Title: Learning to Count

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Abstract:

Learning to Count

Based on work with:

O. DeWolfe (Princeton)
A. Giryavets (Stanford)
W. Taylor (MIT)

to appear

+

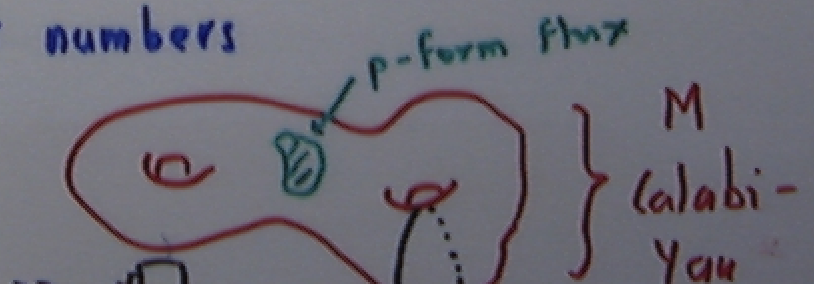
Giryavets, SK, Tripathy hep-th/0404243

+ motivated by GKP, KKLT, ...

Introduction

The study of string theory vacua has reached the point where:

- For models with SUSY at the KK scale
  - $\exists$ a vast # of constructions, both with and without SUSY at tree level
  - known ingredients (fluxes, branes, non-perturbative effects) suffice to yield 'rigid' models w/ no moduli — in vast numbers



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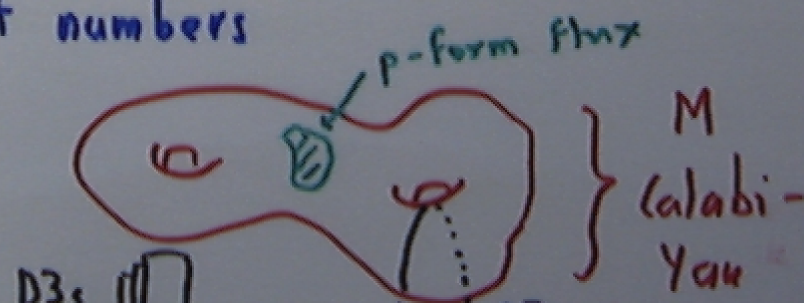

②

This is a good thing:

- Can find Standard-like models, look for natural options for beyond SM physics
- Can try to make contact w/ early universe cosmology -- inflation

But also → a difficult situation:

- Given the wealth of possibilities, how do we usefully describe {vacua}?
- By what criteria are vacua "chosen"? (what is the measure)?
- What does eternal inflation mean?
- CAN WE PREDICT ANYTHING???

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In this talk, I'll describe modest progress on the 1^{st} question, in a large class of type II Calabi-Yau compactifications.

<u>A class of IIB vacua</u>

Giddings
SK
Polchinski
+ ....

· a large class of vacua arising from F-theory on CY 4-folds / IIB on Calabi-Yau orientifolds.

· Assume orientifold action has only fixed pts + surfaces (O3/O7 type)

· Can then show:

- metric is warped, conformally CY :

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- Assume orientifold action has only fixed pts + surfaces (O3/O7 type)
- Can then show:
  - metric is warped, conformally CY:

$$ds_{10}^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}g_{mn}dy^m dy^n$$

④

- $\tilde{F}_5$ satisfies

$$\tilde{F}_5 = (1 + *_{10}) [d\alpha(y) \wedge dx^0 \cdots \wedge dx^3]$$

- The two 3-form field strengths F^{RR}, H^{NS} combine into

$$G_3 = F^{RR} - \phi H^{NS}$$

dilaton

and G_3 satisfies :

$$\boxed{*_6 G_3 = i G_3}$$

Beckers, Dasgupta, Rajesh, Sethi
Gukov-Vafa-Witten

+ standard EOM determine $A(y)$, $\phi(y)$ and

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$$f_i \equiv \int_{\Sigma_i} F^{RR} \in (2\pi)^2 \alpha' \mathbb{Z}$$

$$h_i \equiv \int_{\Sigma_i} H^{NS} \in (2\pi)^2 \alpha' \mathbb{Z}$$

So the correct way to think is :

- Choose M & orientifold action
- Then, choose any fluxes you want.

Must be consistent with :

$$d\tilde{F}_5 = 0 \Rightarrow \boxed{N_{D3} + \int H \wedge F = L}$$

($L \Leftrightarrow$ negative D3 charge from O-planes, ...)

A simple proof shows :

$$*G = iG \Rightarrow \int H \wedge F \geq 0$$

In simple examples, at least, # of possible choices of (f_i, h_i) is

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- Roughly, the "allowed" region is

$$Q(\vec{f}, \vec{h}) \leq L \qquad Q \text{ positive}$$

$L \gg b_3 \Rightarrow$ # pts in $Q \sim (L)^{b_3}$

Easy examples can $\rightarrow$ # $> 10^{300}$.

Typical flux choices do $\rightarrow$ vacua,

so any credible understanding

of {vacua} requires a statistical

theory. We'll talk about this shortly.

c.f. Bousso Polchinski

4d effective QFT

Each choice of f_i, h_i (up to dualities)

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7

Kähler potential & superpotential.

Light modes:

- Complex str moduli: z^a
- Axio-dilaton ϕ
- Kähler moduli ρ

$$K = -\log(-i(\phi - \bar{\phi})) - \log\left(i\int \Omega \wedge \bar{\Omega}\right) - 3\log(-i(\rho - \bar{\rho}))$$

$$\int \Omega \wedge \bar{\Omega} = \log(-i\,\Pi^{\dagger}\Sigma\,\Pi)$$

Π = period vector $\quad \Sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$$= \left(\int_{B^a}\Omega, \int_{A^a}\Omega\right)$$

with A^a, B^a a symplectic basis for H_3.

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⑧

The resulting scalar potential is

$$V = e^{k}\left(\sum_{i,j} D_i W \, g^{i\bar{j}} \bar{D}_j \bar{W} - 3|W|^2\right)$$

Here $\quad D_i W = \partial_i W + k_{,i} W \Rightarrow$

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This "no scale" cancellation persists for models w/ general $h^{1,1}(M)$. In this approximation :

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- SUSY vacua have $D_\rho W = W = 0$

$$\boxed{\begin{array}{l}\text{Since } W(\phi, z^a) = D_a W = D_\phi W \\ \Rightarrow h^{2,1}+2 \text{ eqns in } h^{2,1}+1 \\ \text{vars, } \underline{\text{cannot}} \text{ usually solve}\end{array}}$$

So generic fluxes → no SUSY below the KK scale; ϕ, z^a have isolated vacua.

Beyond "no scale":

- In many models, $\exists$ nonperturbative corrections → W_{np}(kähler moduli)
- $K(\rho, \bar\rho)$ always corrected

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$\Rightarrow$ can solve $D_\rho W = 0$

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(9)

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(10)

A deeper understanding of the structure at the no-scale level for flux vacua is required:

- To get control in full sol'n, need g_s small, $W|_{vac}$ small $\Rightarrow$ want to know how often this happens
- A good toy model for full results may arise by just forgetting ρ, saying

$$V = e^K \left(\sum_{\phi, a} D_a W \, g^{a\bar{b}} \bar{D}_b \bar{W} - 3|W|^2 \right)$$

+ studying resulting distributions of vacua.

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(11)

<u>Questions about {flux vacua}</u>:

1) How many vacua are there?

Already answered; for a given topology M + orientifold action

$$N_{vac} \sim L^{b_3(M)}$$

2) <u>Where</u> on the "former" moduli space $\mathcal{M}$ do vacua arise?

Claim: (Ashok-Douglas)

$$N_{vac} \sim L^{b_3} \int_{\mathcal{M}} \det(-R-\omega)$$

ω = kähler form on $\mathcal{M}$

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assumptions, including a <u>continuous</u> approximation for fluxes f^i, h_i (justified, for some purposes, at $L \gg b_3$). We'll see this approximation is not tenable for certain other questions, though it works beautifully here...

3) What fraction of vacua have $W = 0$ at the level of fluxes ?

This is important for the following reason:

Many authors are debating the 'generic' scale of $SUSY$ in vacua with

$$\Lambda_{4d} \ll M_P^4$$

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(using the potentials described above <u>literally</u>).

- Several have argued that small Λ favors high scale SUSY.

Heuristically

$$V \sim \sum_i |F_i|^2 + \sum |D_a|^2 - 3|W|^2$$

Susskind
Douglas
Arkani-Hamed
Dimopoulos
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W, F_i, D_a uniformly distributed $\rightarrow$ many more vacua with $\Lambda \approx 0$ arise at "large" $|F|$, $|W|$ if # F terms is ≥ 2.

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Similarly, F_i generated by NP effects could have inverse log distribution.

These distributions would $\rightarrow$ opposite conclusion: Small Λ favors low-scale susy.

We remain agnostic about the preferred scale issue, but:

Relevant question is fraction of $W = 0$ vacua. We give simple methods to find this in many cases.

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Note that the 'continuous flux' theory of Ashok-Douglas cannot answer this question :

$$W = D_a W = D_\tau W = 0$$

is over-determined; but if fluxes were continuous, you could regard them as additional <u>fields</u> & then you could <u>always</u> solve this system (determining some fluxes in terms of others).

This question & the next, have more of the flavor of number theory than analysis.

4) What fraction of vacua have enhanced symmetries ?

this question :

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This question & the next, have more of the flavor of number theory than analysis.

4) What fraction of vacua have enhanced discrete symmetries?

Often, $\mathcal{M} = T/G$ T= Teichmüller

Fixed pts of $\mathcal{G}$ → enhanced symmetries.

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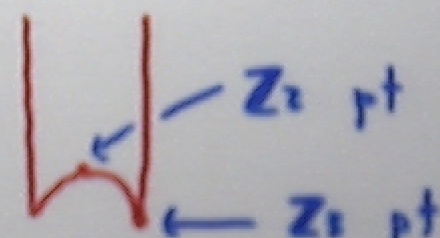

Enhanced symmetries can be of special interest in model building, or cosmology of moduli. But counting vacua at a given ESP cannot be done w/ the Ashok-Douglas theory.

In the rest of the talk, we will check the A-D claim on question 2), and begin to answer 3), 4) by examining simple (but, hopefully, representative) examples.

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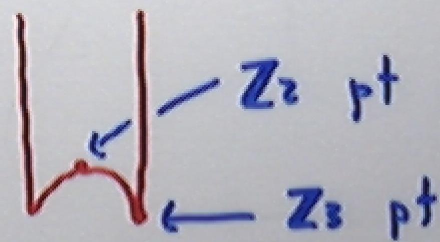

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(17)

Example 1: Rigid CY (c.f. Denef / Douglas)

Consider a Calabi-Yau with $h^{2,1}(M) = 0$.

$$\int_B \Omega = 1 \qquad \int_A \Omega = i$$

The resulting flux superpotential is

$$W = A\phi + B \qquad A = -h_1 - i h_2$$
$$B = f_1 + i f_2$$

The D3 brane charge of the fluxes is

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Let's gauge fix $SL(2,\mathbb{Z})$ by setting :

$$f_2 = 0, \quad 0 \le h_1 < f_1$$

Then: $N_{flux} \le L \Rightarrow f_1 h_2 \le L$

So the total # of inequivalent vacua is:

$$N_{vac}(L) = \sum_{m=1}^{L} \sum_{k|m} k \cong \frac{\pi^2}{12} L^2$$

✓ agrees with L^{b_3} prediction

Distribution of vacua:

Consider the gauge fixing $\phi = -\frac{\bar{B}}{\bar{A}} \in \mathcal{F}$

- The # of complex numbers $z = n + im$ with $n, m \in \mathbb{Z}$ with $|z| \sim \xi$ goes like $2\pi \xi \, d\xi$. For large ξ, the phase is uniformly distributed on a circle.

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(19)

which $\Rightarrow$ ϕ, $\mathrm{Im}(\phi) = y$, $\phi \in F$ as follows.

Let $A = a e^{i\theta}$

$B = b e^{i(\theta+\psi)} = e^{i\theta}\beta$

Then $L \geq ab \sin(\psi)$

$y = \frac{b}{a} \sin(\psi)$

$\Rightarrow \quad a \leq \sqrt{\frac{L}{y}}$.

So distribution of vacua scales as:

$$\rho(\phi) \sim \int_0^{\sqrt{L/y}} 2\pi a \, da \int d^2\beta \; \delta^{(2)}\left(\phi - \frac{1}{a}\beta\right)$$

$$\sim \frac{L^2}{y^2}$$

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get normalization ...

(20)

Vacua with $W=0$:

There are none.

$$W = A\phi + B = 0 \rightarrow \phi = -\frac{B}{A}$$

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$$\phi = \bar{\phi} \rightarrow \mathrm{Im}(\phi) = 0,$$ singular

Discrete symmetries:

$\exists$ two points $\in \mathcal{F}$ with enhanced symmetry:

$$\phi = i \rightarrow \mathbb{Z}_2$$

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$\bar{\phi} = -\frac{B}{A} \rightarrow \mathbb{Z}_3$ pt in-accessible

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So: $N_{vac}(L, \phi = i) \sim \pi L$

And:

$$\boxed{\text{fraction of enhanced sym. vacua} \approx \frac{N(\phi=i)}{N_{tot}} \sim \frac{1}{L}}$$

<u>Example 2</u>: Vacua on $(T^2)^3$ with common modular parameter τ

S.K., Schulz, Trivedi

A basis of $H^3(T_6, \mathbb{Z})$ is: $\left(\begin{smallmatrix} x_i, y_i \\ \text{period } 1 \end{smallmatrix} \right)$

$$\alpha_0 = dx^1 \wedge dx^2 \wedge dx^3$$

$$\alpha_{ij} = \tfrac{1}{2} \epsilon_{ilm}\, dx^l \wedge dx^m \wedge dx^j \qquad 1 \le i,j \le 3$$

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They satisfy $\int \alpha_I \wedge \beta^J = \delta_I{}^J$

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where $dz^i = dx^i + \tau^{ij} dy^j$

(in our examples, $\tau^{ij} = \tau \delta^{ij}$).

The general fluxes are:

$$F_3 = a^0 \alpha_0 + a^{ij} \alpha_{ij} + b_{ij} \beta^{ij} + b_0 \beta^0$$

$$H_3 = c^0 \alpha_0 + c^{ij} \alpha_{ij} + d_{ij} \beta^{ij} + d_0 \beta^0$$

We will look at the special class

$$a^{ij} = a\, \delta^{ij}, \quad b_{ij} = b\, \delta_{ij}$$

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(can show w/o much difficulty that for this class, solutions have $\tau^{ij} = \tau \delta^{ij}$.

Counting generic vacua:

Results of Denef/Douglas $\Rightarrow$

$N_{vac}(L) = \quad (2\pi)^4 \quad L^4$

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W = 0 Vacua?

For SUSY vacua, must have:

$$W = \partial_\phi W = \partial_\tau W = 0 \Rightarrow$$

modular parameter τ of the 2-tori must solve two cubic equations: $(\vec{f}\cdot\vec{\pi} = \vec{h}\cdot\vec{\pi} = 0)$

$$P_1(\tau) = a^0\tau^3 - 3a\tau^2 - 3b\tau - b_0 = 0$$

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Need $\mathrm{Im}(\tau) \neq 0 \Rightarrow P_1, P_2$ must have a common quadratic factor.

$SL(2,\mathbb{Z})$ S-duality $\Rightarrow$ can set $c^0 = 0$, $0 \le b_0 \le d_0$.

$$\Rightarrow \quad P_1(\tau) = (f\tau + g)\,P(\tau)$$

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This is redundant: common factors in

$(-3c, -3d, d_0)$ can be shuffled into $k \Rightarrow$

choose $k = \gcd(-3c, -3d, d_0)$.

Also $\exists$ various mod 3 constraints on

integers, e.g. $-3c = k\ell, \ldots$

So (ignoring mod 3) can:

- sum over relatively prime ℓ, m, n
- sum over f, g and $k < g \leq 0$.

such that tadpole condition

$$(fk)(4\ell n - m^2) \leq 6L$$

is satisfied.

The constraints above, along

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SEE FIGURES

Example III :

The 4 1-parameter CY hypersurfaces,

$M_5 \in \mathbb{P}^4$

$M_6 \in W\mathbb{P}^4_{11112}$

$M_8 \in W\mathbb{P}^4_{11114}$

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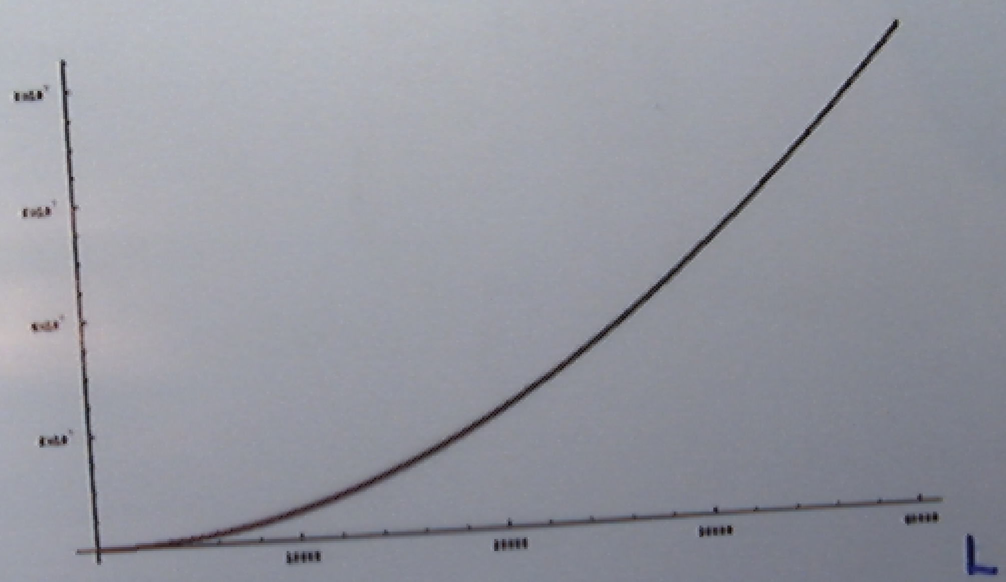
(quotiented by appropriate discrete groups).

$\#$ vacua $\sim L^4$ for all 4

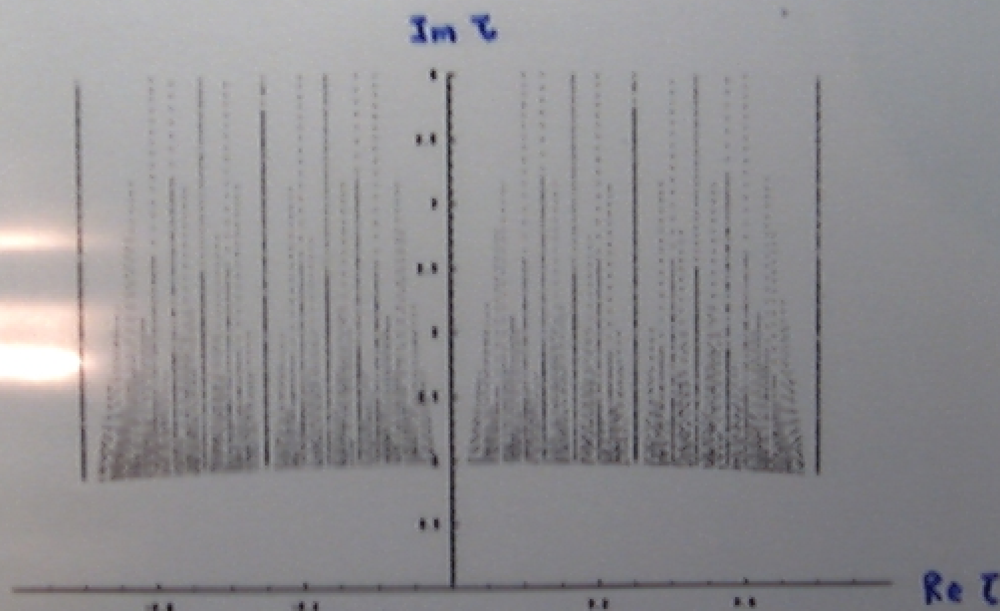
$\exists$ one enhanced symmetry pt $\in \mathcal{M}$ -- the

... point, which for M_k

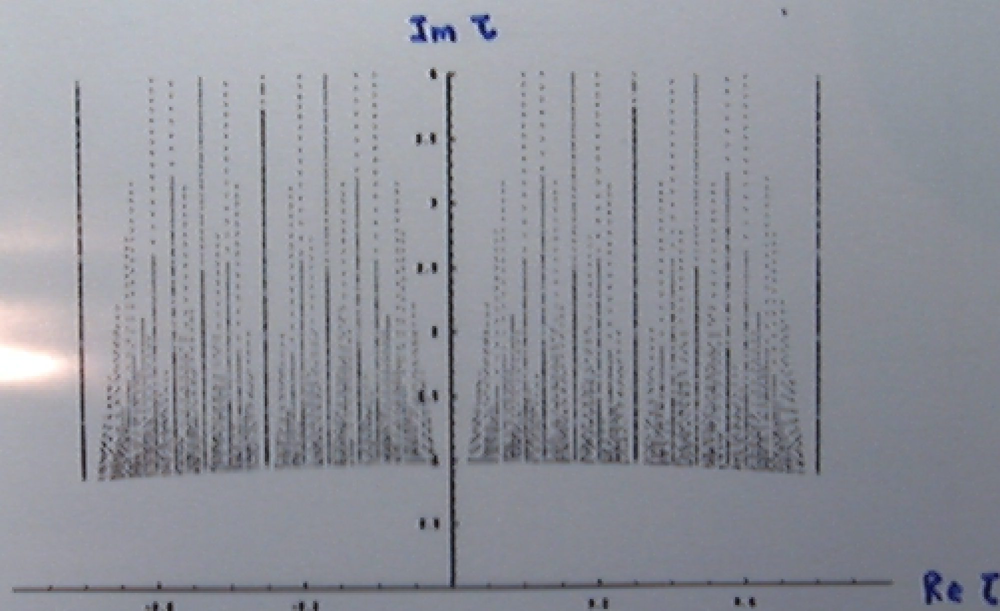
* W=0
vacua



W=0 vacua in $(T^2)^3$



$L = 2000$: SUSY vacua on τ-plane



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Interestingly, at LG in M_K, the periods $\vec{\Pi}$ take values in the cyclotomic field $\mathcal{F}_K$

$\mathcal{F}_K \equiv$ extension of $\mathbb{Q}$ by k^{th} roots of 1

The degree of the extension is given by # of integers $\leq k$ that are relatively prime to k, which can be computed by the Euler totient function

$$\phi(k) = k \prod_i \left(1 - \frac{1}{p_i}\right)$$

(Π runs over prime factors of k).

EOM are $D_\phi W = D_\psi W = 0 \Rightarrow$

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$= 0$

Assuming $\phi \in \mathcal{F}_k$ too → finding soln's @ LG imposes $2\phi(k)$ eqns on the integer fluxes. Since ∃ 8 flux integers for a 1-parameter model:

Solutions will exist if

$$2\phi(k) < 8$$

More generally, at such a point in a model w/ any b_3 if

$$d \equiv 2b_3 - (b_{2,1}+1)\phi(k) + \gamma$$

$\gamma \equiv$ # of relations among flux eqns

$$\Rightarrow \quad N_{vacua}(LG) \sim L^{d/2}$$

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For <u>SUSY</u> solutions $(W=0)$, get one additional eqn $\Rightarrow$

$$N_{susy}(LG) \sim L^{(d-\phi(k))/2}$$

Note that not all soln's at LG preserve the $\mathbb{Z}_k$ symmetry -- need to make sure fluxes don't spontaneously break it.

RESULTS:

M_5, M_{10} : $N_{vac}(LG) = 0$

M_6 : $\frac{N_{vac}(LG)}{N_{vac}} \sim \frac{1}{L^2}$

$\frac{N_{susy}(LG)}{N_{vac}(LG)} \sim \frac{1}{L}$

$\frac{N_{ESP}(LG)}{N_{vac}(LG)} \sim \frac{1}{L}$

M_8 : $\frac{N_{vac}(LG)}{N_{vac}} \sim \frac{1}{L^2}$ $N_{susy}(LG)$

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$$\frac{N_{esp}}{N_{vac}(LG)} \sim 1$$

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Summary:

- Known approximations → lots of vacua
- ∃ a statistical theory for # + distribution of IIB flux vacua -- and the distribution has interesting features

↑ FIGURE

- For 'special' vacua:

$$\left.\begin{array}{l} W = 0 \\ \text{discrete symmetries} \\ \ldots \end{array}\right\} \text{Theory is being developed}$$

Seems like "cost" for e.g. $W=0$ is $\sim (1/L)^m$ with m $\mathcal{O}(1)$

- Theory of heights + "Diophantine geometry" can play a useful role

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↑
FIGURE

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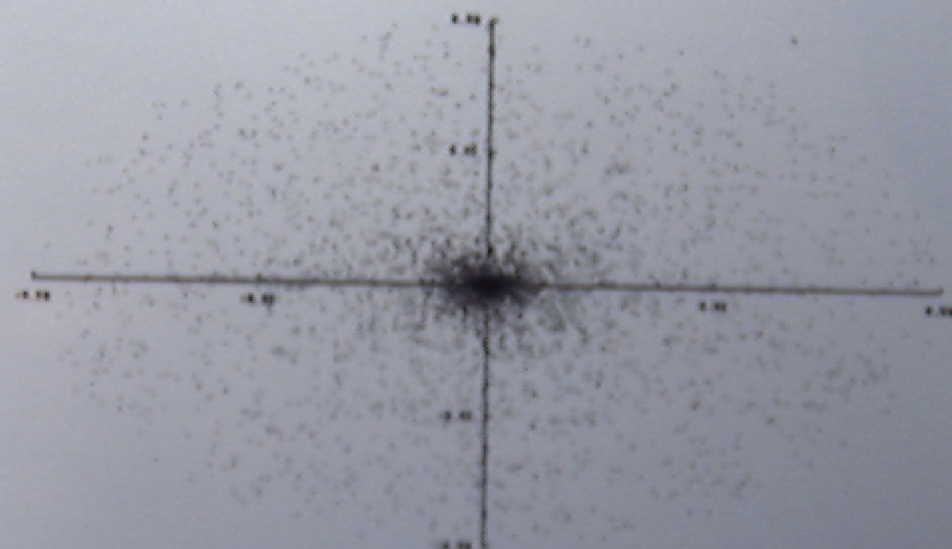

Vacua near conifold $\psi = 1$

Summary:

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Develop similar frameworks in

- IIA on CY with flux
- Heterotic on Non-Kähler, SUSY preserving spaces

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Can then compare to see if samples in each corner are large enough that some feature(s) of distribution are <u>universal</u>.

This would be a hint (though not proof) that we are now looking @ a representative sample of vacua with KK scale SUSY. See also MCS

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