

Title: Discrete Wigner Functions and Quantum Computation

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Abstract:

Discrete Wigner functions and quantum computational speed-up



PERIMETER INSTITUTE
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E.F.G., quant-ph/0405070

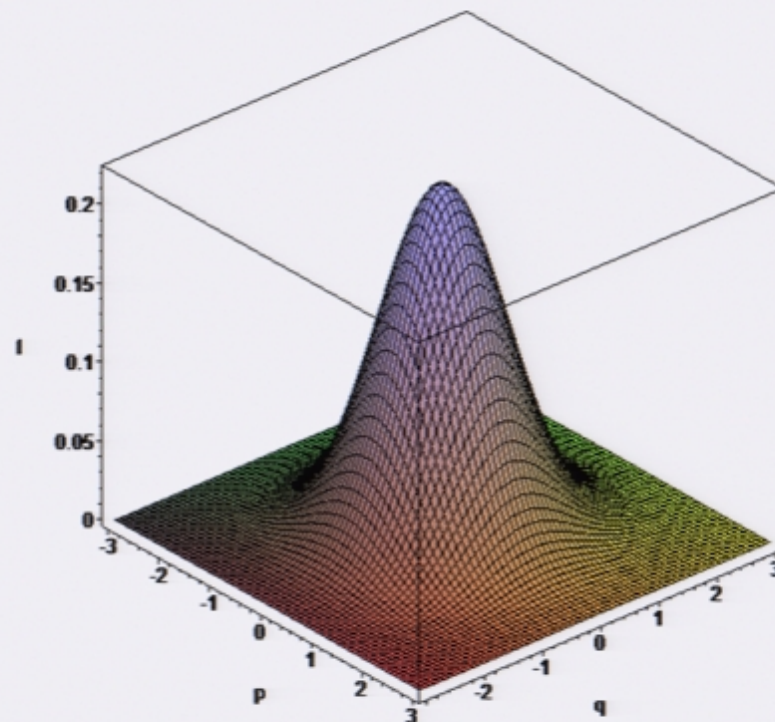
E.F.G., A. O. Pittenger, in preparation

Outline

- Phase space representation of quantum states
 - the Wigner function $W(q,p)$
- Discrete Wigner functions W
 - proposal of Gibbons-Hoffman-Wootters
- Negativity of W and quantum computation
 - in the model of Bravyi and Kitaev
 - in the usual pure-state model
- Open problems

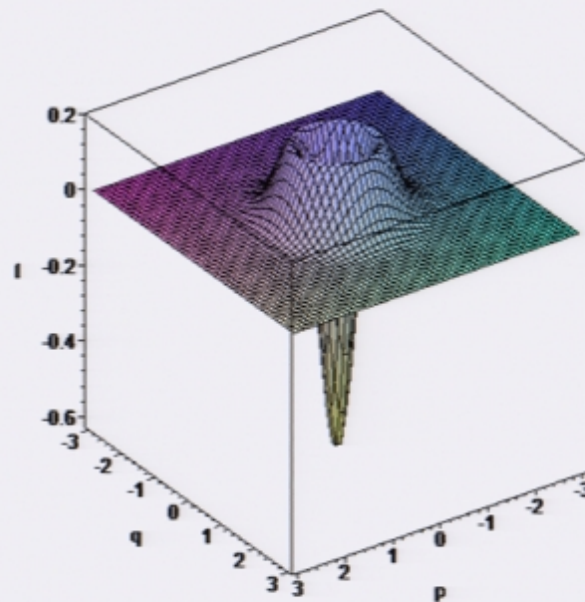
Classical phase space

- (q,p) are position and momentum of the particle
- a classical ensemble is described by a probability density -- the Liouville distribution $f(q,p)$ in phase space
- Integrals over phase space give average properties of the ensemble



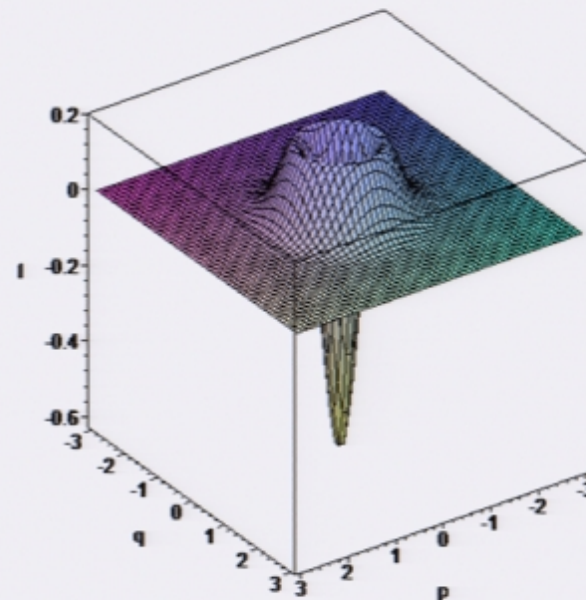
Quantum phase space and the Wigner function $W(q,p)$

- a quantum particle doesn't have (p,q) simultaneously well-defined
- yet, we can define a **quasi**-probability distribution $W(q,p)$ in phase space with useful properties
- $W(q,p)$ can assume **negative** values! Hence the term **quasi**-probability



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Properties of $W(q,p)$

- $W(q,p)$ is tomographically complete, i.e. is equivalent to the density matrix

$$W(q, p) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ipx} \left\langle q - \frac{x}{2} \left| \hat{\rho} \right| q + \frac{x}{2} \right\rangle dx$$

- Correct marginal probability distributions:

$$\int W(q, p) dq = \langle p | \hat{\rho} | p \rangle, \quad \int W(q, p) dp = \langle q | \hat{\rho} | q \rangle$$

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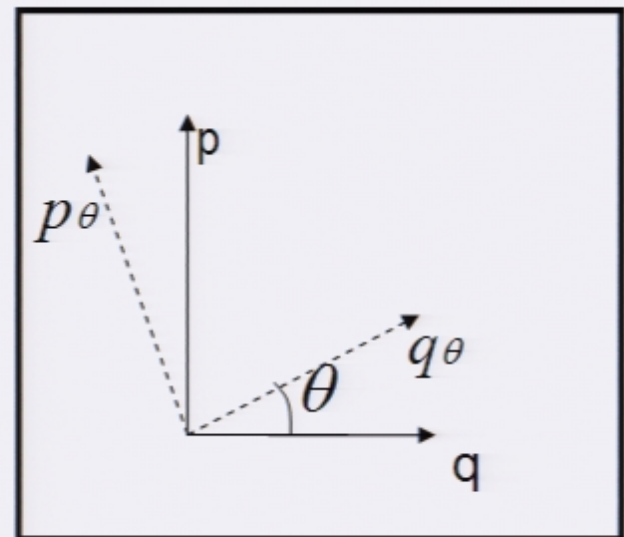
$$\int W(q, p) dq = \langle p | \hat{\rho} | p \rangle, \quad \int W(q, p) dp = \langle q | \hat{\rho} | q \rangle$$

- This is true even for other quadratures:

$$\begin{cases} q_\theta = q \cos \theta + p \sin \theta \\ p_\theta = -q \sin \theta + p \cos \theta \end{cases}$$

$$\Rightarrow \langle q_\theta | \hat{\rho} | q_\theta \rangle =$$

$$\int W(q_\theta \cos \theta - p_\theta \sin \theta, q_\theta \sin \theta + p_\theta \cos \theta) dp_\theta$$

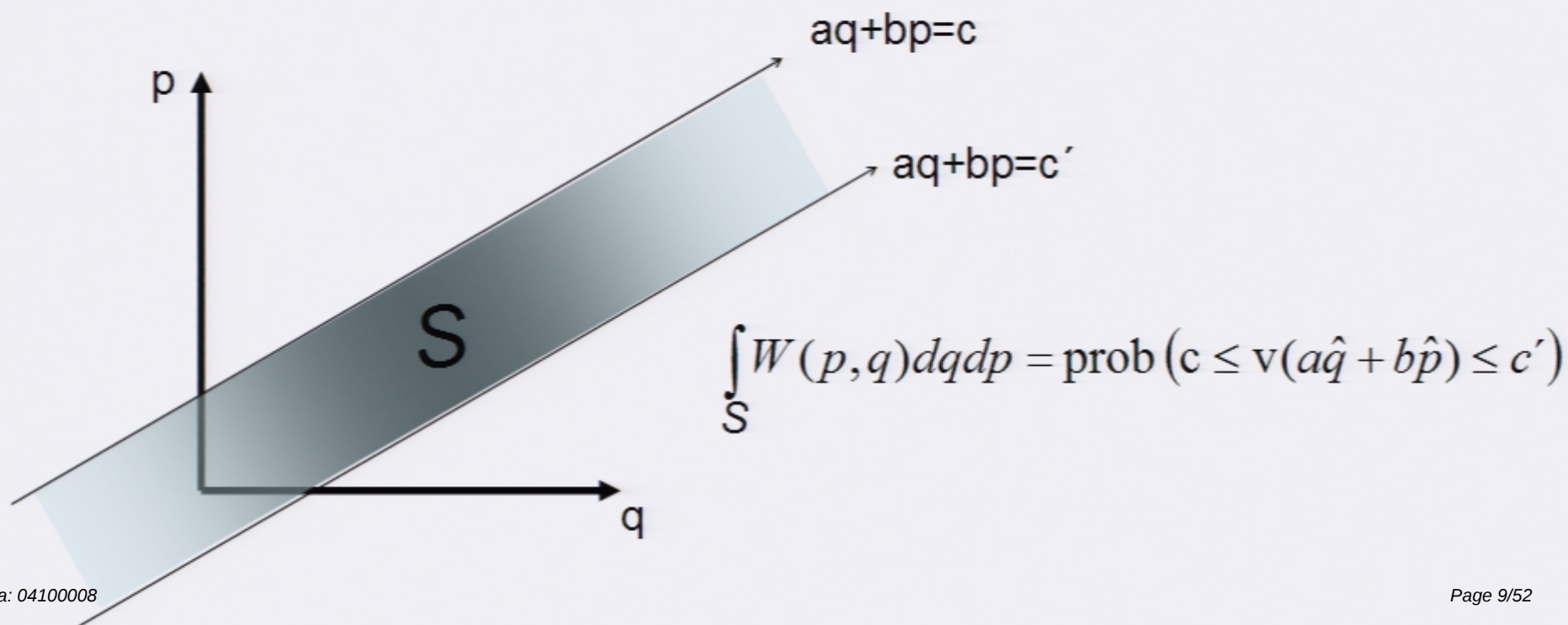


More properties of $W(q,p)$

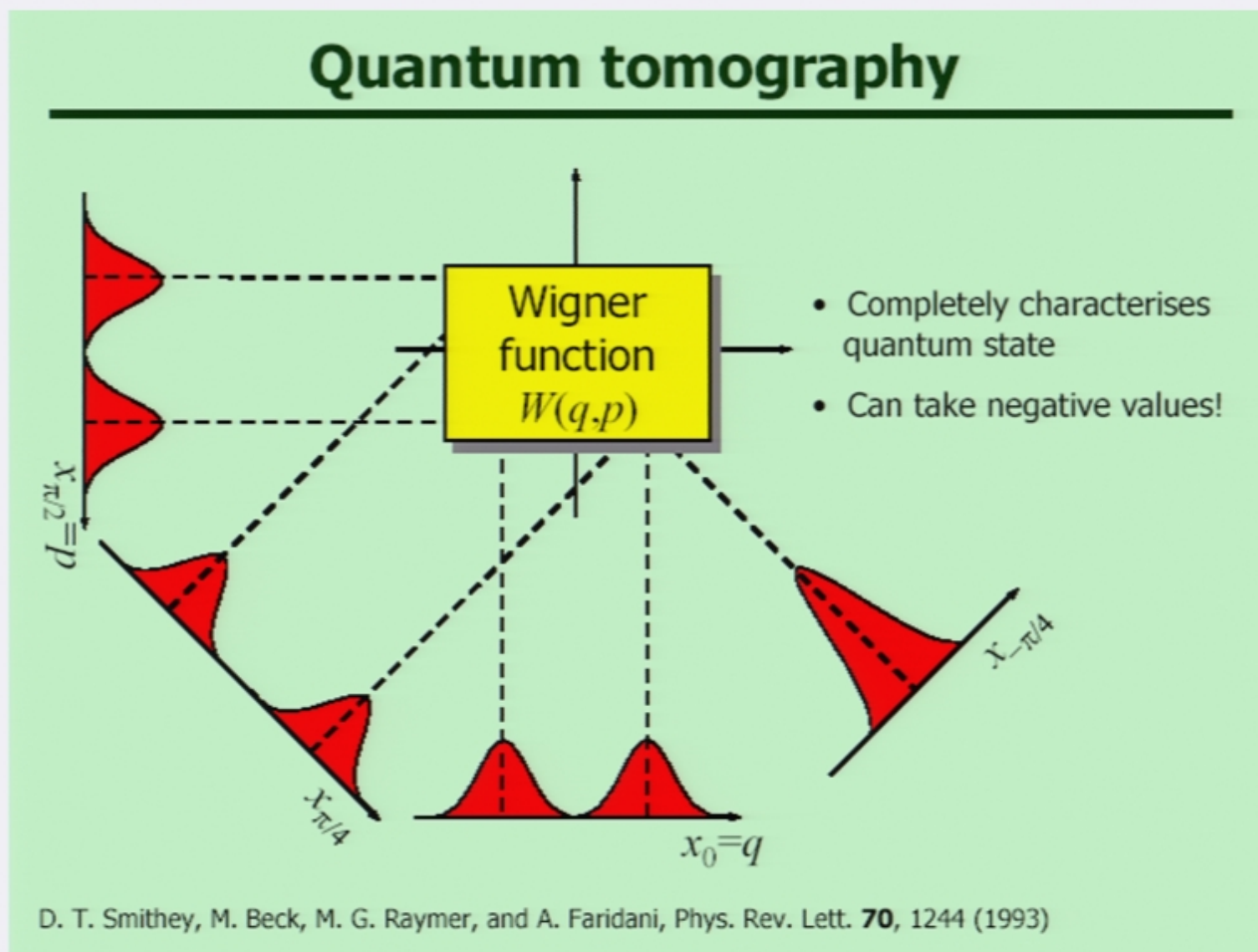
- Expected values of symmetrical operators can be calculated as in the classical case. Example:

$$\text{Tr}[\hat{\rho}(\hat{q}^2 \hat{p} + \hat{q} \hat{p} \hat{q} + \hat{p} \hat{q}^2) / 3] = \int dq dp W(q, p) q^2 p$$

- Also true for integrals over strips in phase space:



Tomography: reconstructing quantum 'stuff' from shadows



Motivating question

- What is non-classical about states with **negative** values of $W(q,p)$?

Motivating question

- What is non-classical about states with **negative** values of $W(q,p)$?



Here I investigate this question using Wigner functions defined for **discrete** Hilbert spaces, i.e. qubits, qutrits, etc.

Discrete Wigner functions for d -dimensional Hilbert space

- Different proposals for discrete Wigner functions: Feynman '87, Wootters '87, Galetti and de Toledo Pisa '88, ...
- A particularly elegant formulation is the one by Wootters, extended recently by **Gibbons, Hoffman and Wootters** (quant-ph/0401155).

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Ingredients:

1. Discrete phase space ($d \times d$ array)
2. $(d+1)$ striations of the phase space
(= partitions of phase space into parallel lines)
3. $(d+1)$ Mutually Unbiased Bases (MUB's)

Parallel lines in a $d \times d$ discrete phase space

- **Line**: set of d phase-space points.
- **Striation**: partition of the discrete phase-space into parallel lines.
- Gibbons *et al.* use **finite fields** to define $(d+1)$ striations of the $d \times d$ phase space with properties:
 - there exists only one parallel line containing an arbitrary pair of points;
 - given a line L and a point P outside it, there exists only one line parallel to L containing P ;
 - two non-parallel lines intersect at exactly one point.

- Examples

- the 3 striations for a qubit ($d=2$):



- the 5 striations for $d=4$:



Mutually unbiased bases in d-dimensional Hilbert space

- Let's take different bases for a d-dimensional Hilbert space:

$$\text{MUB1} = \{|\alpha_{1,1}\rangle, |\alpha_{1,2}\rangle, \dots, |\alpha_{1,d}\rangle\}, \quad |\langle\alpha_{1,k} | \alpha_{1,l}\rangle|^2 = \delta_{k,l}$$

$$\text{MUB2} = \{|\alpha_{2,1}\rangle, |\alpha_{2,2}\rangle, \dots, |\alpha_{2,d}\rangle\}, \quad |\langle\alpha_{2,k} | \alpha_{2,l}\rangle|^2 = \delta_{k,l}$$

$\vdots$

- They are called mutually unbiased bases (MUB's) if:

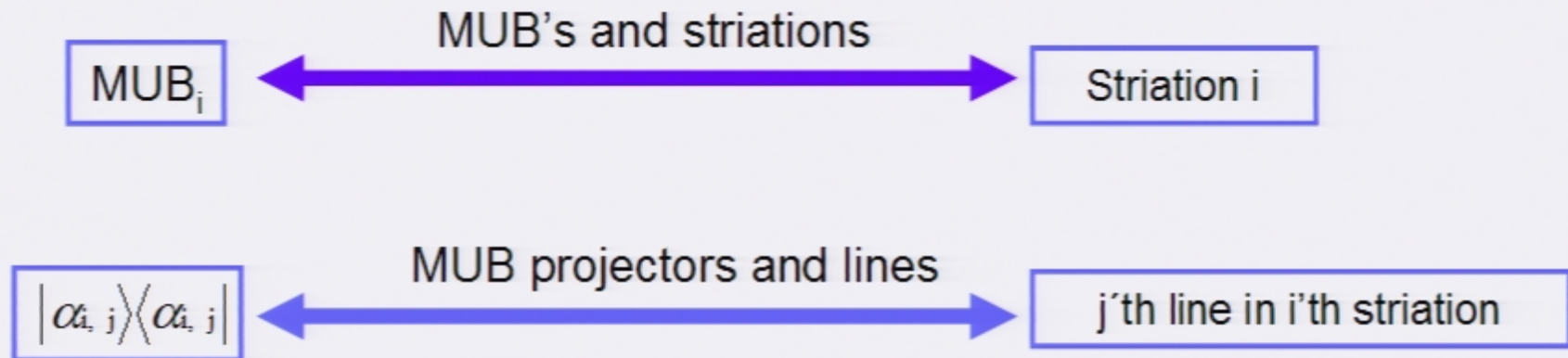
$$|\langle\alpha_{i,k} | \alpha_{j,l}\rangle|^2 = \begin{cases} \frac{1}{d} & \text{for } i \neq j \\ \delta_{k,l} & \text{for } i = j \end{cases}$$

- Facts:**

- (Wootters and Fields): for $d=p^k$ (p prime), there exist (d+1) MUB's.
- Projectors onto the basis states of these (d+1) MUB's form a tomographically complete set of observables.

Defining a class of discrete Wigner functions

- Associations:



- Imposing the following requirement defines W uniquely:

$$p_{i,j} \equiv Tr(\hat{\rho} |\alpha_{i,j}\rangle\langle\alpha_{i,j}|) = \sum_{r \in line(i,j)} W(r)$$

Each choice of associations picks one definition of Wigner function. Gibbons-Hoffman-Wootters use the recipe above to define a **class** of Wigner functions.

Example: W for a qubit

Striations

Requirements:

1:

$W_{1,1}$	$W_{1,2}$
$W_{2,1}$	$W_{2,2}$

$$\sum \blacksquare = \text{prob}(\sigma_x = +1)$$

$$\sum \square = \text{prob}(\sigma_x = -1)$$

2:

$W_{1,1}$	$W_{1,2}$
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Striations

Requirements

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$W_{1,1}$	$W_{1,2}$
$W_{2,1}$	$W_{2,2}$

$$W_{1,1} + W_{1,2} = p_{1,1}$$

$$W_{2,1} + W_{2,2} = p_{1,2}$$

2:

$W_{1,1}$	$W_{1,2}$
$W_{2,1}$	$W_{2,2}$

$$W_{1,1} + W_{2,1} = p_{2,1}$$

$$W_{1,2} + W_{2,2} = p_{2,2}$$

3:

$W_{1,1}$	$W_{1,2}$
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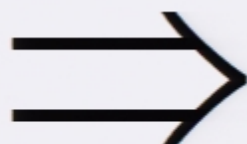
$$W_{1,1} + W_{2,2} = p_{3,1}$$

$$W_{1,2} + W_{2,1} = p_{3,2}$$

Finding the resulting Wigner function W

- We solve the system of equations:

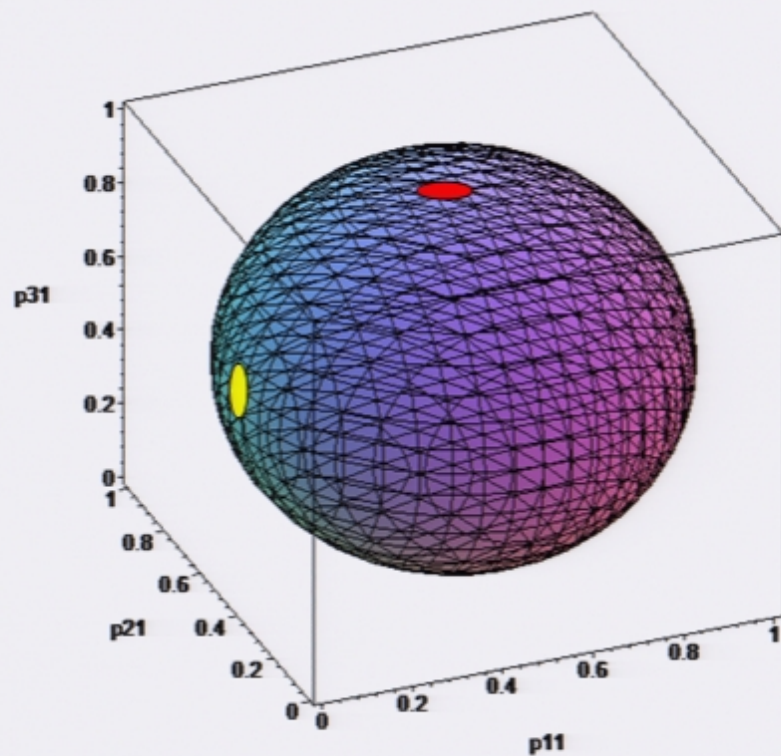
$$\begin{cases} W_{1,1} + W_{1,2} = p_{1,1} \\ W_{2,1} + W_{2,2} = p_{1,2} = 1 - p_{1,1} \\ W_{1,1} + W_{2,1} = p_{2,1} \\ W_{1,2} + W_{2,2} = p_{2,2} = 1 - p_{2,1} \\ W_{1,1} + W_{2,2} = p_{3,1} \\ W_{1,2} + W_{2,1} = p_{3,2} = 1 - p_{3,1} \end{cases}$$



$$\begin{aligned} W_{1,1} &= \frac{1}{2} (p_{1,1} + p_{2,1} + p_{3,1} - 1) \\ W_{1,2} &= \frac{1}{2} (p_{1,1} + p_{2,2} + p_{3,2} - 1) \\ W_{2,1} &= \frac{1}{2} (p_{1,2} + p_{2,1} + p_{3,2} - 1) \\ W_{2,2} &= \frac{1}{2} (p_{1,2} + p_{2,2} + p_{3,1} - 1) \end{aligned}$$

$$W = \begin{bmatrix} \frac{1}{2} (p_{1,1} + p_{2,1} + p_{3,1} - 1) & \frac{1}{2} (p_{1,1} + p_{2,2} + p_{3,2} - 1) \\ \frac{1}{2} (p_{1,2} + p_{2,1} + p_{3,2} - 1) & \frac{1}{2} (p_{1,2} + p_{2,2} + p_{3,1} - 1) \end{bmatrix}$$

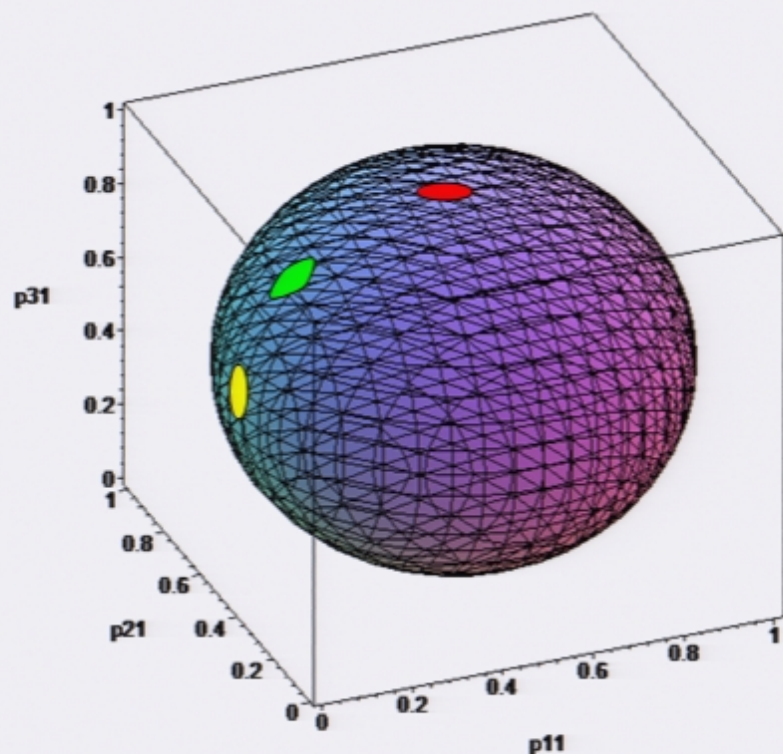
Wigner function for some states



$$W(\bullet) = \begin{array}{|c|c|} \hline \frac{1}{2} & 0 \\ \hline 0 & \frac{1}{2} \\ \hline \end{array}$$

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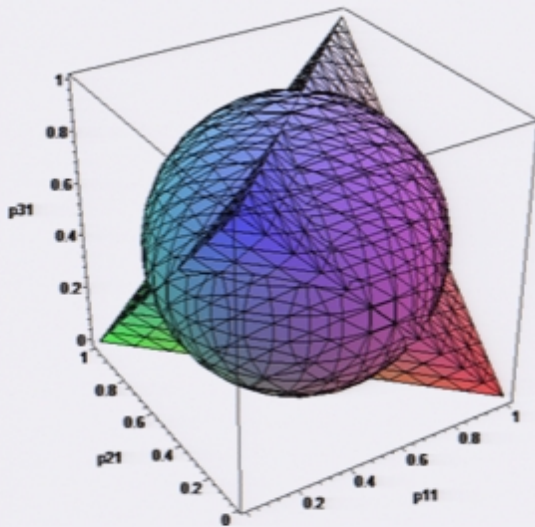
$$W(\bullet) = \begin{array}{|c|c|} \hline 0.25 & -0.10 \\ \hline 0.25 & 0.60 \\ \hline \end{array}$$

Negativity of W !

One-qubit states with non-negative W

For one particular definition:

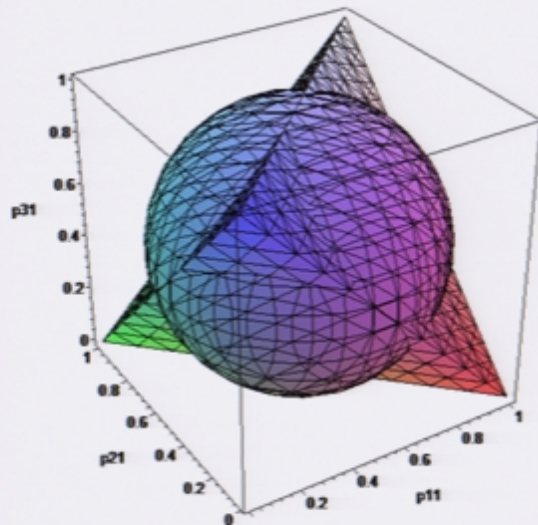
$$\begin{cases} p_{1,1} + p_{2,1} + p_{3,1} \geq 1 \\ p_{1,1} - p_{2,1} - p_{3,1} \geq -1 \\ -p_{1,1} + p_{2,1} - p_{3,1} \geq -1 \\ -p_{1,1} - p_{2,1} + p_{3,1} \geq -1 \end{cases}$$



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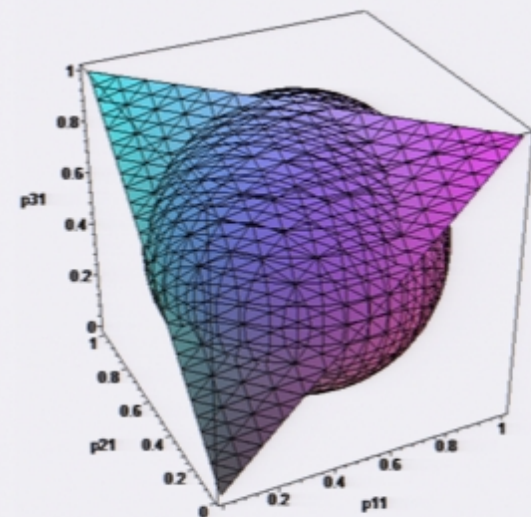
Swap lines in
3rd striation



$$p_{3,1} \leftrightarrow 1 - p_{3,1}$$

For a different definition of W :

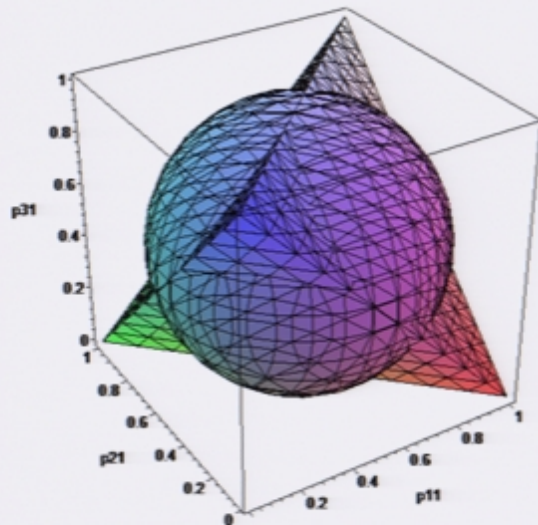
$$\begin{cases} p_{1,1} + p_{2,1} - p_{3,1} \geq 0 \\ p_{1,1} - p_{2,1} + p_{3,1} \geq 0 \\ -p_{1,1} + p_{2,1} + p_{3,1} \geq 0 \\ -p_{1,1} - p_{2,1} - p_{3,1} \geq -2 \end{cases}$$



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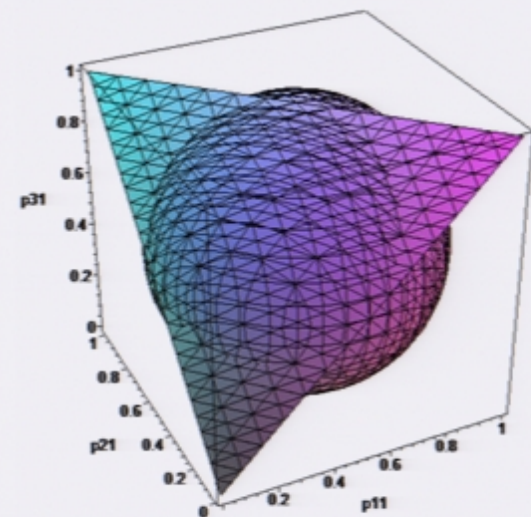
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The set of states
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'Classical' states in d dimensions -- definition

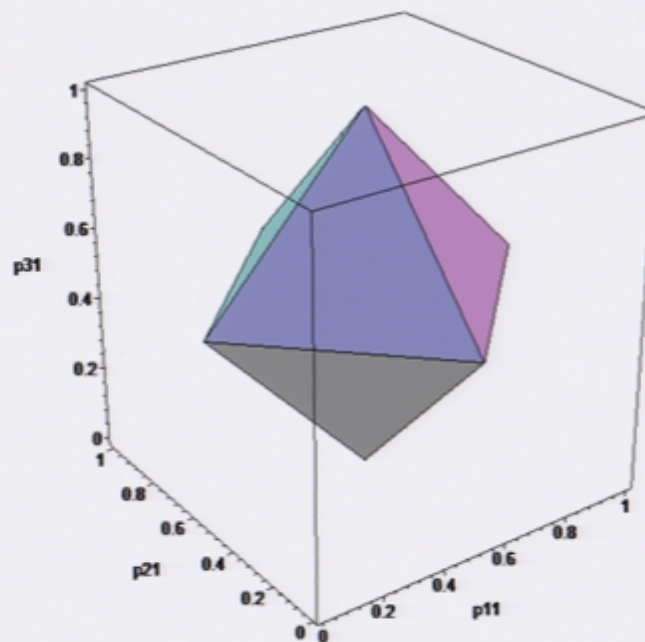
- Let's define a set C_d of 'classical' states with properties:
 - 1- is based on non-negativity of W ('no negative probabilities');
 - 2- is independent of the particular definition of W we pick;
 - 3- includes only physical states;
 - 4- states in C_d display 'classicality' in a concrete computational sense.

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Definition: the set C_d of 'classical' states are those with non-negative W in *all* definitions of W (using a fixed set of MUB's).

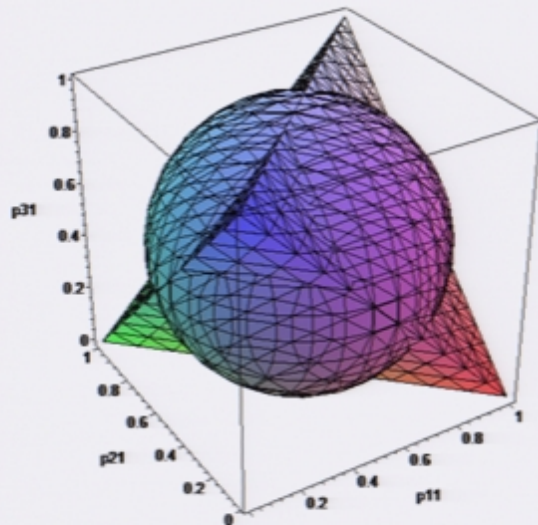
- The set C_2 (one-qubit 'classical' states) is the intersection of the two tetrahedra above – an octahedron.



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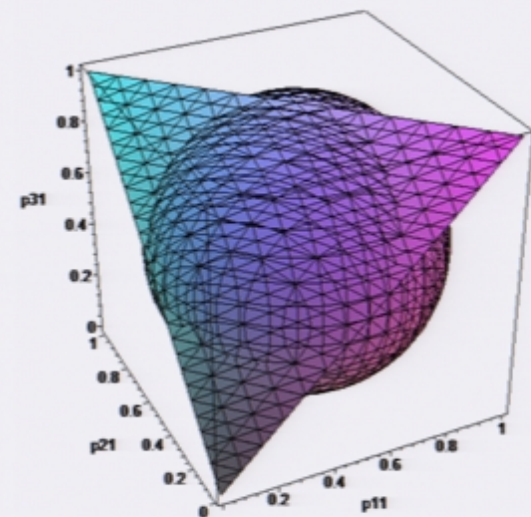
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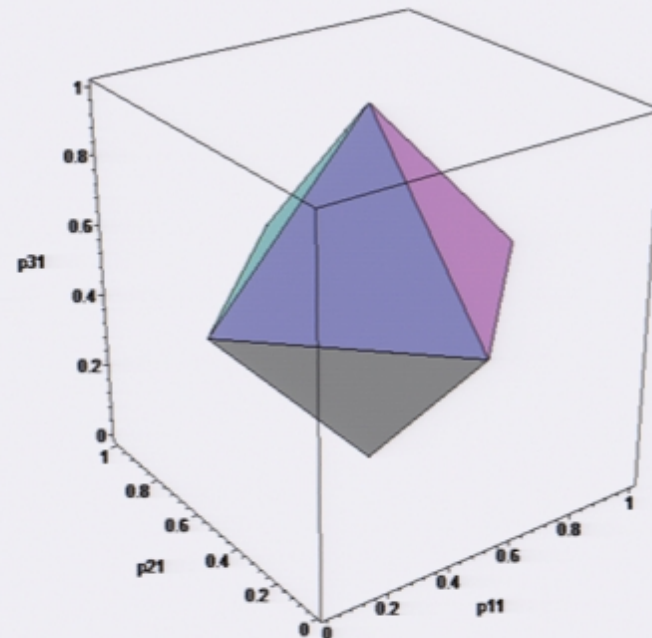


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Why do I call states in C_d 'classical'?

$d=2$ (one qubit case)

- One-qubit ancillas in states in C_2 are *useless* to obtain universal quantum computation in a recent model by Bravyi and Kitaev.
- Bravyi-Kitaev QC Model (quant-ph/0403025):

Perfect operations

- Initialization in $|00 \dots 0\rangle$
- Unitaries from the Clifford group (eg. CNOT, H, phase gate)
- Measure single-qubit Paulis.

Imperfect operations

- Preparation of a one-qubit ancilla in a mixed state $\hat{\rho}$

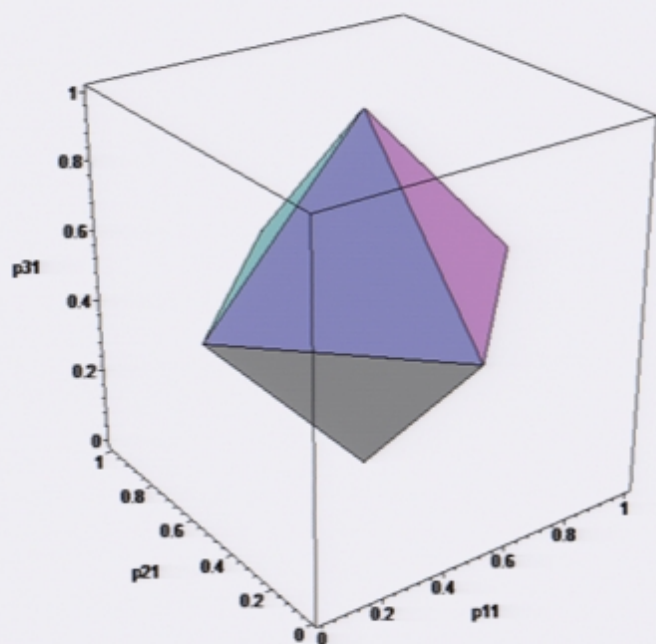
- States in C_2 **fail** to provide for universal quantum computation;
- Bravyi-Kitaev showed that many states outside C_2 are 'non-classical' in the sense of attaining universal quantum computation in this model.

Why do I call states in C_d 'classical'?

general d case

- Theorem:** for any power-of-prime Hilbert space dimension d , C_d is the convex hull of the $d(d+1)$ MUB projectors, i.e.

$$\hat{\rho} \in C_d \Leftrightarrow \hat{\rho} = \sum_{i,j} q_{i,j} |\alpha_{i,j}\rangle\langle\alpha_{i,j}|, \quad \text{with } 0 \leq q_{i,j} \leq 1 \text{ and } \sum_{i,j} q_{i,j} = 1$$



$$\hat{\rho} \in C_d \Leftrightarrow$$

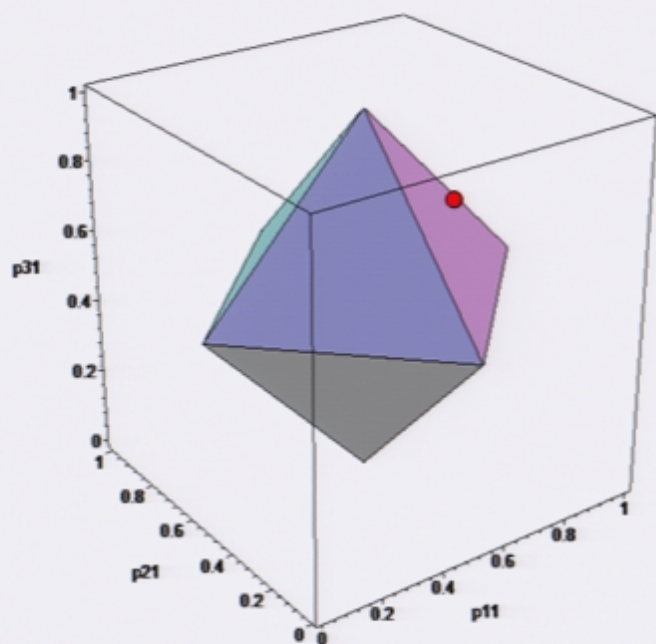
'shadows' (i.e. measurement outcomes) compatible with classical model in which **only one** MUB has sharp value. Complexity of model (in bits) = $\log_2(d)$

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$$\hat{\rho} \in C_d \Leftrightarrow$$

'shadows' (i.e. measurement outcomes) compatible with classical model in which **only one** MUB has sharp value.

$$\hat{\rho} \text{ has negative } W \Leftrightarrow$$

classical model reconstructed from 'shadows' indicates there are other states with sharp values.

Why do I call states in C_d 'classical'?

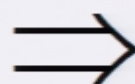
general d case

- Another reason: pure states in C_d have an efficient (size $\log(d)$) classical description using the stabilizer formalism.
- How?

Fact: MUB basis vectors can always be chosen to be stabilizer states (eigenstates of generalized Pauli operators)

- Example: 5 MUB's for 2 qubits can be defined as joint eigenstates of Paulis in each line. Efficient description: generators of the subgroup {each line, identity}, and their eigenvalues.

MUB 1	σ_z^1	σ_z^2	$\sigma_z^1 \sigma_z^2$
MUB 2	σ_x^1	σ_x^2	$\sigma_x^1 \sigma_x^2$
MUB 3	σ_y^1	σ_y^2	$\sigma_y^1 \sigma_y^2$
MUB 4	$\sigma_x^1 \sigma_z^2$	$\sigma_y^1 \sigma_x^2$	$\sigma_z^1 \sigma_y^2$
MUB 5	$\sigma_x^1 \sigma_y^2$	$\sigma_y^1 \sigma_z^2$	$\sigma_z^1 \sigma_x^2$



Negativity of W is required for pure-state universal quantum computing...

Open problems

- Apply insights from the discrete case to the continuous Wigner function $W(q,p)$, and other quasi-probability distributions. In what computational sense are non-negative $W(q,p)$ `classical`?
- Characterize unitaries (and general operations) which keep W non-negative.
- Visualize quantum computing algorithms (cf. Bianutti *et al.*, Miquel *et al.*, Paz) and other quantum information protocols.

Conclusions

- I characterized the set C_d of states with non-negative discrete Wigner function W in **all** definitions of W . The set C_d is the convex hull of a set of stabilizer states.
- States in C_d are 'classical' in many senses:
 - they can be described with non-negative quasi-probability distributions;
 - they have an efficient (size $\log(d)$) classical description;
 - one-qubit states outside of C_2 are necessary for universal quantum computation in the Bravyi-Kitaev model;
 - d -dimensional states outside of C_d are necessary for pure-state universal quantum computation.

References:

- this talk: E. F. Galvão, quant-ph/0405070; Galvão and Pittenger, in preparation.
- defining discrete Wigner functions: K. S. Gibbons, M. J. Hoffman, W. K. Wootters, quant-ph/0401155.
- mutually unbiased bases: Wootters and Fields, Ann. Phys. (N.Y.) 191(2), 363 (1989); Bandyopadhyay *et al.*, quant-ph/0103162; Lawrence *et al.* PRA 65, 032320 (2002); Pittenger and Rubin, quant-ph/0308142; Klappenecker and Roetteler, quant-ph/0309121.
- one-qubit non-classical states as a resource : Bravyi and Kitaev, quant-ph/0403025.
- visualizing quantum computing with discrete Wigner functions: Bianucci *et al.*, quant-ph/0106091; Miquel *et al.*, Nature 418, 59 (2002); Paz, PRA 65, 062311 (2002).

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E.F.G., A. O. Pittenger, in preparation

Discrete Wigner functions and quantum computational speed-up

Outline

- Phase space representation of quantum states
 - the Wigner function $W(q,p)$
- Discrete Wigner functions W
 - proposal of Gibbons-Hoffman-Walsh
- Negativity of W and quantum computation
 - in the model of Shor and Kitaev
 - in the usual pure state model
- Open problems

Classical phase space

- (q,p) as position and momentum of the particle
- a classical ensemble is described by a probability density - the Liouville distribution $\rho(q,p)$ in phase space
- Integrals over phase space give average properties of the ensemble

Quantum phase space and the Wigner function $W(q,p)$

- a quantum state doesn't have (q,p) simultaneously well defined
- yet we can define a probability distribution $W(q,p)$ in phase space with useful properties
- $W(q,p)$ can assume negative values hence the term *quasi-probability*

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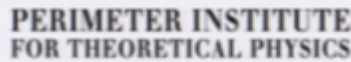
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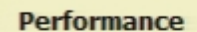
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- (q,p) as position and momentum of the particle
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Quantum phase space and the Wigner function $W(q,p)$

- a quantum state doesn't have (q,p) (it's not a classical state)
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Shut down

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