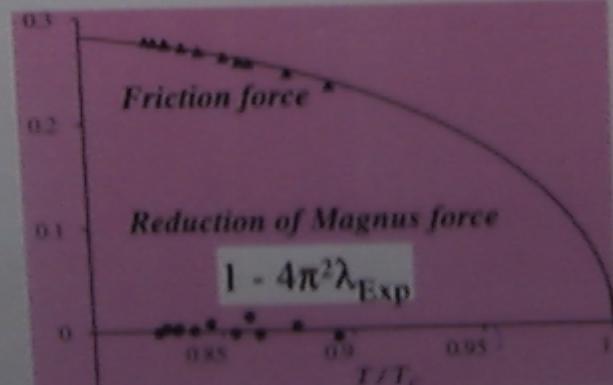
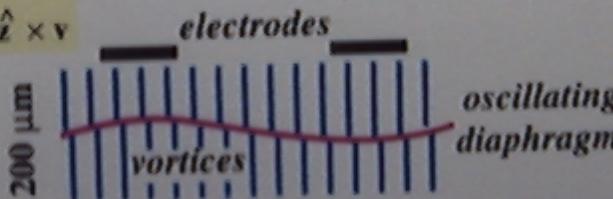
vortex \mathbf{l} -texture (skyrmion)

* Extra force on moving vortex due to chiral anomaly

$$\mathbf{F} = \int d^3r \dot{\mathbf{P}} = \hbar (1/3\pi^2) p_F^3 \hat{\mathbf{z}} \times \mathbf{v}$$

* Experimental set-up

* Experimental result



$$\dot{\mathbf{P}} = \lambda p_F \mathbf{l} (\mathbf{B} \cdot \mathbf{E})$$

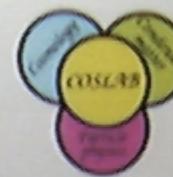
λ_{Exp}
measured parameter
in anomaly equation

$\lambda_{\text{Theory}} = 1/4\pi^2$
theoretical value



Experimental verification of Adler-Bell-Jackiw equation in $^3\text{He}-\text{A}$

Bevan, et al, Nature 386, 689 (1997)



$$\dot{\mathbf{P}} = (1/4\pi^2) \mathbf{B} \cdot \mathbf{E} \sum_a \mathbf{P}_a \mathbf{C}_a e_a^2$$

Momentogenesis

$$\dot{\mathbf{B}} = (1/4\pi^2) \mathbf{B}_Y \cdot \mathbf{E}_Y \sum_a \dot{\mathbf{B}}_a \mathbf{C}_a Y_a^2$$

baryogenesis

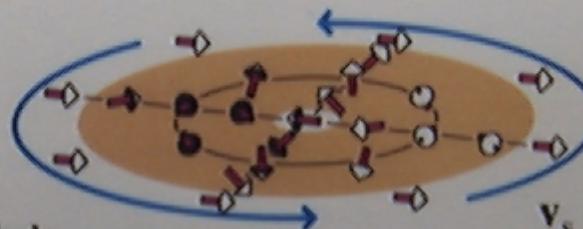
\mathbf{P}_a -- momentum

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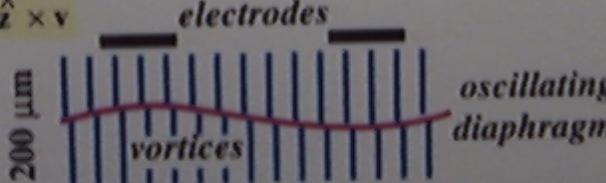


vortex l-texture (skyrmion)

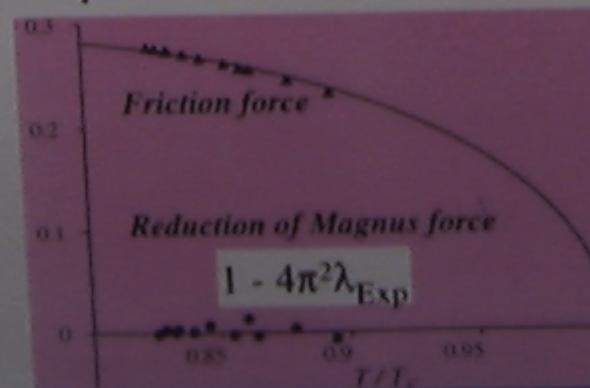
* Extra force on moving vortex due to chiral anomaly

$$\mathbf{F} = \int d\Omega_T \dot{\mathbf{P}} = \hbar (1/3\pi^2) p_F^3 \hat{\mathbf{z}} \times \mathbf{v}$$

* Experimental set-up



* Experimental result



$$\dot{\mathbf{P}} = \lambda p_F \mathbf{l} (\mathbf{B} \cdot \mathbf{E})$$

λ_{Exp}
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$\lambda_{\text{Theory}} = 1/4\pi^2$
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Conclusion



The momentum-space topology determines universality classes of QFT vacua

Vacuum of Standard Model belongs to Fermi-point universality class

Elementary (quasi)particles in vacua of this universality class are chiral fermions emerging near Fermi points

Gravity and gauge fields are low-energy collective modes, either fundamental or emerging due to Fermi points

If RQFT is emergent, spin and speed of light are not fundamental, but fundamental for low-energy observers

If RQFT is emergent, quantum gravity does not exist:
gravity is the classical output of the quantum vacuum
and one should not quantize gravity again (except for gravitons)

Equilibrium quantum vacuum does not gravitate ($\Lambda = 0$)

$\Lambda = 0$ before and after cosmological phase transition

Λ is on order of E_{matter} or of other perturbations of vacuum

Horizon can be constructed at AB-brane

Vacuum behind horizon can be unstable
due to interaction with extra-dimensional environment

Vacuum can resist to formation of horizon