Title: SU(N) Yang-Mills Thermodynamics and its Cosmological Consequences

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Abstract:

Analytical and nonperturbative approach to SU(2) / SU(3) Yang-Mills thermodynamics

CTP, M.I.T., 1 October 2004
[hep-ph/0404265]

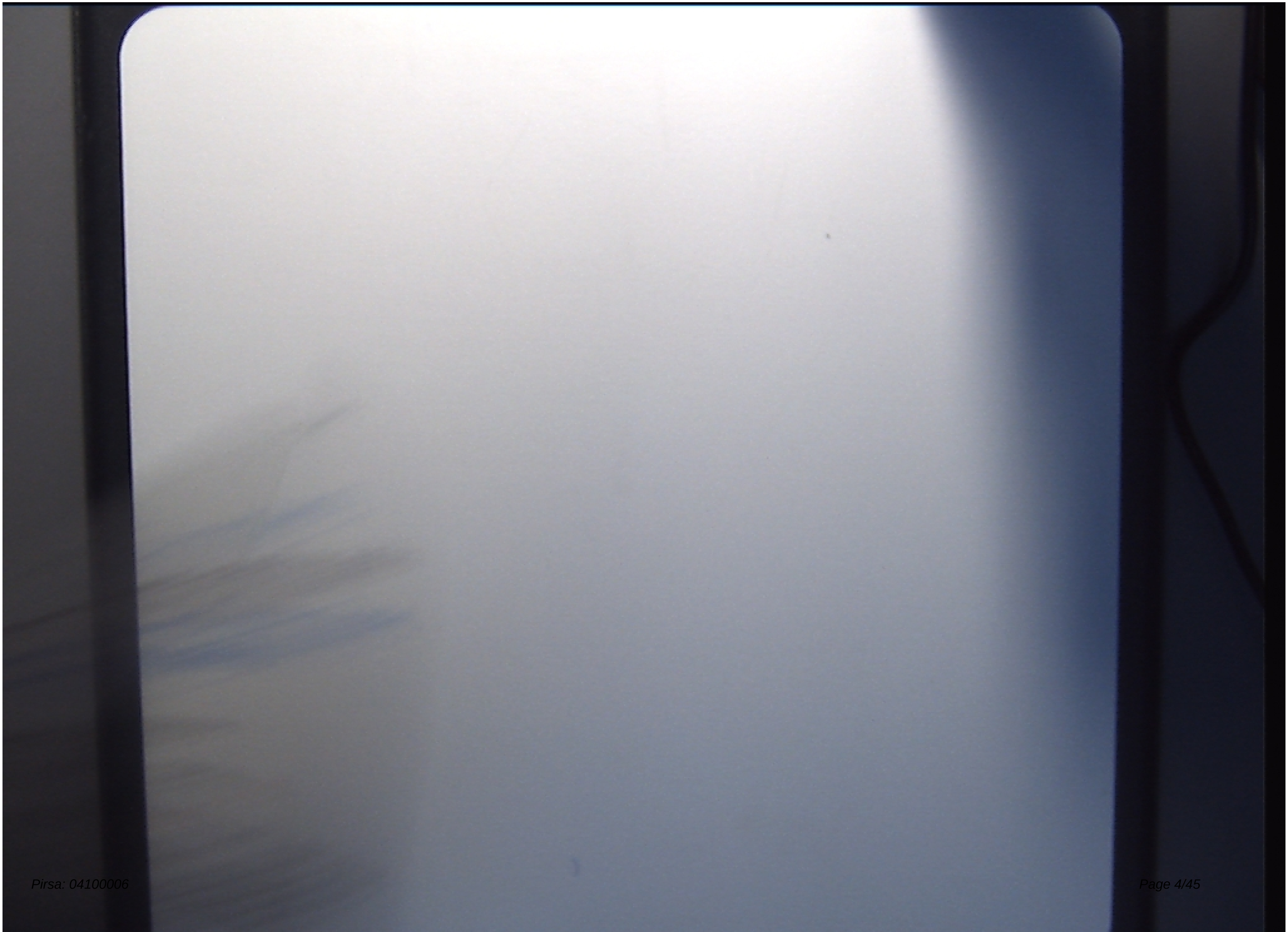
„If an idea does not appear absurd at first then there is no hope for it."
— Albert Einstein

R. Hofmann
Universität Heidelberg/Frankfurt

①

Outline

- motivation
- preview on main results
- construction for SU(2), T↘
  - electric phase (deconfining)
  - magnetic phase ('preconfining')
  - center phase (confining)
- thermodynamical quantities
  - pressure
  - energy density
  - entropy density
- conclusions



③

Motivation

experimental:

- RHIC : hydrodynamics of elliptic flow

  ⇒ QP most perfect fluid in Nature

  $\eta/s \ll 1$

  [Shuryak 2003]

- success of thermal models for particle yields

  [Braun-Munzinger, Cleymans, Stachel, Redlich 1995-2004]

- cosmological expansion:

  Hubble expansion vs.
  expansion of fire ball in early stage of heavy-ion collision ?

  [WMap 2003, Shuryak, Kolb, Teany 2003]

(4) Motivation, cntd...

theoretical:

- thermal perturbation theory:
  - breaks down at g^6 (weakly screened magnetic gluons)

    [Linde 1980]

- resummation attempts have various pathologies

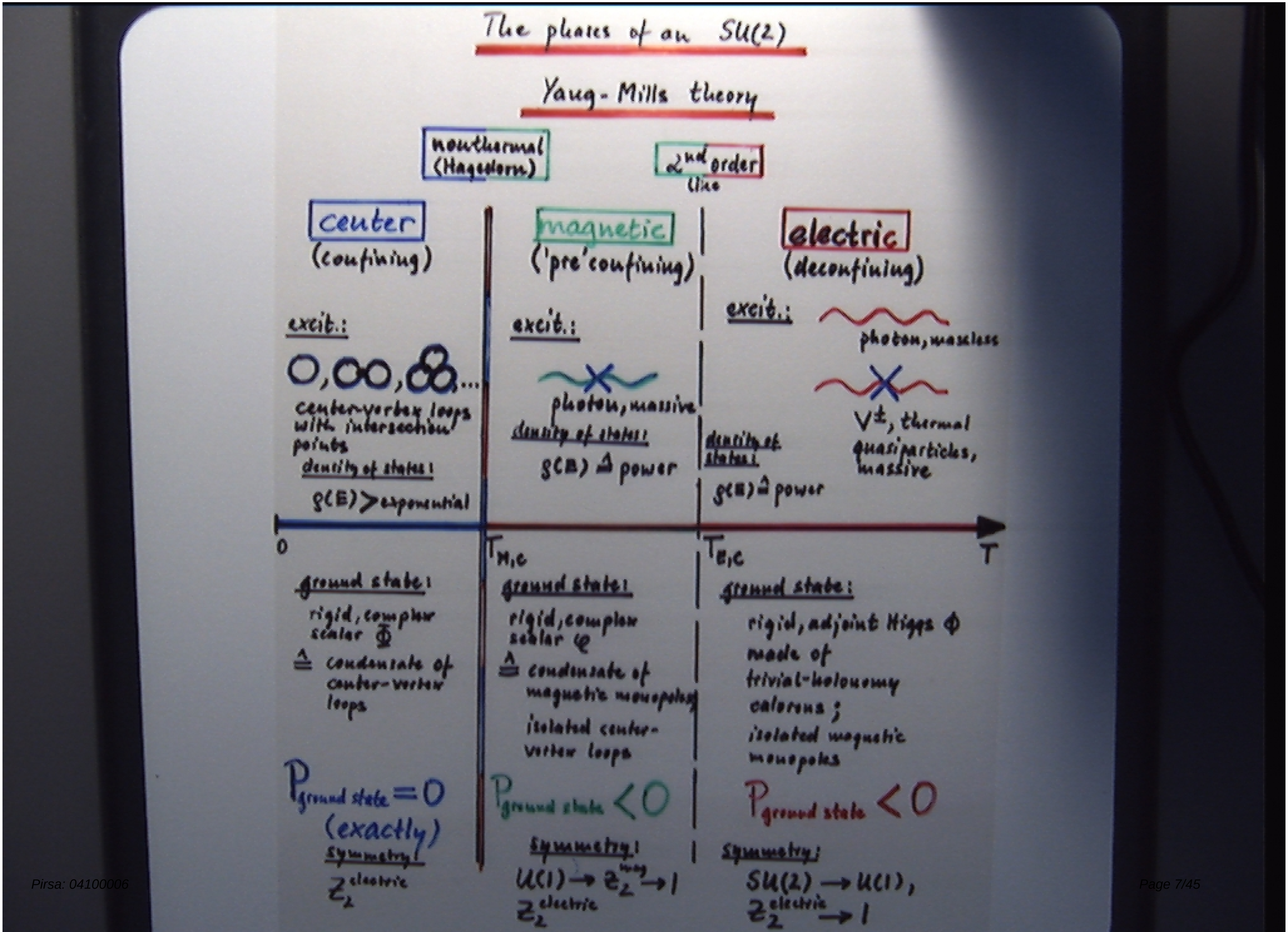
  [Braaten, Pisarski, Rebhan, Blaizot, 1992 – present]

- lattice:

  for $T \gg \Lambda \Rightarrow$

  $\sqrt{\sigma_{spat.}} \sim T$

  [Hart, Philipsen 1999,


The phases of an SU(2)
Yang-Mills theory
nonthermal (Hagedorn)
2nd order like
center (confining)
magnetic ('pre'confining)
electric (deconfining)
excit.:
center-vortex loops with intersection points
density of states:
ρ(E) > exponential
excit.:
photon, massive
density of states:
ρ(E) ≙ power
excit.:
photon, massless
V±, thermal quasiparticles, massive
density of states:
ρ(E) ≙ power
0
T_{M,c}
T_{E,c}
T
ground state:
rigid, complex scalar Φ
≙ condensate of center-vortex loops
ground state:
rigid, complex scalar φ
≙ condensate of magnetic monopoles
isolated center-vortex loops
ground state:
rigid, adjoint Higgs φ
made of trivial-holonomy calorons;
isolated magnetic monopoles
P_{ground state} = 0 (exactly)
P_{ground state} < 0
P_{ground state} < 0
Symmetry:
Z_2^{electric}
Symmetry:
U(1) → Z_2^{mag} → 1
Z_2^{electric}
Symmetry:
SU(2) → U(1),
Z_2^{electric} → 1

assumption:

SU(2)/SU(3) YMTD, condenses 'n.i. trivial-holonomy calorons (embedded) into macroscopic, adjoint Higgs field ϕ^a at $T \gg \Lambda_{YM}$.

subject to proof in terms of gap equation:

$$\phi^a |\phi|^{-1} = \mathrm{tr} \left\langle \int_0^{|x|=|\phi|^{-1}} d|x|' \, F_{\mu\nu}(x') [x',0] \, t^a \, F_{\mu\nu}(0) [0,x'] \right\rangle_{\text{zero modes, t.h. caloron}}$$

⑤

What are calorons?

- $\boxed{F_{\mu\nu} = \pm \tilde{F}_{\mu\nu}}$ on $A_\mu^{cal.}$

  $(0 \le \tau \le \frac{1}{T})$

$\Rightarrow \quad \Theta_{\mu\nu}[A_\mu^{cal.}] = 0$

trivial holonomy	nontrivial holonomy
(no magn. monoples)	(magnetic monopoles)
$\mathcal{P}(\lvert\vec{x}\rvert \to \infty) = 1$	$\mathcal{P}(\lvert\vec{x}\rvert \to \infty) \neq 1$
⇓	⇓
$S_{eff}^{1\text{-loop}} \sim \frac{8\pi^2}{g^2} + \frac{2}{3} N (\pi g T)^2$	$S_{eff}^{1\text{-loop}} \sim T^3 V$
[Gross, Pisarski, Yaffe 1981]	[Diakonov et al. 2004]
stable	unstable

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[Gross, Pisarski, Yaffe 1981]	[Diakonov et al. 2004]
stable	unstable

nontrivial holonomy

- classical solution -

Figure 1

action density on a 2D spatial slice

③

The concept

borrow from:

macroscopic theory for superconductivity
[Ginzburg, Landau, Abrikosov 1950's]

- fundamental U(1) gauge field A_μ:

A_μ + A_μ^C

res. fluct.

A_μ^C → existence of Cooper pairs

- $\phi \neq 0$ by a potential [U(1) → 1]
- pure gauge solution $a_\mu^{b.g.}$

} ground state

- compute mass for fluctuations δa_μ in $a_\mu = a_\mu^{b.g.} + \delta a_\mu$

thermodynamics below T_c ⟹ self-consistent T-dependence of parameters

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$\xrightarrow{\text{thermodynamics below } T_c}$ self-consistent T-dependence of parameters

④

The road ahead ...

- find potential $V_E = \mathrm{tr}\, v_E^\dagger v_E$ such that

  1) solut to BPS equation

     v_E (energy free!)

     with $\phi(\tau=0) = \phi(\tau=\frac{1}{T})$ (periodic)

     exists, with $|\phi| = |\phi|(T)$ (g.s.)

- 2) take interactions of t.h. calorons into account macroscopically by solving

  $$D_\mu G_{\mu\nu} = 2ie[\phi, D_\nu \phi]$$

  generation of macr. holonomy → with $G_{\mu\nu} = D_\nu \phi = 0$ (g.s.)

- 3) make sure that ϕ does not fluctuate

- find potential $V_B = \mathrm{tr}\, v_B^\dagger v_E$ such that
  1) solution to BPS equation

$$\boxed{\partial_\tau \phi = v_E} \quad \text{(energy free!)}$$

with $\boxed{\phi(\tau=0) = \phi(\tau=\frac{1}{T})}$ (periodic)

with $\boxed{|\phi| = |\phi|(T)}$ (g.s.)

- 2) take interactions of t.l. calorons into account macroscopically by solving

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- 3) make sure that ϕ does not fluctuate
- 4) compute $a_\mu^{\text{triv.}}$ spectrum in unitary gauge
- 5) impose thermodynamical

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- 3) make sure that ϕ does not fluctuate

- 4) compute $a_\mu^{\text{triv.}}$ spectrum in unitary gauge

- 5) impose thermodynamical self-consistency

(6)

Construction for SU(2)

electric phase: $SU(2) \rightarrow U(1)$

$Z_2^e \rightarrow 1$

- $S_E = \int_0^{1/T} d\tau \int d^3x \left\{ \frac{1}{2} \mathrm{tr}\, G_{\mu\nu} G_{\mu\nu} + \mathrm{tr}(D_\mu \phi)^2 + V_E \right\}$

  $D_\mu \phi = \partial_\mu \phi + ie[\phi, a_\mu]$ (effective coupl.)

- V_E uniquely determined as: (up to global g.t.)

  $V_E = \Lambda_E^6\, \mathrm{tr}(\phi^2)^{-1} = \mathrm{tr}\, v_E^\dagger v_E$ with $v_E = i\Lambda_E^3 \lambda_1 \frac{\phi}{|\phi|^2}$

  (Λ_E: scale), $|\phi|^2 \equiv \frac{1}{2}\mathrm{tr}\,\phi^2$, λ_i ... Pauli matr. $(i=1,2,3)$

- Solution: $\partial_\tau \phi = v_E$ (ϕ pressure-free, energy-free):

  $$\phi_e(\tau) = \sqrt{\frac{\Lambda_E^3}{2\pi T |e|}}\, \lambda_3 \exp[-2\pi i T e \lambda_1 \tau] \quad (e \neq 0, e \in \mathbb{Z})$$

  $$\Rightarrow |\phi_e| = |\phi_e|(T) = \sqrt{\frac{\Lambda_E^3}{2\pi T |e|}}, \quad \phi(0) = \phi(\tfrac{1}{T})$$

  ↳ compositeness scale

- Does ϕ_e fluctuate?

  q.m.: $\frac{\partial^2_{|\phi|} V_E \big|_{\phi=\phi_e}}{|\phi_e|^2} = 3e^3 \lambda_E^3 \gg 1 \Rightarrow$ No!

  [since $\lambda_E \gg 1$ (later)]

  th.d.: $\partial^2_{|\phi|} V_E \big|_{\phi=\phi_e}$ ...

SU(2)

electric phase: $SU(2) \to U(1)$

$Z_2^e \to 1$

- $$S_E = \int_0^{1/T} d\tau \int d^3x \left\{ \frac{1}{2} \mathrm{tr}\, G_{\mu\nu} G_{\mu\nu} + \mathrm{tr}(D_\mu \phi)^2 + V_E \right\}$$
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  $$V_E = \Lambda_E^6 \, \mathrm{tr}(\phi^2)^{-1} = \mathrm{tr}\, v_E^\dagger v_E$$
  (scale) with $v_E = i \Lambda_E^3 \lambda_1 \frac{\phi}{|\phi|^2}$
  and $|\phi|^2 \equiv \frac{1}{2} \mathrm{tr}\, \phi^2$, λ_i ... Pauli matr. $(i=1,2,3)$

- Solutions to [illegible] v_E (ϕ pressure-free, energy-free):
  $$\phi_e(\tau) = \frac{[illegible]}{[illegible]\pi T|e|} \lambda_3 \exp[-2\pi i T e \lambda_1 \tau] \quad (e \neq 0, e \in \mathbb{Z})$$
  $$\Rightarrow |\phi_e| = |\phi_e|(T) = \sqrt{\frac{\Lambda_E^3}{2\pi T|e|}}, \quad \phi(0) = \phi\left(\frac{1}{T}\right)$$
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  [since $\lambda_E \gg 1$ (later)]

  th.d.: $$\frac{\partial^2_{|\phi|} V_E \big|_{\phi=\phi_e}}{T^2} = 12\pi^2 e^2 \gg 1 \Rightarrow \text{No!}$$

⑦ • $D_\tau \phi_\ell = G_{0i} = 0$ on:

$$a^{b.g.}_{\mu,\ell} = \frac{\pi}{e} T \delta_{\mu 0} \quad \ell \cdot \lambda_1$$

⇒ g.s. energy density (− g.s. pressure) lifted from [zero] to

$$V_E^\ell = 4\pi |\ell| \Lambda_E^3 \cdot T \overset{\ell=1}{=} 4\pi \Lambda_E^3 \cdot T$$

(domain boundaries of $\phi[2]$, isolated magnetic monopoles)

• now: $\mathcal{P}[a_{\mu,i}] \equiv P e^{ie \int_0^{1/T} d\tau\, a^{b.g.}_{0,1}} = -1$ (winding gauge)

to compute mass spectrum for $a_\mu^{trivial}$ excitations:

Go to unitary gauge $\tilde{a}^{b.g.}_{0,1} = 0$!

– involves singular periodic or regular nonperiodic g.t.:

$$\Omega_1 = e^{i\theta} \quad \text{with } \theta = -\pi \lambda_1 T \tau$$

⇒ $\mathcal{P}[\tilde{a}^{b.g.}_{\mu,1}] = +1$ (different vac.)

however: periodicity of fluctuations

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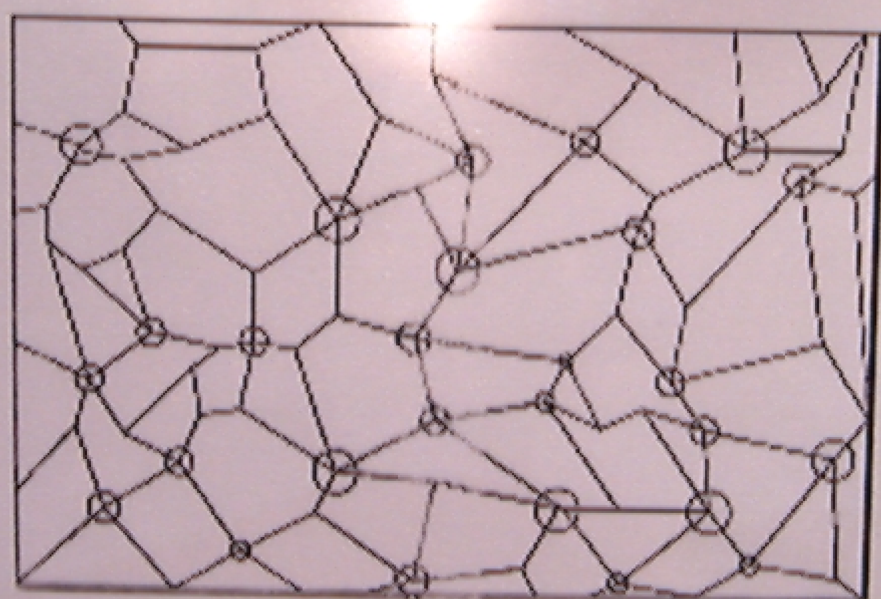
however: periodicity of fluctuations $a_\mu^{trivial}$ not affected!

snapshot of SU(2) YM ground state at $T \gg \Lambda_E$

domains and magnetic monopoles

different color orientations of ϕ form domain boundaries at which nontrivial-holonomy calorons [illegible] and dissociate (Diakonov et al, 2004)



$$a^{b.g.}_{\mu,l} = \frac{\pi}{e} T \delta_{\mu 0} \; l \cdot \lambda_1$$

⇒ g.s. energy density (− g.s. pressure) lifted from zero to

$$V_E^l = 4\pi |l| \Lambda_E^3 \cdot T \overset{l=1}{=} 4\pi \Lambda_E^3 \cdot T$$

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$$a^{b.g.}_{\mu,\ell} = \frac{\pi}{e} T \delta_{\mu 0} \quad \ell \cdot \lambda_1$$

$\Rightarrow$ g.s. energy density (− g.s. pressure) lifted from zero to

$$V_E^{\ell} = 4\pi |\ell| \Lambda_E^3 \cdot T \overset{\ell=1}{=} 4\pi \Lambda_E^3 \cdot T$$

(domain boundaries, isolated magnetic monopoles)
$\Phi[1] + \Phi[2]$

- now: $\mathcal{P}[a^{b.g.}] \equiv \mathcal{P} e^{ie \int_0^{1/T} d\tau\, a^{b.g.}_{0,1}} = -1$

(winding gauge)

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⑧ • mass spectrum:

$$m_{W^\pm} = 2e\sqrt{\frac{\Lambda_E^3}{2\pi T}}\,, \quad m_{a_0} = 0$$

• pressure (1-loop): $\bigcirc + 2\bigcirc + \times$

$$P(\lambda_E) = -\Lambda_E^4 \left[\frac{2\lambda_E^4}{(2\pi)^6}\left\{2\,\bar{P}(0) + 6\,P(a)\right\} + 2\lambda_E \right]$$

$$a \equiv \frac{m_W}{T}\,, \quad \lambda_E \equiv \frac{2\pi T}{\Lambda_E}$$

$$(a) \equiv \int_0^\infty dx\, x^2 \log\left[1 - \exp\left(-\sqrt{x^2+a^2}\right)\right]$$

(quantum fluctuations negligible)

• thermodynamical self-consistency:

$$\partial_a P \equiv 0$$

$$\Rightarrow \quad \partial_a \lambda_E = -\frac{24\,\lambda_E^4\, a}{(2\pi)^6}\, D(2a)$$

$$D(a) \equiv \int_0^\infty dx\, \frac{x^2}{\sqrt{x^2+a^2}}\, \frac{1}{\exp\sqrt{x^2+a^2} - 1}$$

→ fixed points at $a = 0, \infty$.

(8) • mass spectrum:

$$m_{W^\pm} = 2e\sqrt{\frac{\Lambda_E^3}{2\pi T}}\,, \quad m_{Z_0} = 0$$

• pressure (1-loop): ○ + 2○ + ×

$$P(\lambda_E) = -\Lambda_E^4\left[\frac{2\lambda_E^4}{(2\pi)^6}\left\{2\bar{P}(0) + 6P(a)\right\} + 2\lambda_E\right]$$

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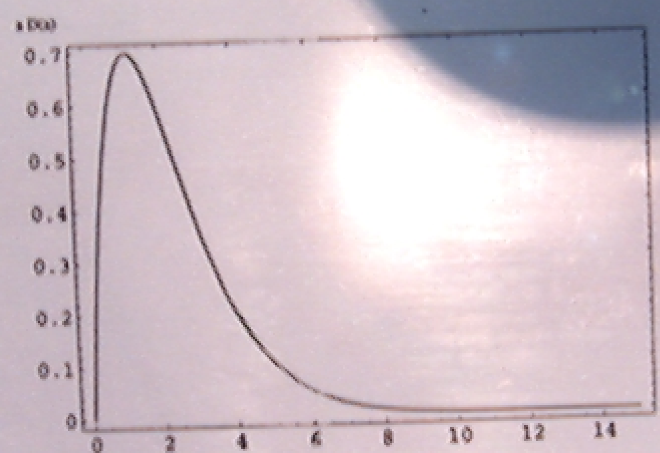
↳ fixed points at $a = 0, \infty$.

$\Rightarrow$ highest ($a=0$) and lowest ($a=\infty$)

⑨



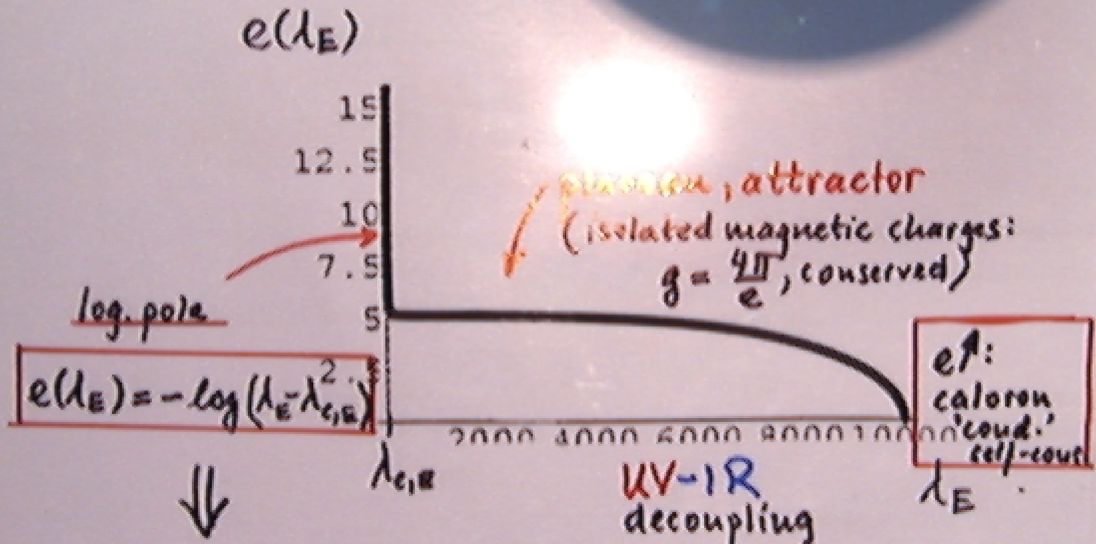
$a = 0$
(highest T)

Figure 3:

$a = \infty$
(lowest T)

⑩

Evolution of effective gauge coupling e



$\Downarrow$

- $m_{mon} \sim \frac{|\phi|(\lambda_E)}{e(\lambda_E)} \longrightarrow 0$ (monopole condensation)
- $m_{W\pm} = 2e|\phi|(\lambda_E) \longrightarrow \infty$ (thermal decoupl.)

$\Rightarrow$ $U(1) \longrightarrow 1$ (2^{nd} order)

$\Delta P^{2\text{-loop}} =$

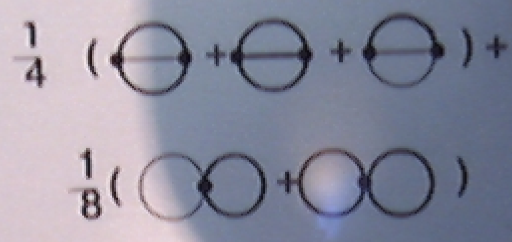


Figure 7:

subject to co… leness constraints

(i) p_e^2 … $|\phi|^2$

(ii) $(p_e + q_e)^2 \leq |\phi|^2$

(in unitary gauge)

$$\Rightarrow \boxed{\frac{\Delta P^{2\text{-loop}}}{\Delta P^{1\text{-loop}}} \leq 0.1\%}$$

$\Rightarrow$ practically noninteracting quasiparticles

(12)

Magnetic phase

$$\frac{q}{|e|} = \left\langle \oint d\Sigma_{ij} F_{ij}^{ext} \right\rangle$$

- same philosophy as in electric phase but now with an Abelian Higgs model

- one can show that $\langle \mathcal{P} \rangle = 0$

  $\Rightarrow$ con[illegible] fundamental test charges

- evolution equation for magnetic coupling

$$\partial_a \lambda_M = -\frac{12}{(2\pi)^6} \lambda_M^4 \, a \, D(a)$$

$\Rightarrow$ highest and lowest T

$\Rightarrow$ 'photon' mass is order parameter

$\left[\lambda_M = \frac{2\pi T}{\ldots} \right]$

$g(\lambda_M) = -\log(\lambda_M - \lambda_{M,c})$, center trans.

$[S_{ANO} \sim \frac{1}{g^2}]$

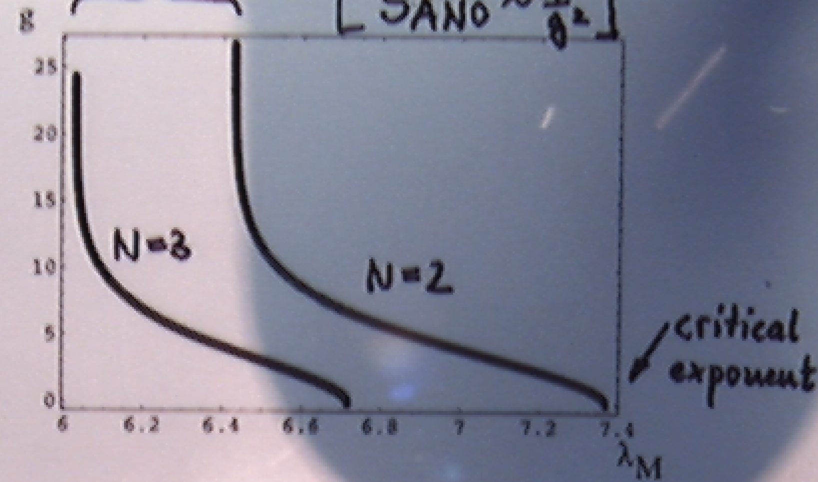


Figure 8

- typical correlation length

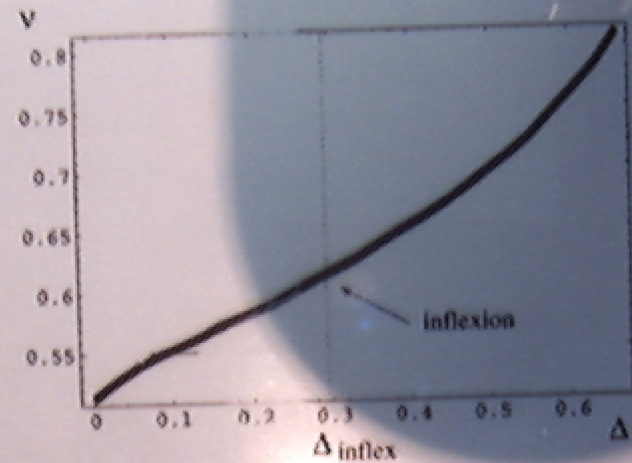
$$\ell_{cor} \approx g_{dec} \Lambda_{YM}^{-1},$$

since $g_{dec} \gg 1$

⇒ infrared sensitive quantities not measurable on lattice in magnetic phase

(14)

Critical exponent for SU(2) (electric-magnetic trans.)



$$\nu = 0.61 + 0.02 - 0.01$$

(Ising 3D: $\nu \sim 0.63$)

[2 inflexion points for SU(3) !!]

(15)

Center phase

- $$\Phi(x)\,|\Phi(x)|^{-1} = \left\langle \exp\left[ig \oint_{R=|\Phi|^{-1}} dz_\mu A^D_\mu\right]\right\rangle$$

- potential [illegible] $\Phi(x)$:

$$V_c = \bar{v}_c v_c \quad \text{where}$$

$$v_c = i\left(\frac{\Lambda_c^3}{\Phi} - \Phi \Lambda_c\right)$$

↑ no global $U(1)$, $Z_{2,\text{mag}} \to 1$

- BPS equation

$$\partial_\tau \Phi = \bar{v}_c(\bar{\Phi})$$

⑮

Center phase

- $$\Phi(x)\,|\Phi(x)|^{-1} = \left\langle \exp\left[ig \oint_{R=|\Phi|^{-1}} dz_\mu\, A_\mu^D\right]\right\rangle$$

- potential for $\Phi(x)$:

$$V_c = \text{[illegible]} \quad \text{where}$$

$$v_c = i\left(\frac{\Lambda_c^3}{\Phi} - \Phi\Lambda_c\right)$$

  ↑ no global $U(1)$, $Z_{2,mag} \to 1$

- BPS equation

$$\partial_{\bar{z}}\Phi = \bar{v}_c(\bar{\Phi})$$

⑯

Solutions

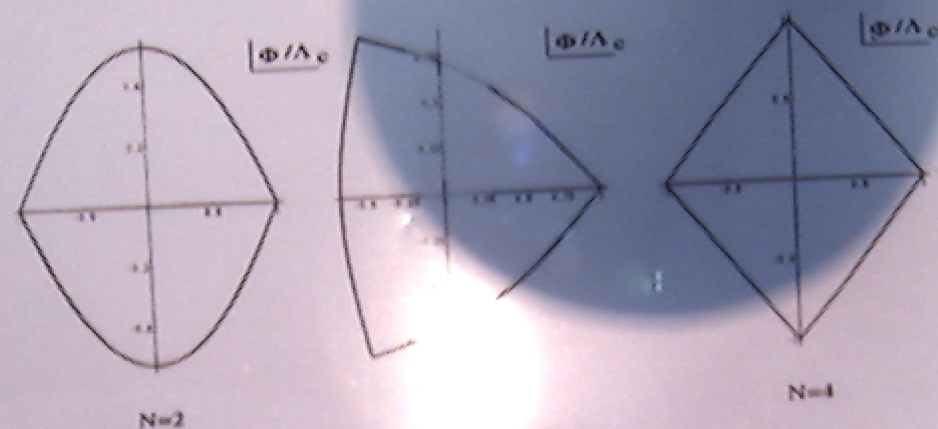


Figure 14:

- periodic <u>but</u>
- $|\Phi| = f(\tau)$

$\Rightarrow$ violation of thermal equilibrium

(17)

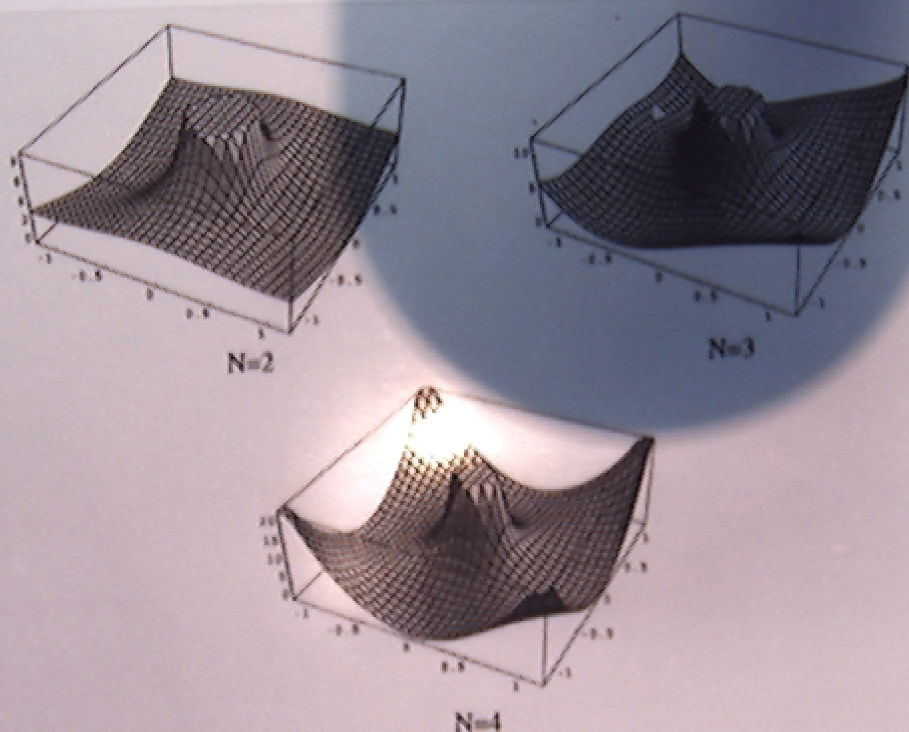
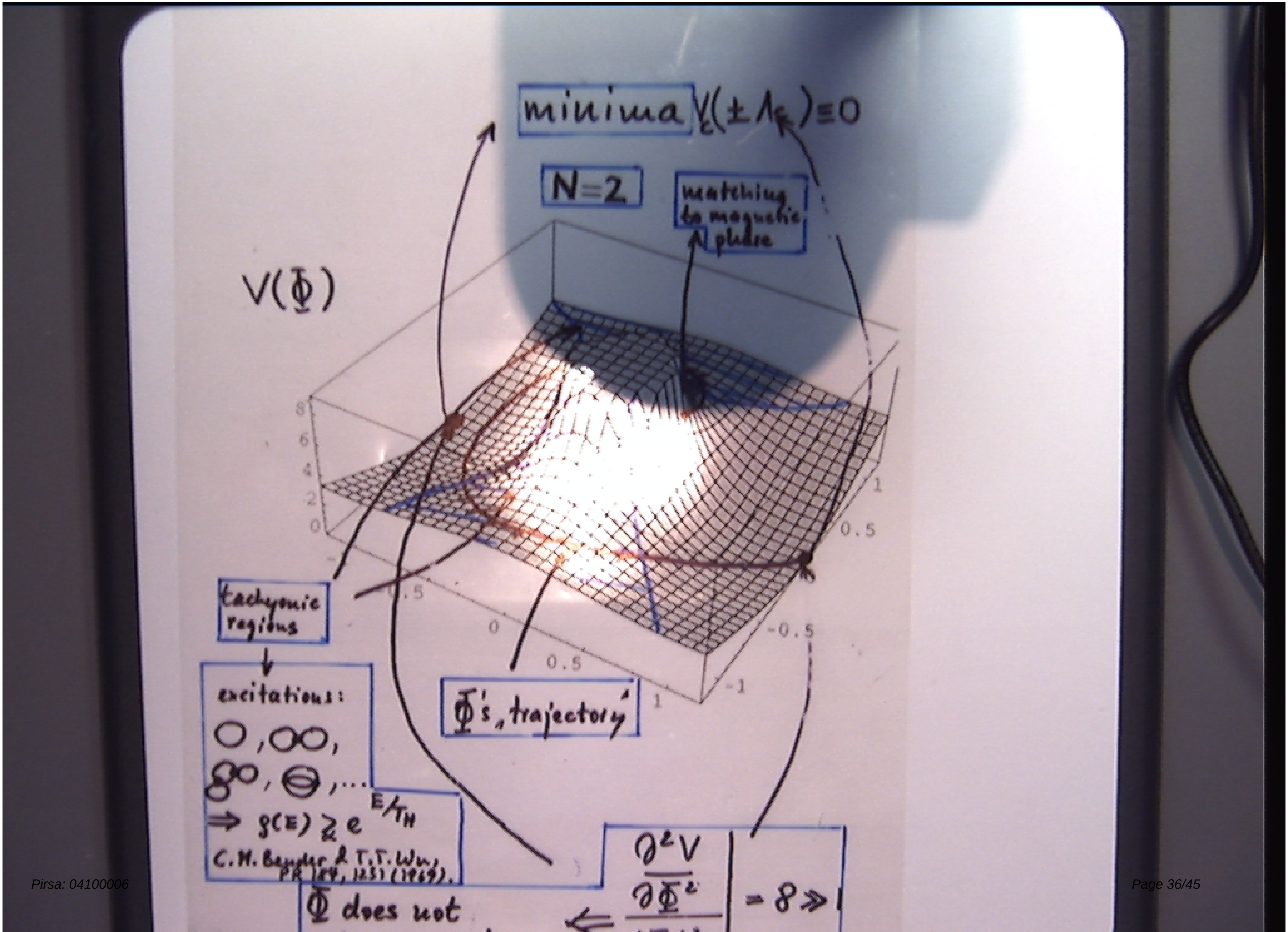
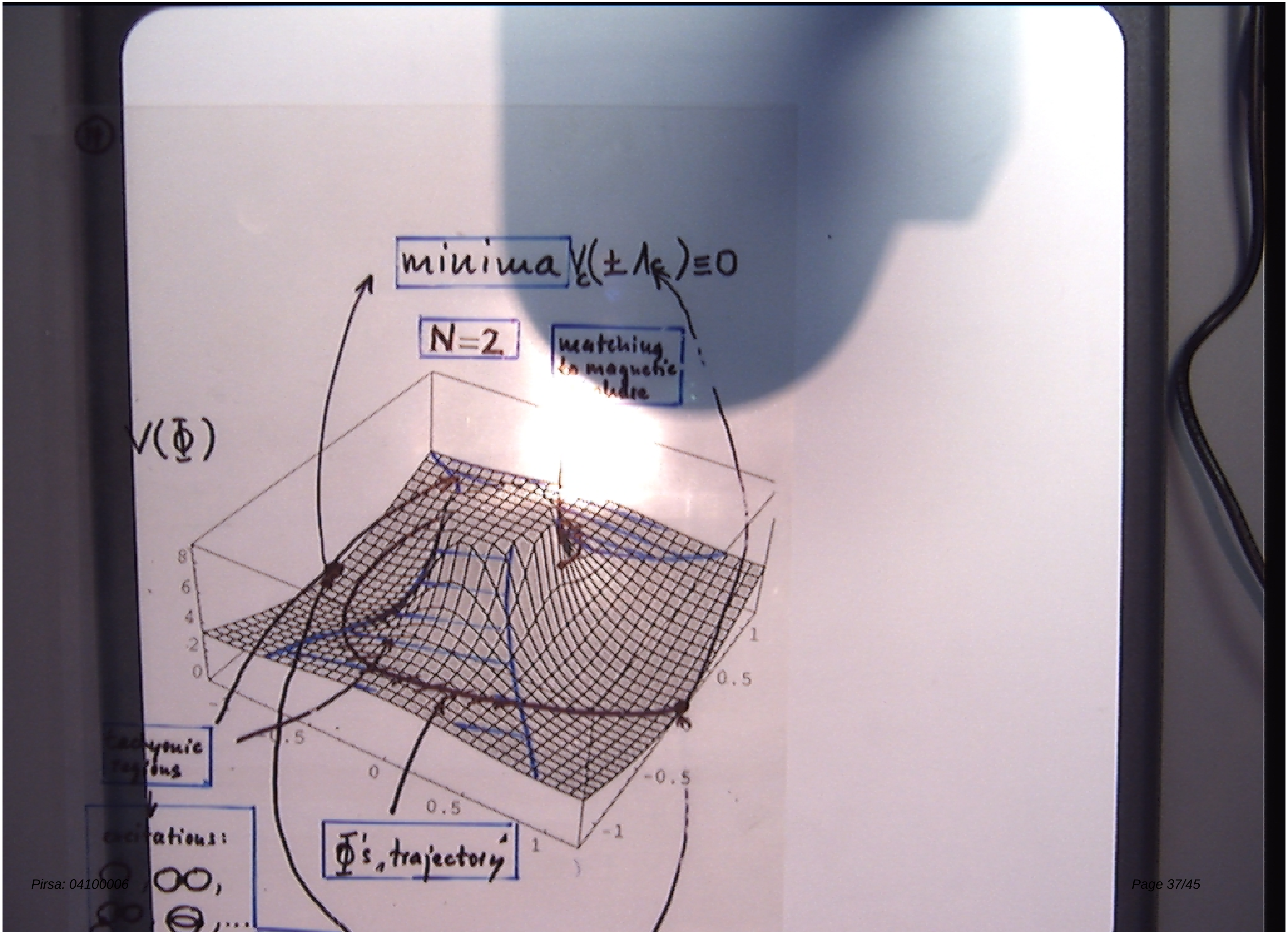


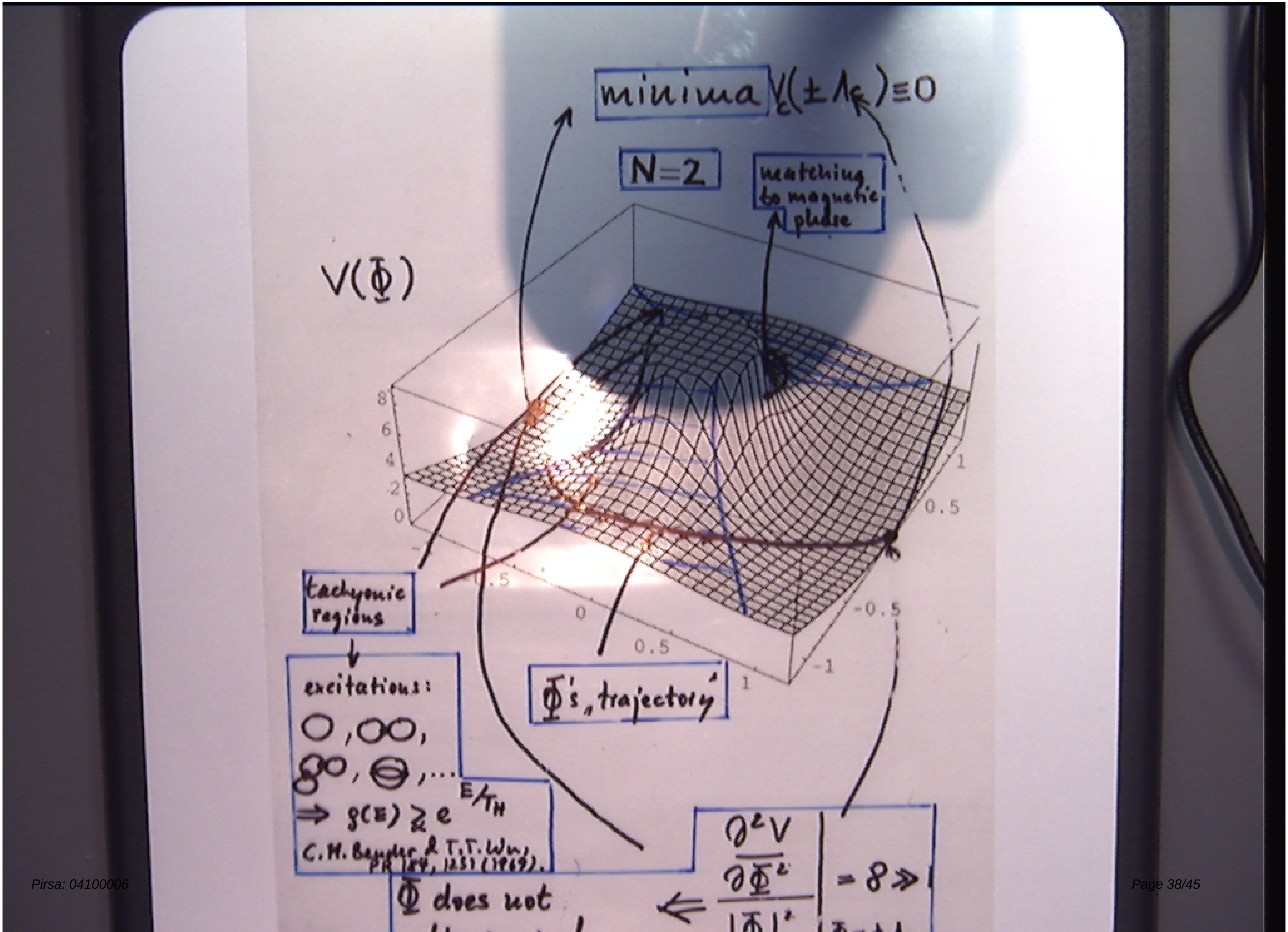
Figure 13:

Indeed: tachyonic,
tangential modes


minima $V_c(\pm\Lambda_c)\equiv 0$
N=2
matching to magnetic phase
$V(\Phi)$
tachyonic regions
excitations:
$\Rightarrow \varrho(E) \gtrsim e^{E/T_H}$
C. M. Bender & T. T. Wu, PR 184, 1231 (1969).
Φ's „trajectory"
$\frac{\partial^2 V}{\partial \Phi^2}$
$= 8 \gg 1$
Φ does not



minima
$V_c(\pm\Lambda_c)\equiv 0$
N=2
matching to magnetic solution
V(Φ)
Φ's „trajectory"
excitations:

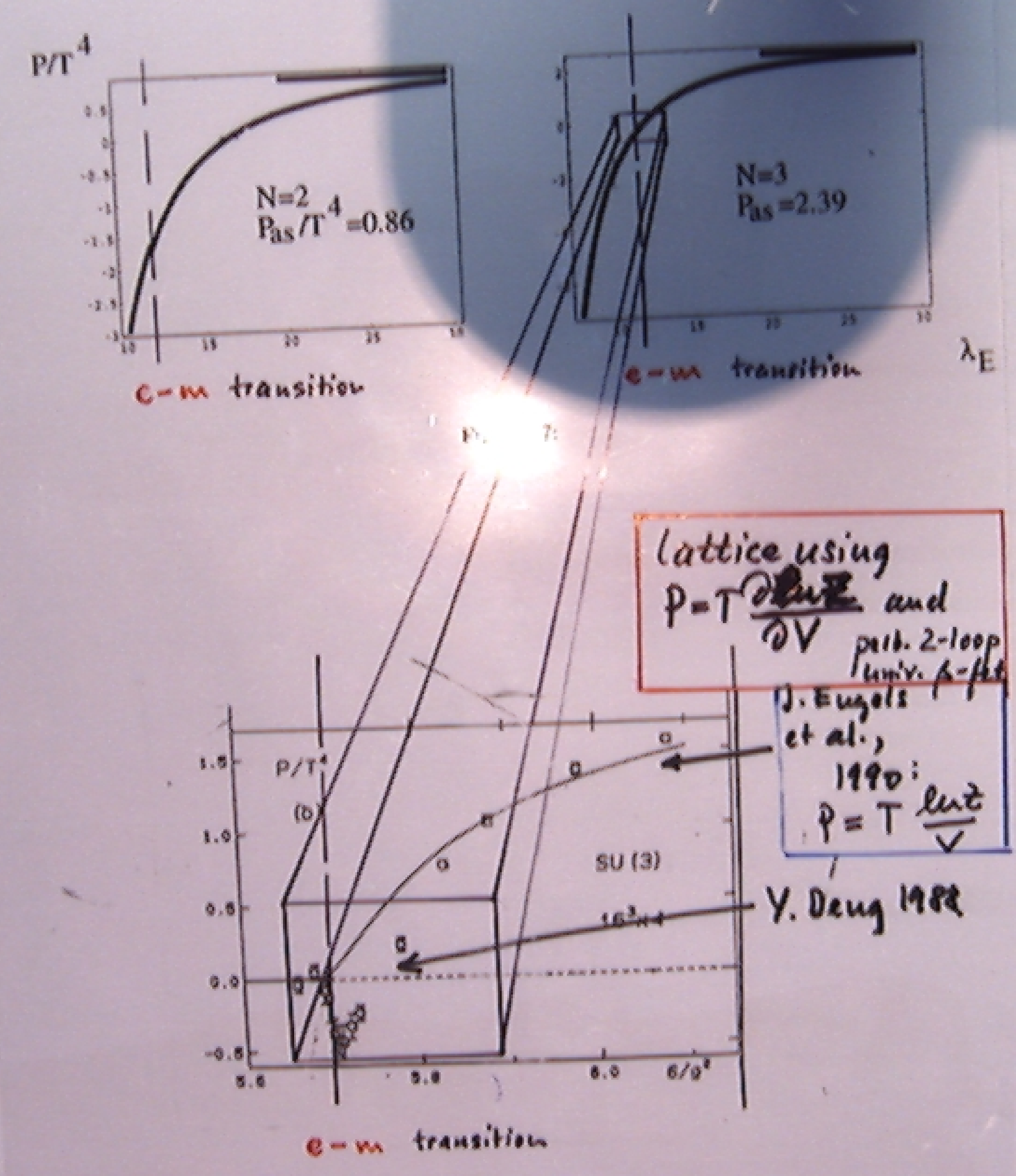

minima $V_c(\pm\Lambda_c)\equiv 0$
N=2
matching to magnetic phase
V(Φ)
tachyonic regions
excitations:
⇒ g(E) ≳ e^{E/T_H}
C.M. Bender & T.T. Wu, PR 184, 1231 (1969).
Φ's „trajectory"
Φ does not
∂²V/∂Φ² / |Φ|² = 8 ≫ 1

(15)

T-dependence of thermodynamical quantities in electric and magnetic phases

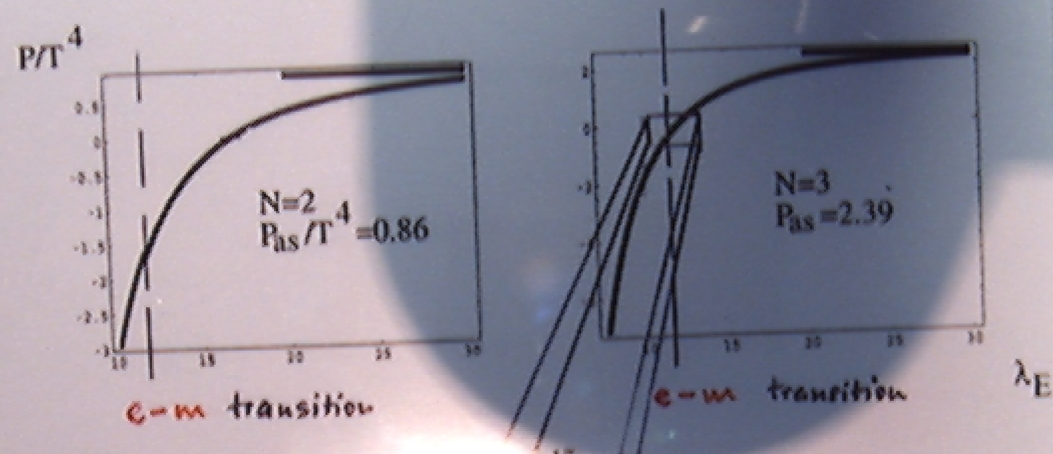
(16)

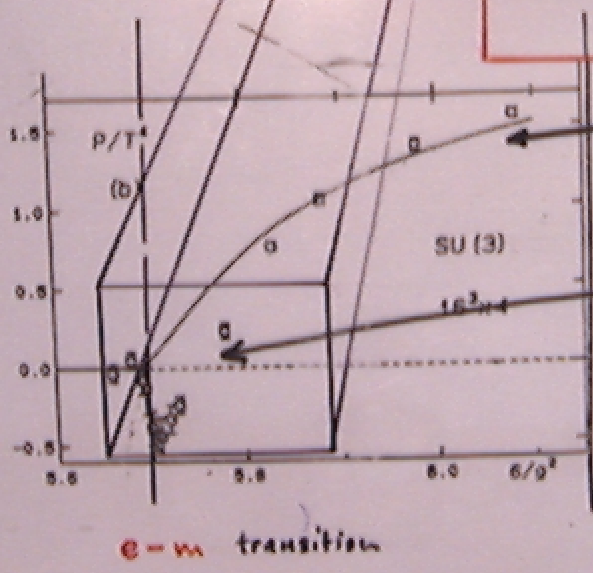
Pressure


P/T^4
N=2
Pas/T^4 =0.86
e-m transition
N=3
Pas =2.39
λE
e-m transition
lattice using
p = T ∂lnZ/∂V and
pert. 2-loop
univ. β-fct
J. Engels
et al.,
1990:
p = T lnZ/V
Y. Deng 1988
P/T^4
(b)
SU (3)
6/g^2
e-m transition

⑯

Pressure





lattice using $P = T\frac{\partial \ln Z}{\partial V}$ and pert. 2-loop univ. β-fct

J. Engels et al., 1990: $P = T\frac{\ln Z}{V}$

Y. Deng 1988

17

(16)

Energy density

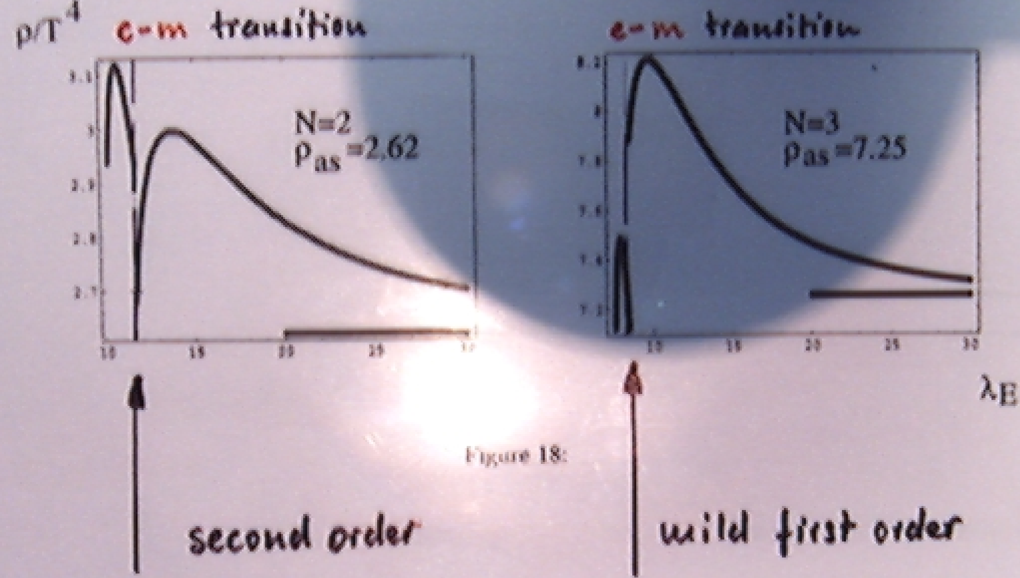


Figure 18:

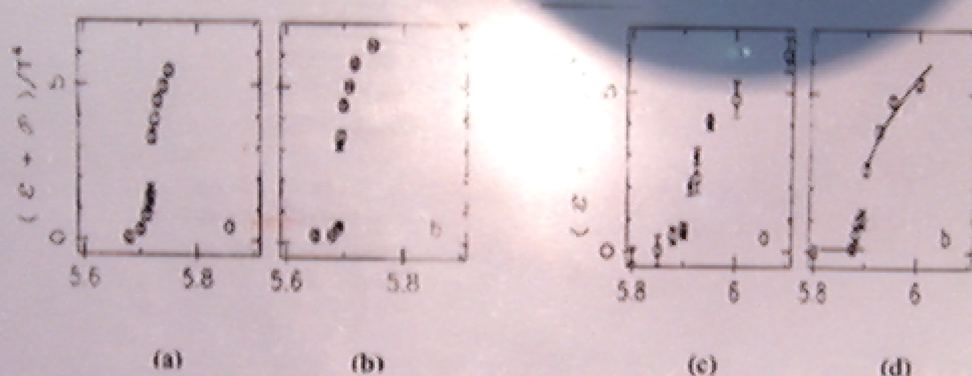
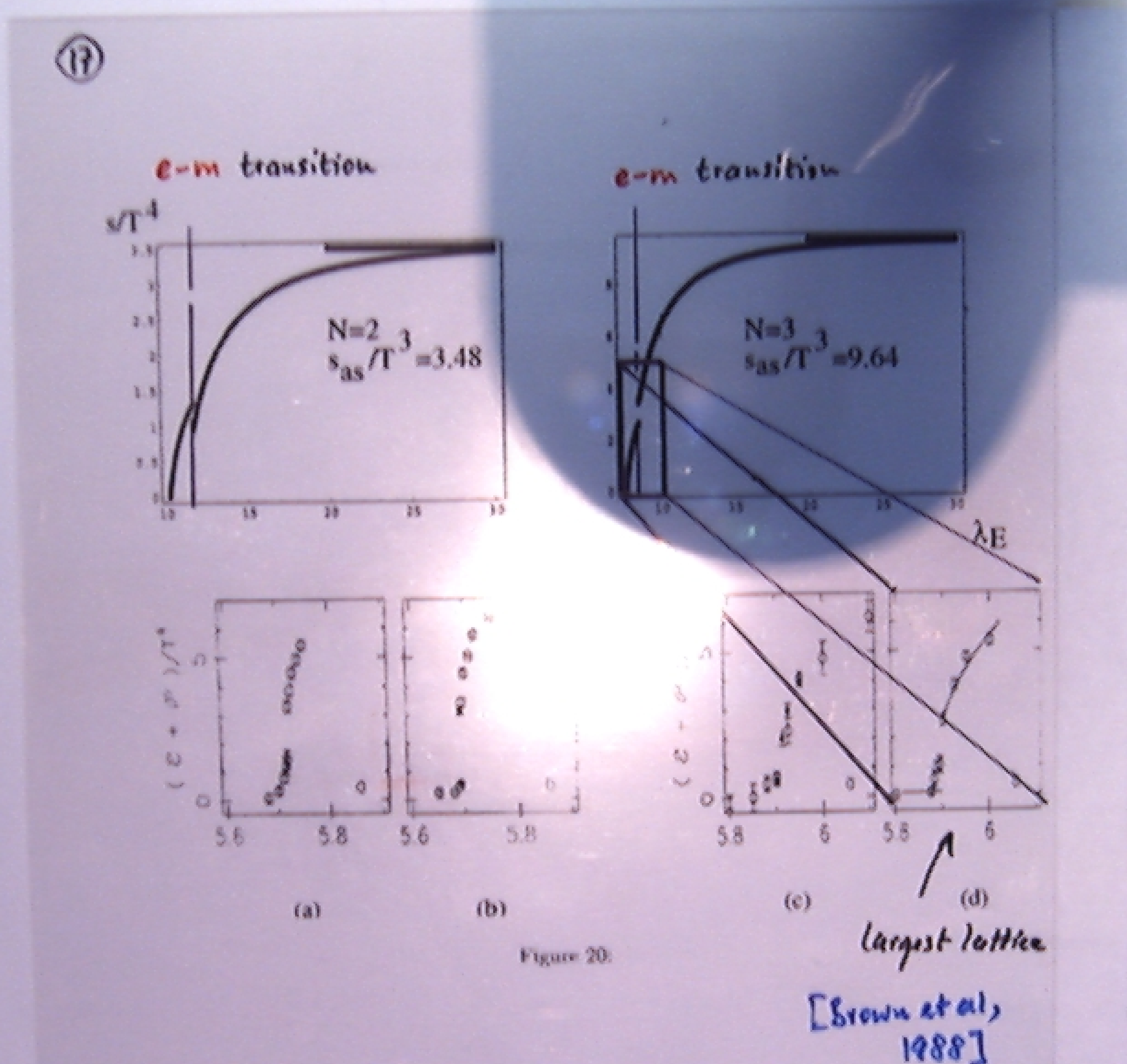


Figure 20:



17
e-m transition
e-m transition
s/T^4
N=2
s_as/T^3=3.48
N=3
s_as/T^3=9.64
λE
5.6
5.8
5.6
5.8
5.8
6
5.8
6
(a)
(b)
(c)
(d)
largest lattice
[Brown et al, 1988]

Figure 20:

18

Summary & Conclusions

- thermodynamical approach to strong interactions fruitful [Hagedorn]

- if gap equation can be solved
  $\longrightarrow$ SU(N) YM solved thermodyn.

- strong indications that macroscopic approach correct

- - lattice can measure infrared sensitive quantities only down to
  $\boxed{T_{E,c}}$ [beyond that: $\ell \sim \ell_{dec} \Lambda^{-1} \nearrow \infty$]
  - infrared insensitive stuff is o.k.

- various cosmological implications

- QCD: light quarks should not deform the ground state

- properties of ‚elementary' particles