Title: Supersymmetric Black Rings and Supertubes

Date: Sep 30, 2004 01:00 PM

URL: http://pirsa.org/04090004

Abstract:

Supersymmetric Black Rings and Supertubes

David Mateos

Perimeter Institute

Based on:

A Supersymmetric Black Ring, hep-th/0407065;
Supersymmetric Black Rings and Three-charge Supertubes, hep-th/0408120,

with H. Elvang, R. Emparan, and H. Reall

Supertubes, hep-th/0103030,

with P. Townsend

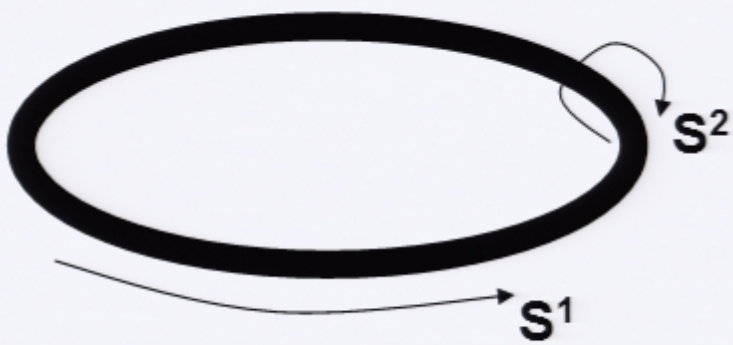
Supergravity Supertubes, hep-th/0106012,

with R. Emparan and P. Townsend

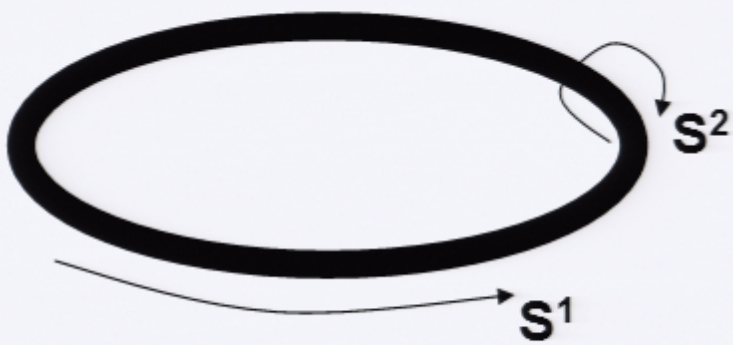
Tachyons, Supertubes and Brane/Anti-Brane Systems, hep-th/0112054,

with S. Ng and P. Townsend

Supersymmetric Black Ring = Supersymmetric,



Supersymmetric Black Ring = Supersymmetric,
Asymptotically Flat,
Stationary Black Hole Solution of 5D Supergravity



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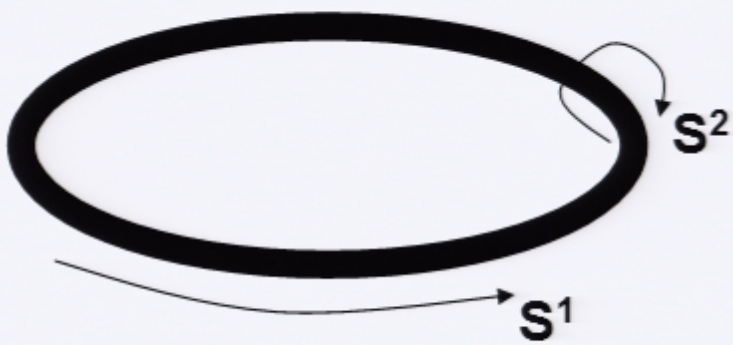

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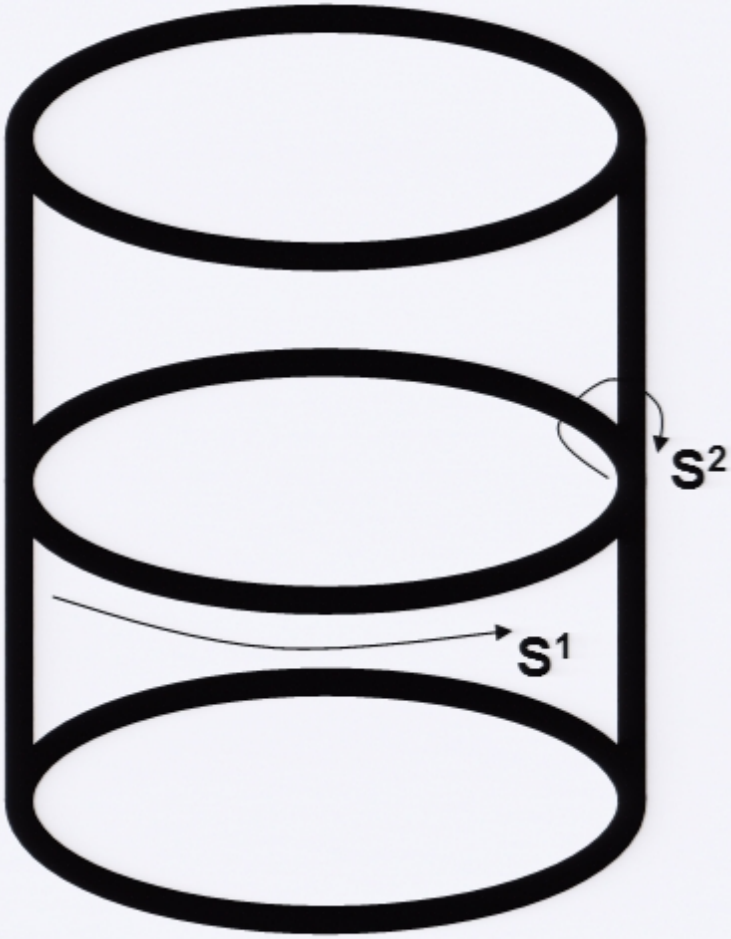
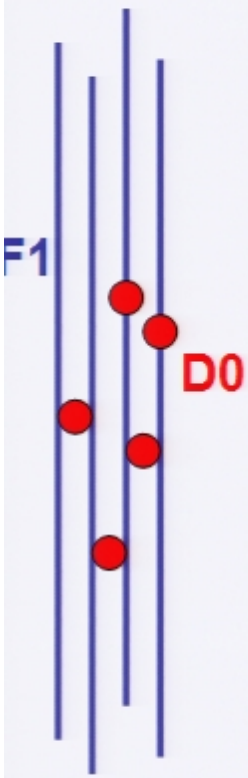


Times flat directions = Black Supertube

2-charge Supertubes: DBI (Worldvolume) Description

Townsend & D.M.

Supersymmetric Brane Expansion in Flat Space by Angular Momentum



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→ ¼-SUSY

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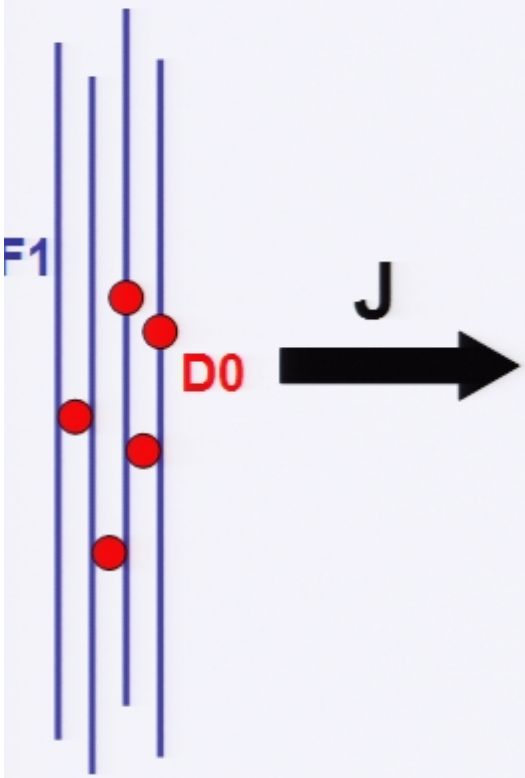

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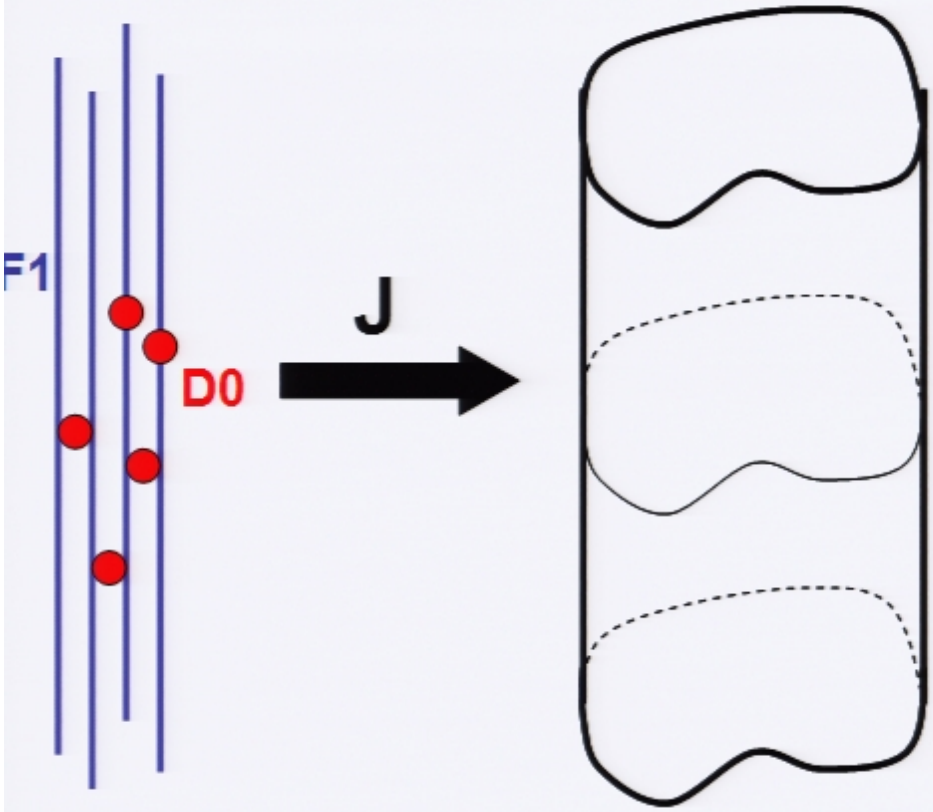

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Tubular D2/F1/D0 Bound State

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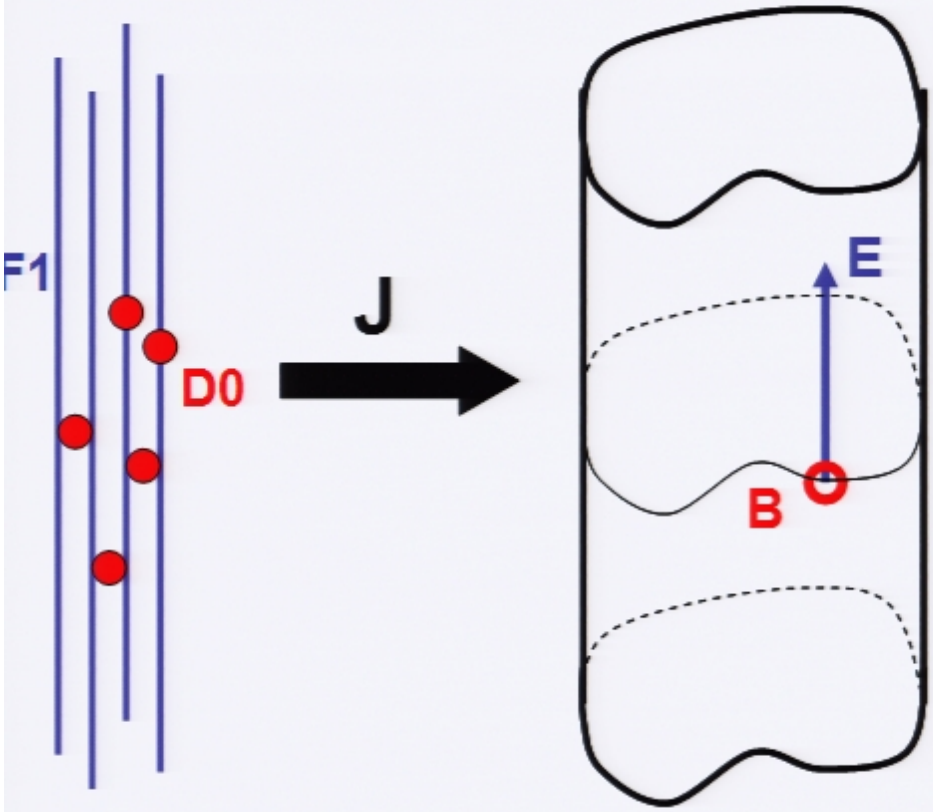
Q_{F1} and Q_{D0} dissolved as fluxes



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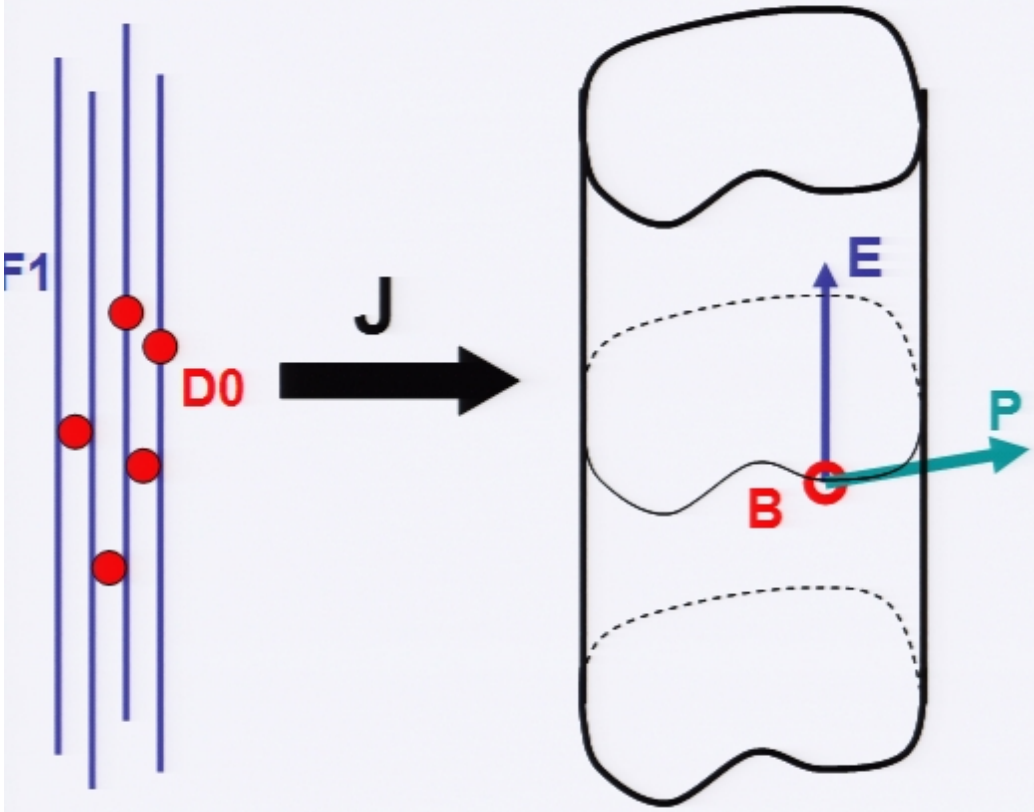

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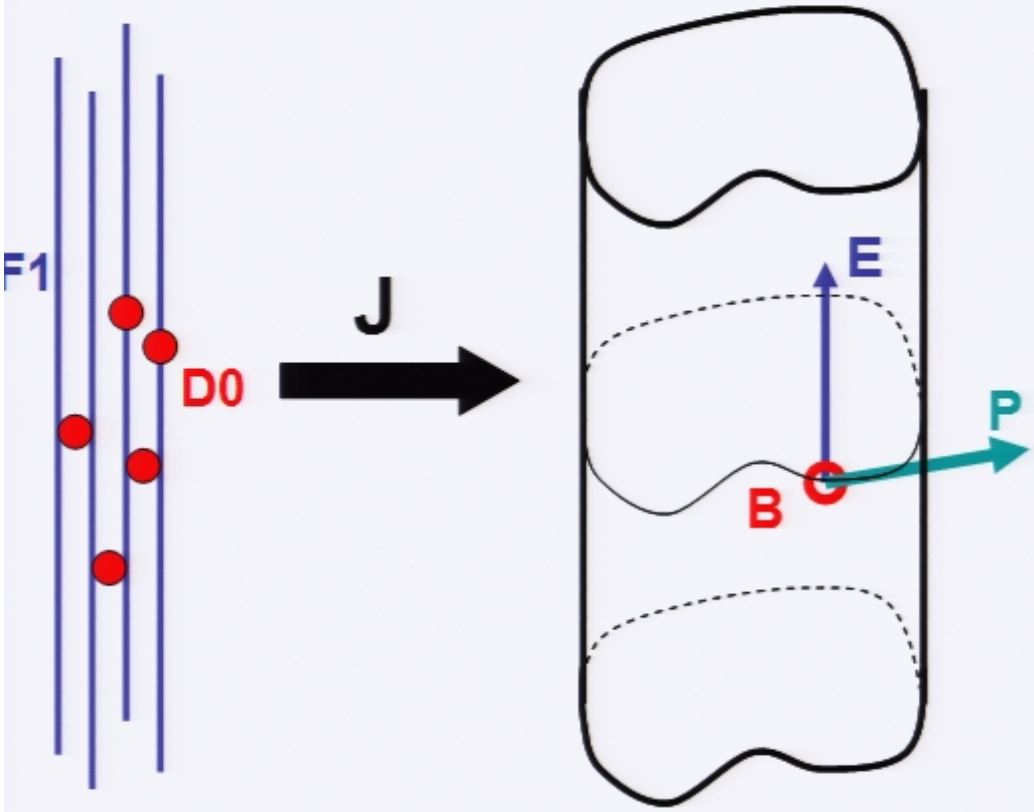

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$E = Q_{F1} + Q_{D0}$

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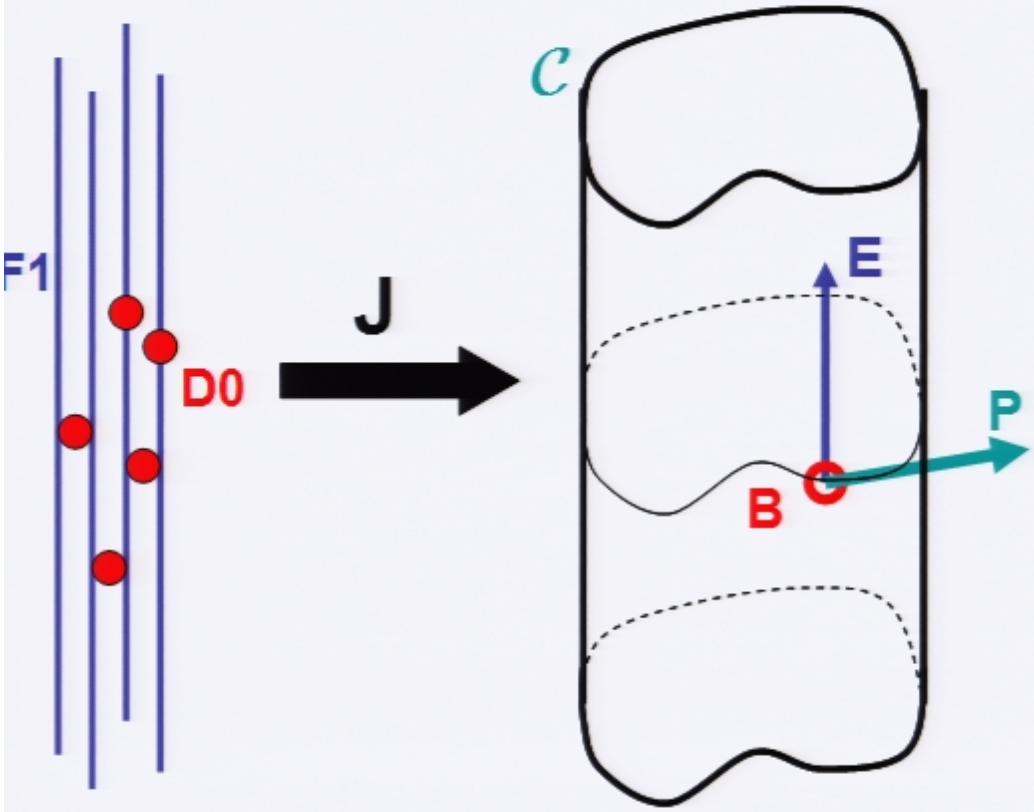

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Arbitrary Cross-section $\mathcal{C}$ in $\mathbb{E}^8$:
(and charge densities)

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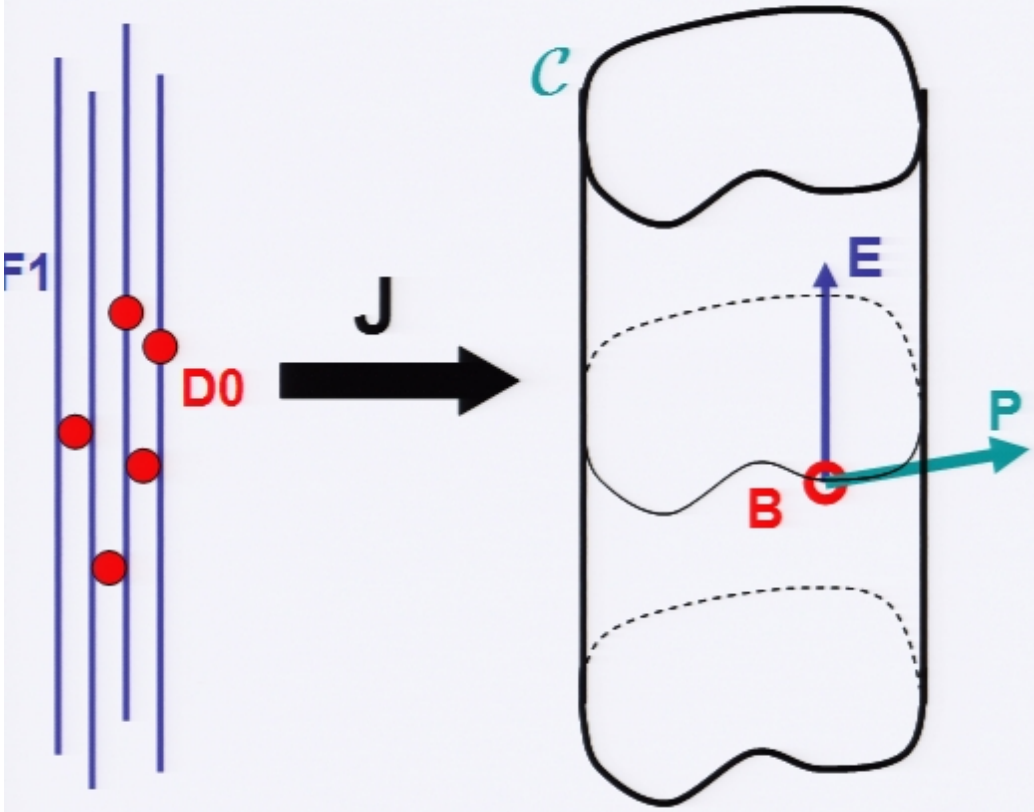

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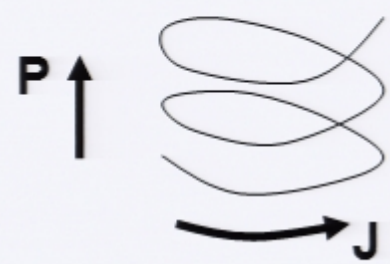
TS-Dualizing = `Helical' String with Left-moving wave on it



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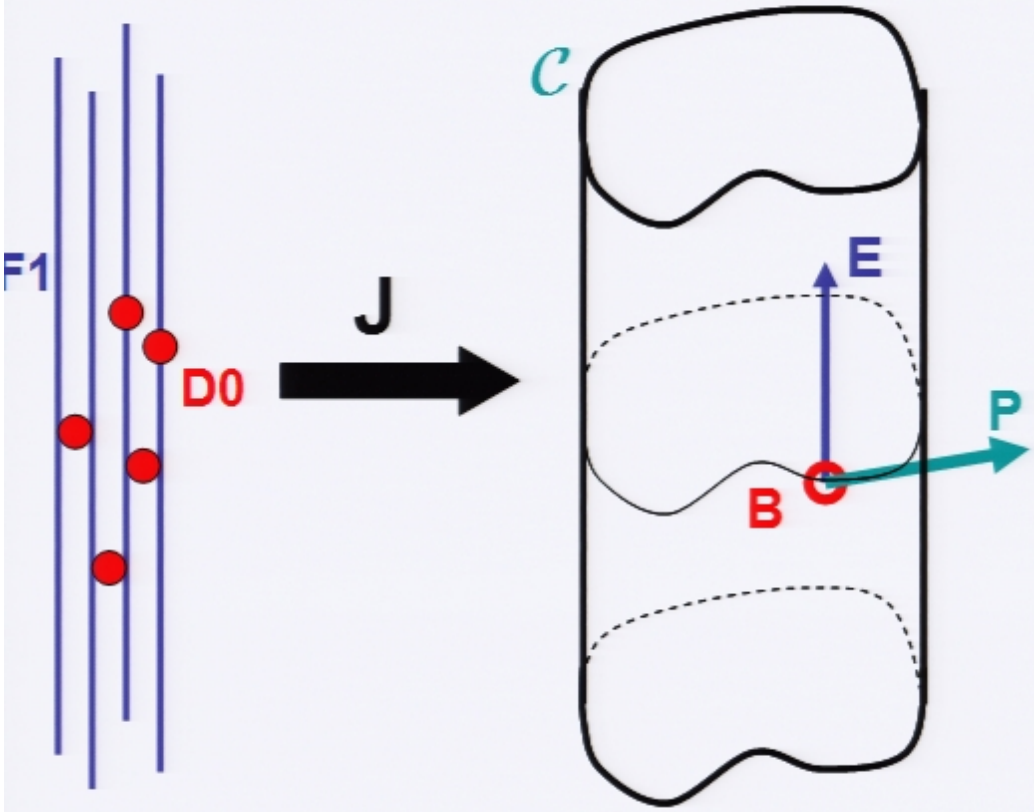

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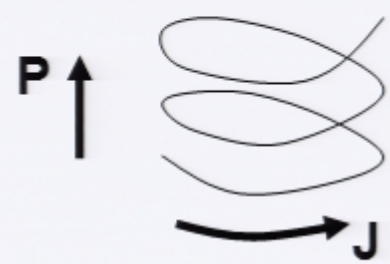

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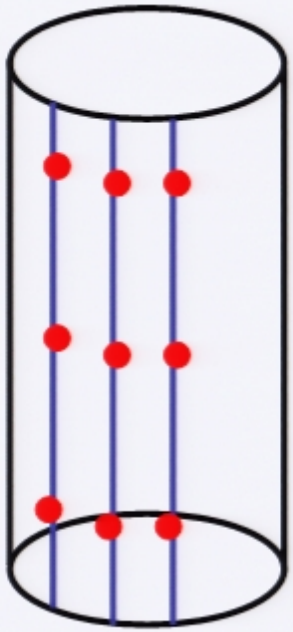

For circular Supertube of radius R:

$$R = \frac{\sqrt{Q_{F1}Q_{D0}}}{q_{D2}} \qquad J = \frac{Q_{F1}Q_{D0}}{q_{D2}} = q_{D2}R^2$$

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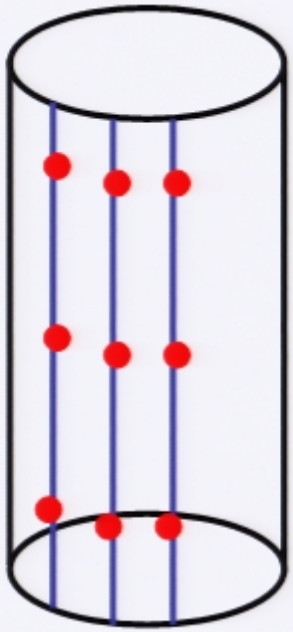

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$$R \leq \frac{\sqrt{Q_{F1}Q_{D0}}}{q_{D2}} \qquad J \leq \frac{Q_{F1}Q_{D0}}{q_{D2}}$$

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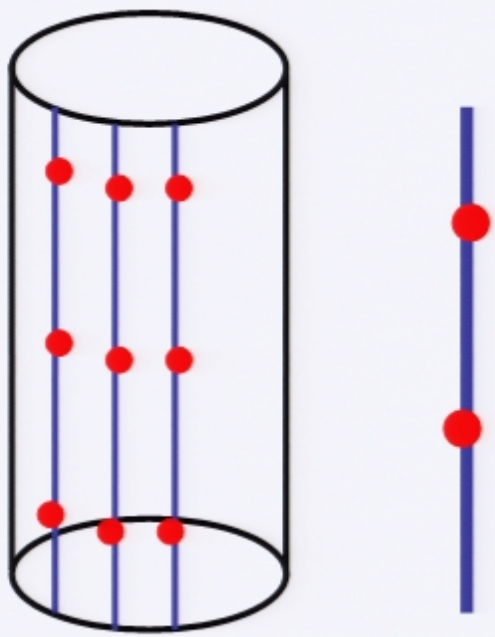

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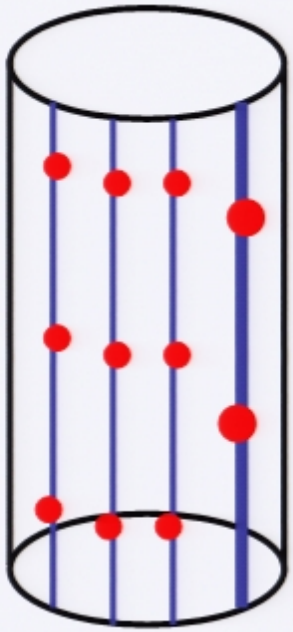

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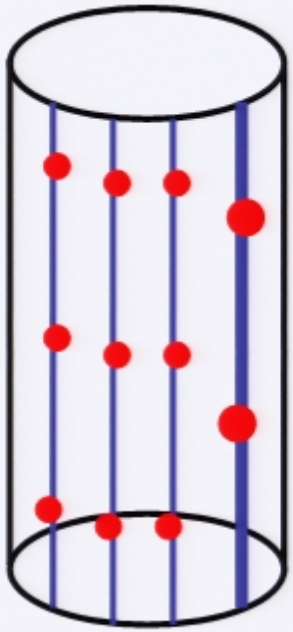

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Or eliminating q_{D2}:

$$J^2 \leq R^2\, Q_{F1}\, Q_{D0}$$

2-charge Supertubes: Supergravity Description

Emparan, Townsend & D.M.

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\begin{aligned}
ds^2 &= -U^{-1}V^{-1/2}(dt+\omega)^2 + U^{-1}V^{1/2}dz^2 + V^{1/2}d\vec{x}\cdot d\vec{x} \\
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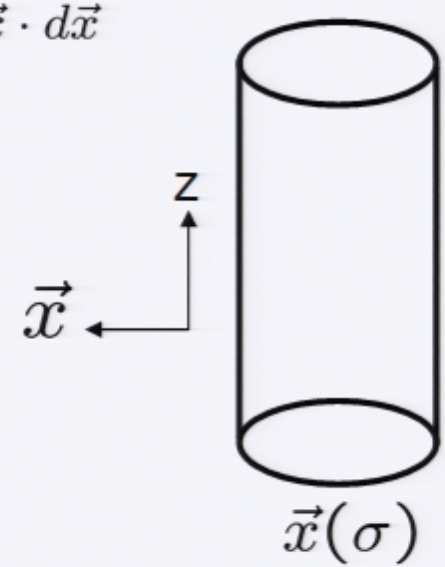

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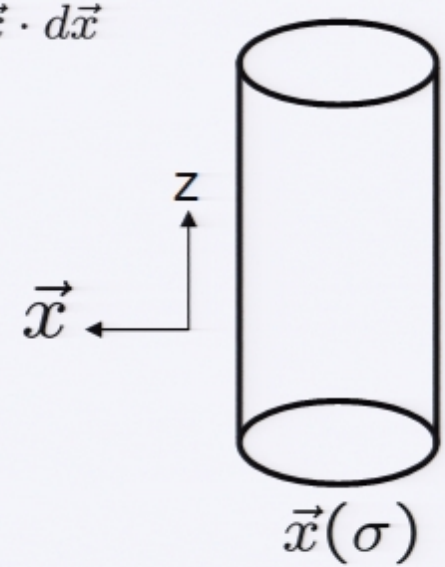

U(x) and **V**(x) harmonic functions on $\mathbb{E}^8$

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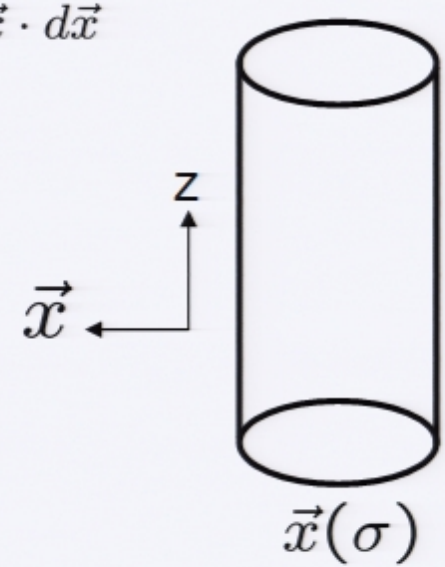

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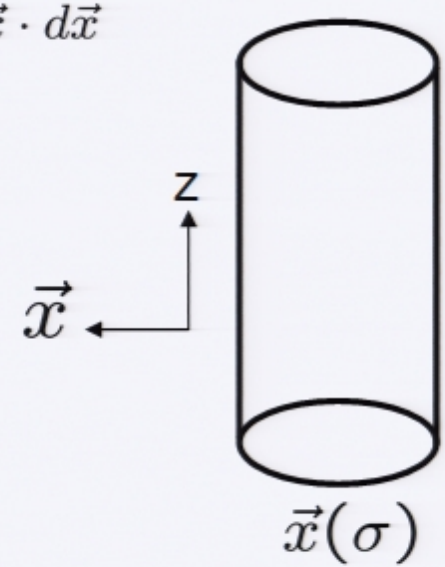

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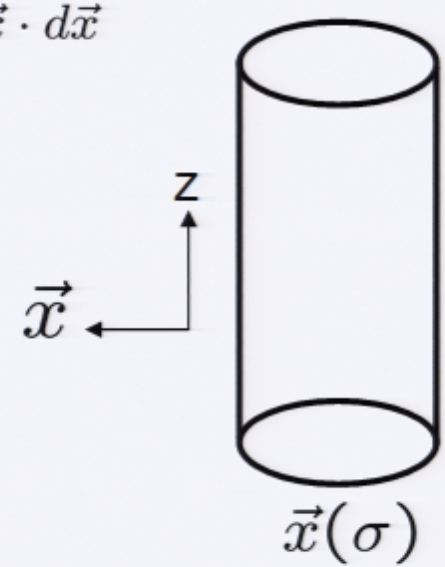

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Net charges: $\int_{S^7_\infty} \star H_3 = Q_{F1}$ $\int_{S^8_\infty} \star G_2 = Q_{D0}$ **$J = q_{D2} R^2$**

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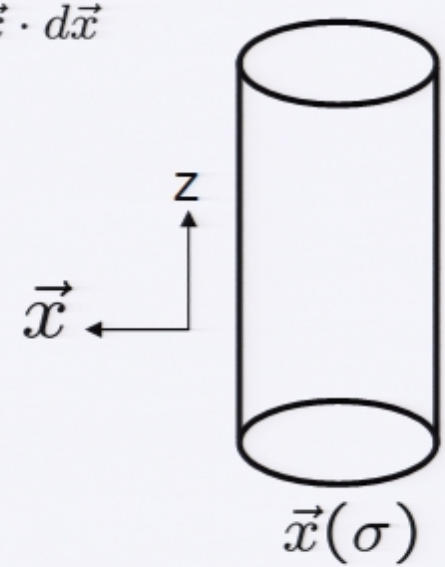

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M = Q_{F1} + Q_{D0} $\longleftrightarrow$ ¼-SUSY

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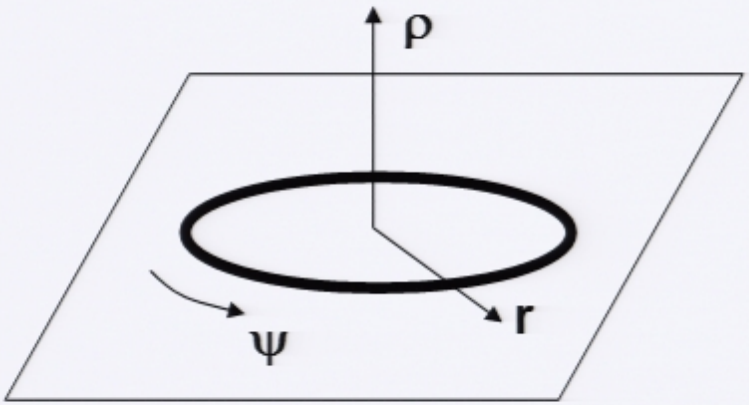

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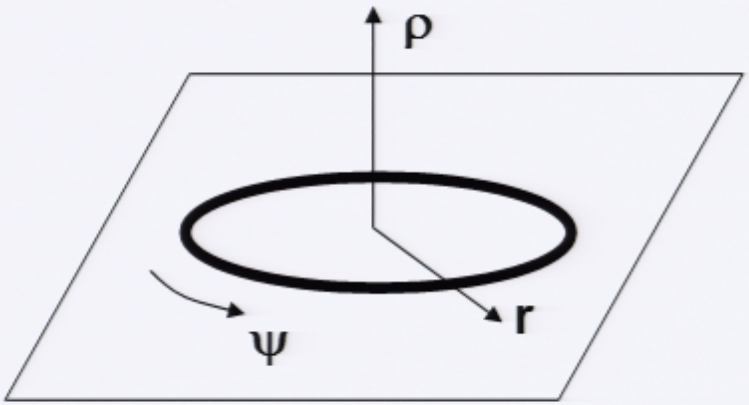

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$$\Rightarrow \quad \int_{S^6_\infty} \star G_4 = 0$$

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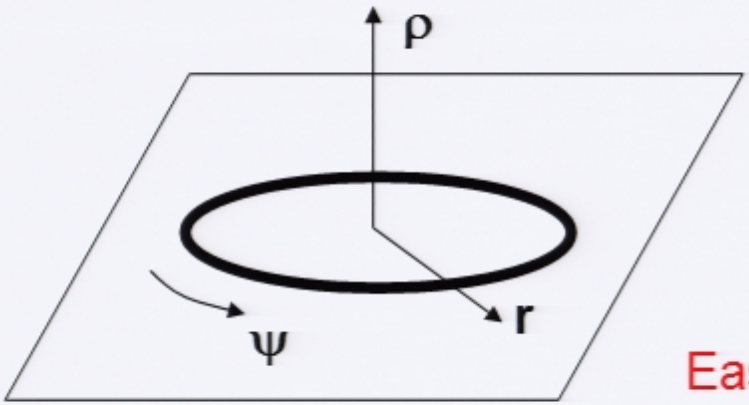

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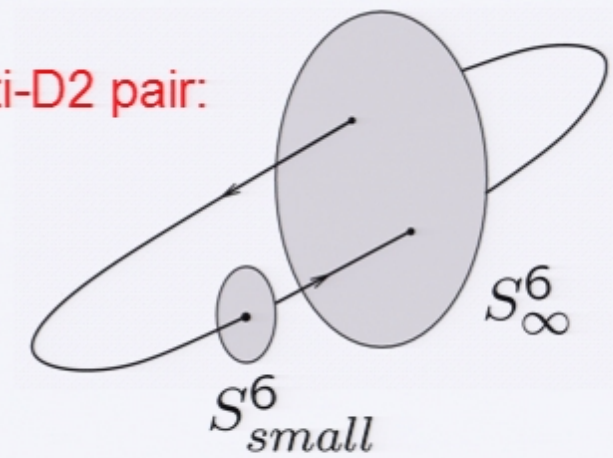
$$\rightarrow \quad \int_{S^6_\infty} \star G_4 = 0$$

Easily understood ~ D2/anti-D2 pair:

$$\rightarrow \quad \int_{S^6_{small}} \star G_4 = q_{D2}$$



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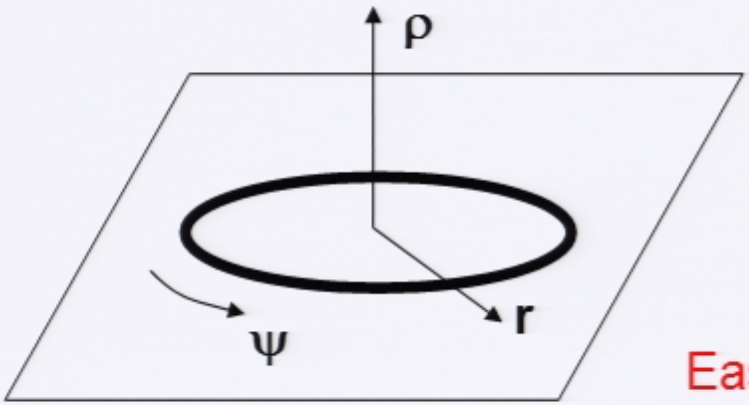
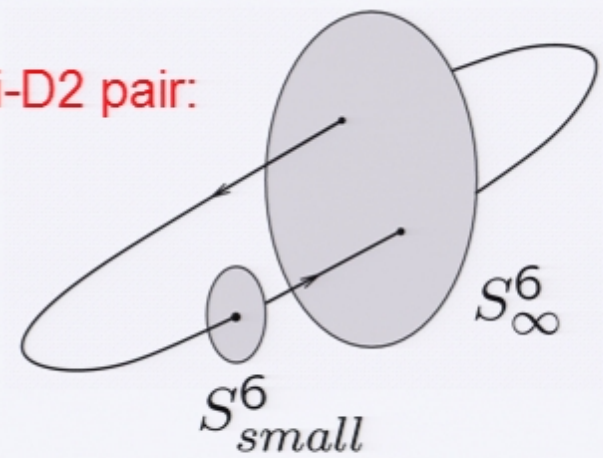


$$G_4 = d\,[\,\frac{J\,r^2}{(r^2+\rho^2)^4}\,dt \wedge dz \wedge d\psi] + \cdots$$

$$\rightarrow \quad \int_{S^6_\infty} \star G_4 = 0$$

Easily understood ~ D2/anti-D2 pair:

$$\rightarrow \quad \int_{S^6_{small}} \star G_4 = q_{D2}$$



Bound recovered: Absence of CCCs $\rightarrow$ $J^2 \leq R^2\,Q_{F1}\,Q_{D0}$

2-charge Supertubes: Supergravity Description

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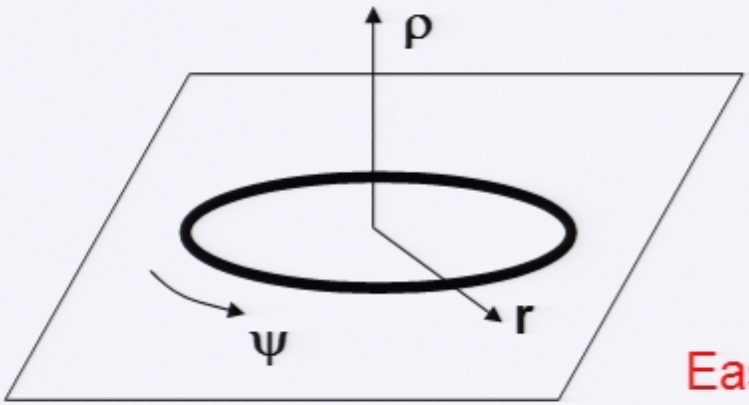

$$G_4 = d\,[\,\frac{J\,r^2}{(r^2+\rho^2)^4}\,dt \wedge dz \wedge d\psi] + \cdots$$

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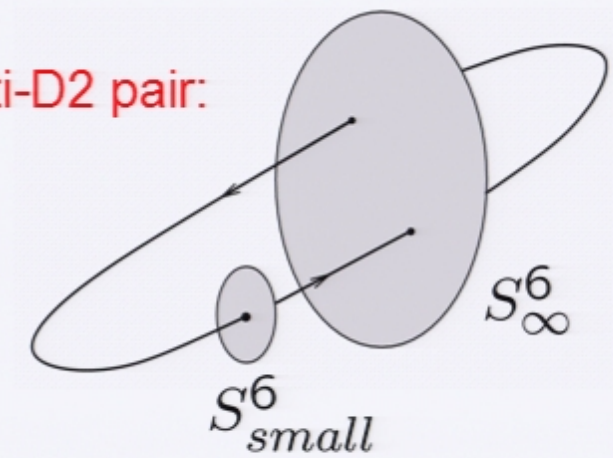
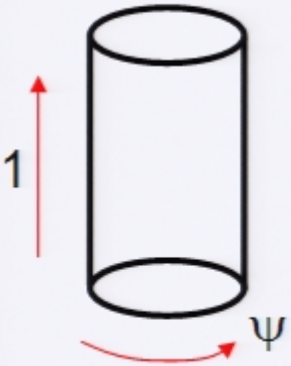


Bound recovered: Absence of CCCs $\rightarrow$ $J^2 \leq R^2\,Q_{F1}\,Q_{D0}$

Linearity of eqs $\rightarrow$ Multiple-tube solutions

2-charge Supertubes → D1/D5 System

Lunin&Mathur
Maoz&Maldacena
Lunin,Maldacena&Maoz
.....



F1:	_	1
D0:	_	_
d2:	ψ	1

2-charge Supertubes → D1/D5 System

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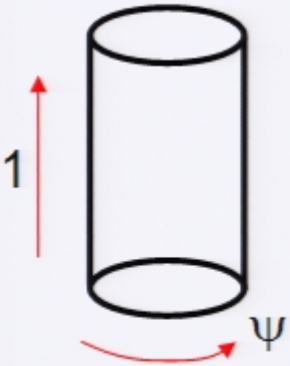
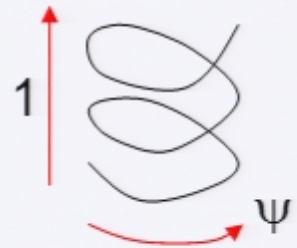
.....



F1:	_	1
D0:	_	_
d2:	ψ	1

T1 →



P:	_	1
D1:	_	1
d1:	ψ	_

2-charge Supertubes → D1/D5 System

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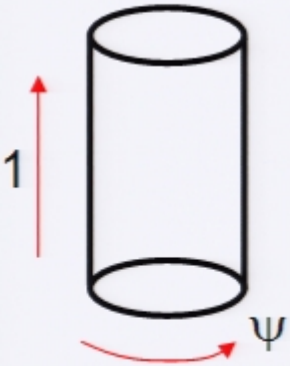
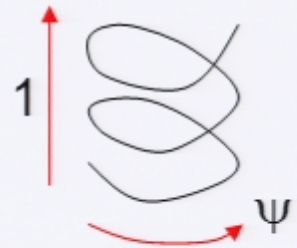
.....



F1:	_	1
D0:	_	_
d2:	ψ	1

T1 →



P:	_	1
D1:	_	1
d1:	ψ	_

T2345 →

P:	_	1	_	_	_	_
D5:	_	1	2	3	4	5
d5:	ψ	_	2	3	4	5

2-charge Supertubes → D1/D5 System

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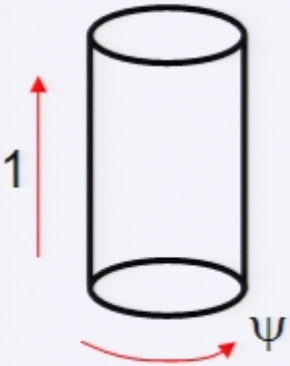

F1:	_	1
D0:	_	_
d2:	ψ	1

T1 →

P:	_	1
D1:	_	1
d1:	ψ	_

T2345 →

P:	_	1	_	_	_	_
D5:	_	1	2	3	4	5
d5:	ψ	_	2	3	4	5

S →

P:	_	1	_	_	_	_
NS5:	_	1	2	3	4	5
ns5:	ψ	_	2	3	4	5

2-charge Supertubes → D1/D5 System

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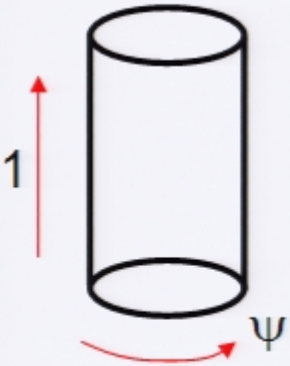
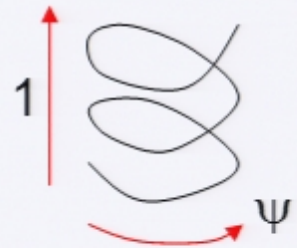


F1:	_	1
D0:	_	_
d2:	ψ	1

T1 →



P:	_	1
D1:	_	1
d1:	ψ	_

T2345 →

P:	_	1	_	_	_	_
D5:	_	1	2	3	4	5
d5:	ψ	_	2	3	4	5

S →

P:	_	1	_	_	_	_
NS5:	_	1	2	3	4	5
ns5:	ψ	_	2	3	4	5

T1 →

F1:	_	1	_	_	_	_
NS5:	_	1	2	3	4	5
kk5:	ψ	•	2	3	4	5

2-charge Supertubes → D1/D5 System

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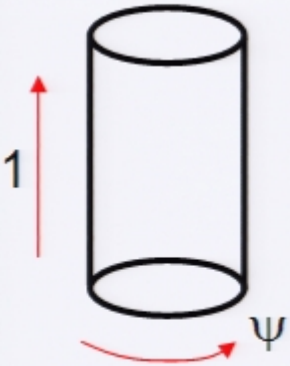
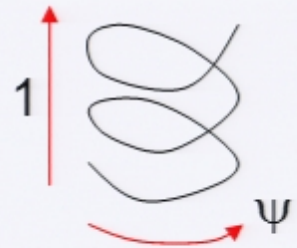


F1:	_	1
D0:	_	_
d2:	ψ	1

T1 →



P:	_	1
D1:	_	1
d1:	ψ	_

T2345 →

P:	_	1	_	_	_	_
D5:	_	1	2	3	4	5
d5:	ψ	_	2	3	4	5

S →

P:	_	1	_	_	_	_
NS5:	_	1	2	3	4	5
ns5:	ψ	_	2	3	4	5

T1 →

F1:	_	1	_	_	_	_
NS5:	_	1	2	3	4	5
kk5:	ψ	•	2	3	4	5

(Trivial T) **S** →

D1:	_	1	_	_	_	_
D5:	_	1	2	3	4	5
kk5:	ψ	•	2	3	4	5

3-charge Supertubes and Supersymmetric Black Rings: Supergravity Description

Elvang, Emparan, Reall & D.M.
Bena & Warner
Gauntlett & Gutowski

3-charge Supertubes and Supersymmetric Black Rings: Supergravity Description

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First, lift 2-charge supertube to M-theory:

F1:	_	1
D0:	_	_
d2:	ψ	1

3-charge Supertubes and Supersymmetric Black Rings: Supergravity Description

Elvang, Emparan, Reall & D.M.
Bena & Warner
Gauntlett & Gutowski

First, lift 2-charge supertube to M-theory:

F1:	_	1		F1:	_	1	_	_	_
D0:	_	_	**T34** →	D2:	_	_	_	3	4
d2:	ψ	1		d4:	ψ	1	_	3	4

3-charge Supertubes and Supersymmetric Black Rings: Supergravity Description

Elvang, Emparan, Reall & D.M.
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First, lift 2-charge supertube to M-theory:

F1:	_	1		F1:	_	1	_	_	_		M2:	_	1	2	_	_
D0:	_	_	**T34** →	D2:	_	_	_	3	4	**Lift** →	M2:	_	_	_	3	4
d2:	ψ	1		d4:	ψ	1	_	3	4		m5:	ψ	1	2	3	4

With 3 charges, each pair expands:

Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

With 3 charges, each pair expands:

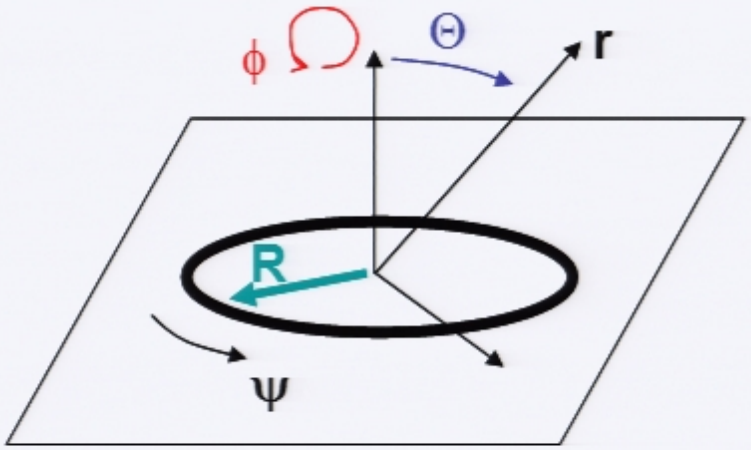
Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

$$\underbrace{\qquad\qquad\qquad\qquad}_{\mathbb{T}^6 = \{ z^1, \ldots , z^6\}}$$

With 3 charges, each pair expands:

Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

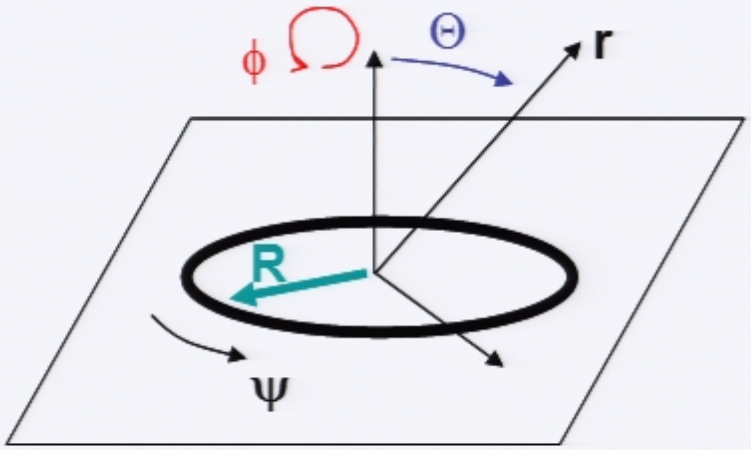
$\mathbb{T}^6 = \{ z^1, \ldots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$



With 3 charges, each pair expands:

Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$



$$ds^2 = ds_5^2 + X^1\left(dz_1^2 + dz_2^2\right) + X^2\left(dz_3^2 + dz_4^2\right) + X^3\left(dz_5^2 + dz_6^2\right)$$

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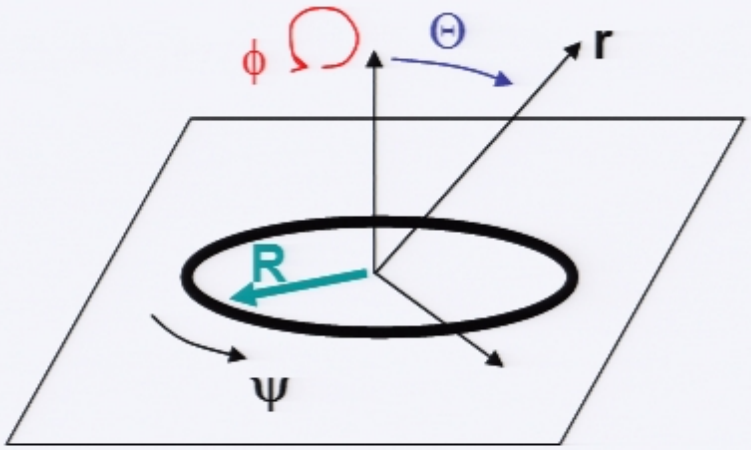
Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$



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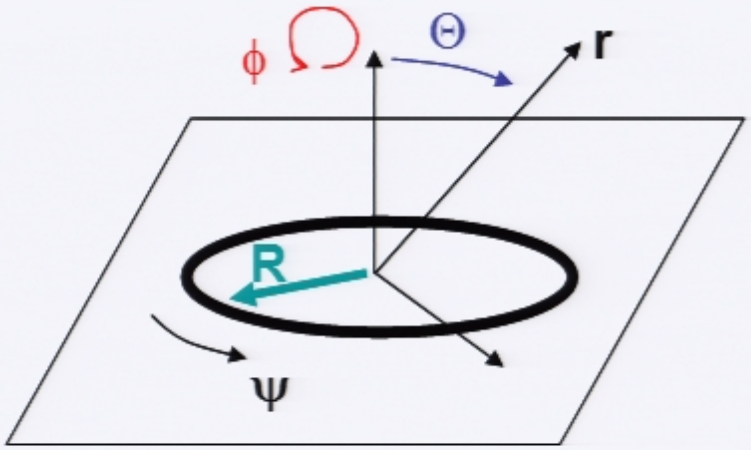
$$X^i = H_i^{-1}\left(H_1 H_2 H_3\right)^{1/3} \quad , \qquad X^1 X^2 X^3 = 1$$

With 3 charges, each pair expands:

Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$

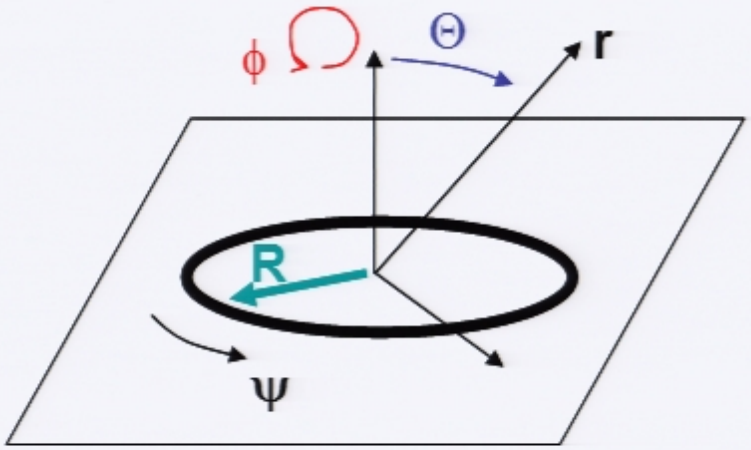


$$ds^2 = ds_5^2 + X^1\left(dz_1^2 + dz_2^2\right) + X^2\left(dz_3^2 + dz_4^2\right) + X^3\left(dz_5^2 + dz_6^2\right)$$

$$X^i = H_i^{-1}\left(H_1 H_2 H_3\right)^{1/3} \quad , \qquad X^1 X^2 X^3 = 1$$

$$ds_5^2 \;=\; -(H_1 H_2 H_3)^{-2/3}(dt + \omega)^2 + (H_1 H_2 H_3)^{1/3} dx_4^2$$

With 3 charges, each pair expands:



Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$

$$ds^2 = ds_5^2 + X^1\left(dz_1^2 + dz_2^2\right) + X^2\left(dz_3^2 + dz_4^2\right) + X^3\left(dz_5^2 + dz_6^2\right)$$

$$X^i = H_i^{-1}\left(H_1 H_2 H_3\right)^{1/3} \quad , \qquad X^1 X^2 X^3 = 1$$

$$ds_5^2 \;=\; -(H_1 H_2 H_3)^{-2/3}(dt + \omega)^2 + (H_1 H_2 H_3)^{1/3} dx_4^2$$

$$H_1 = 1 + \frac{Q_1 - q_2 q_3}{\Sigma} + q_2 q_3 \frac{r^2}{\Sigma^2}$$

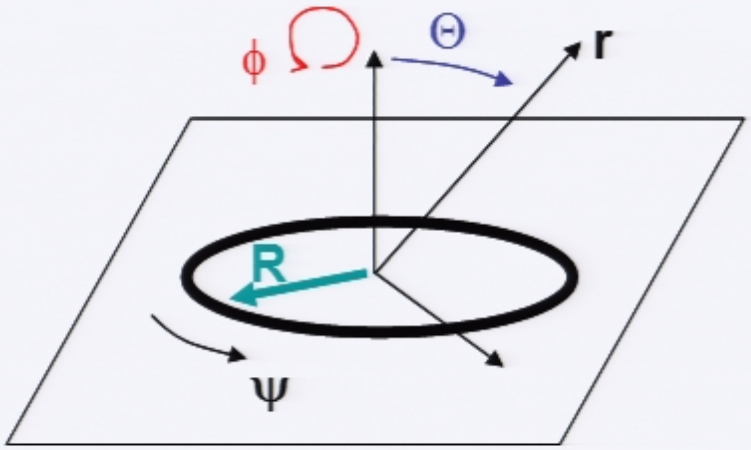
$$H_2 = 1 + \frac{Q_2 - q_1 q_3}{\Sigma} + q_1 q_3 \frac{r^2}{\Sigma^2}$$

$$H_3 = 1 + \frac{Q_3 - q_1 q_2}{\Sigma} + q_1 q_2 \frac{r^2}{\Sigma^2}$$

With 3 charges, each pair expands:

Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$



$$ds^2 = ds_5^2 + X^1\left(dz_1^2 + dz_2^2\right) + X^2\left(dz_3^2 + dz_4^2\right) + X^3\left(dz_5^2 + dz_6^2\right)$$

$$X^i = H_i^{-1}\left(H_1 H_2 H_3\right)^{1/3} \quad , \qquad X^1 X^2 X^3 = 1$$

$$ds_5^2 \;=\; -(H_1 H_2 H_3)^{-2/3}(dt + \omega)^2 + (H_1 H_2 H_3)^{1/3} dx_4^2$$

$$H_1 = 1 + \frac{Q_1 - q_2 q_3}{\Sigma} + q_2 q_3 \frac{r^2}{\Sigma^2} \qquad \Sigma(r,\Theta) = \sqrt{(r^2 - R^2)^2 + 4R^2 r^2 \cos^2\Theta}$$

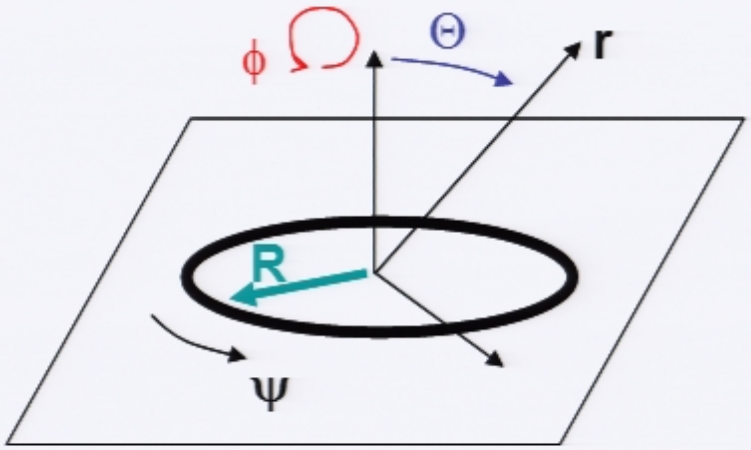
$$H_2 = 1 + \frac{Q_2 - q_1 q_3}{\Sigma} + q_1 q_3 \frac{r^2}{\Sigma^2}$$

$$H_3 = 1 + \frac{Q_3 - q_1 q_2}{\Sigma} + q_1 q_2 \frac{r^2}{\Sigma^2}$$

With 3 charges, each pair expands:

Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$



$$ds^2 = ds_5^2 + X^1\left(dz_1^2 + dz_2^2\right) + X^2\left(dz_3^2 + dz_4^2\right) + X^3\left(dz_5^2 + dz_6^2\right)$$

$$X^i = H_i^{-1}\left(H_1 H_2 H_3\right)^{1/3} \quad , \qquad X^1 X^2 X^3 = 1$$

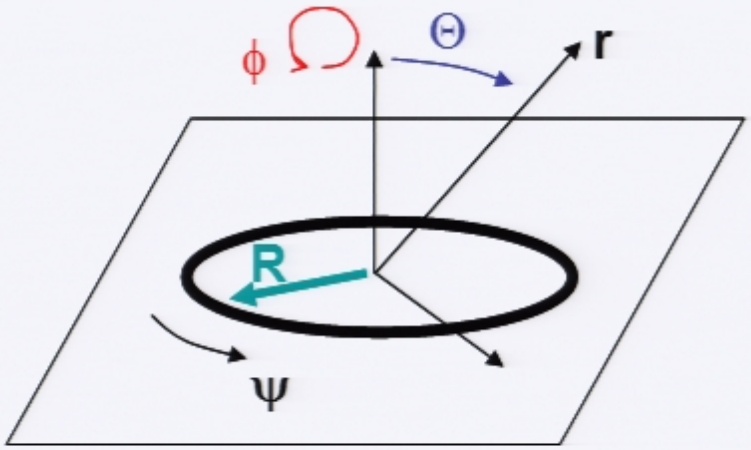
$$ds_5^2 \;=\; -(H_1 H_2 H_3)^{-2/3}(dt + \omega)^2 + (H_1 H_2 H_3)^{1/3} dx_4^2$$

$$H_1 = 1 + \frac{Q_1 - q_2 q_3}{\Sigma} + q_2 q_3 \frac{r^2}{\Sigma^2} \qquad \Sigma(r,\Theta) = \sqrt{(r^2 - R^2)^2 + 4R^2 r^2 \cos^2\Theta}$$

$$H_2 = 1 + \frac{Q_2 - q_1 q_3}{\Sigma} + q_1 q_3 \frac{r^2}{\Sigma^2}$$

$$H_3 = 1 + \underbrace{\frac{Q_3 - q_1 q_2}{\Sigma}}_{\text{Harmonic with ring source}} + q_1 q_2 \frac{r^2}{\Sigma^2}$$

With 3 charges, each pair expands:



Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$

$$ds^2 = ds_5^2 + X^1\left(dz_1^2 + dz_2^2\right) + X^2\left(dz_3^2 + dz_4^2\right) + X^3\left(dz_5^2 + dz_6^2\right)$$

$$X^i = H_i^{-1}\left(H_1 H_2 H_3\right)^{1/3} \quad , \qquad X^1 X^2 X^3 = 1$$

$$ds_5^2 \;=\; -(H_1 H_2 H_3)^{-2/3}(dt + \omega)^2 + (H_1 H_2 H_3)^{1/3} dx_4^2$$

$$H_1 = 1 + \frac{Q_1 - q_2 q_3}{\Sigma} + q_2 q_3 \frac{r^2}{\Sigma^2}$$

$$\Sigma(r,\Theta) = \sqrt{(r^2 - R^2)^2 + 4R^2 r^2 \cos^2\Theta}$$

$$H_2 = 1 + \frac{Q_2 - q_1 q_3}{\Sigma} + q_1 q_3 \frac{r^2}{\Sigma^2}$$

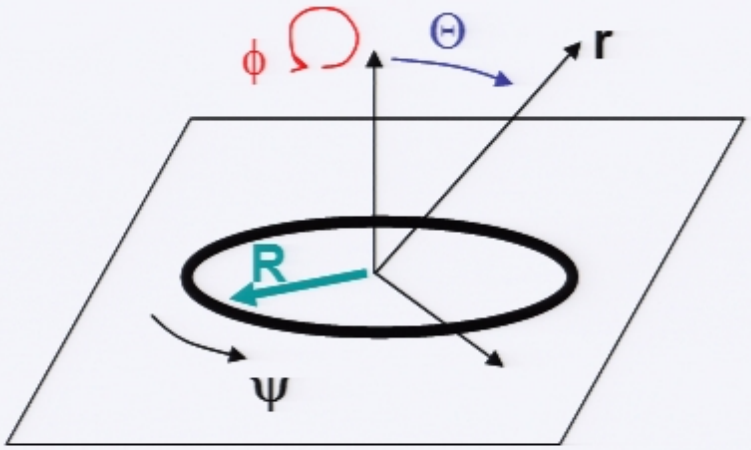
$$\omega(r,\Theta; Q_i, q_i, R) = \omega_\psi\, d\psi + \underbrace{\omega_\phi\, d\phi}_{\text{Extra angular momentum}}$$

$$H_3 = 1 + \underbrace{\frac{Q_3 - q_1 q_2}{\Sigma}}_{\text{Harmonic with ring source}} + q_1 q_2 \frac{r^2}{\Sigma^2}$$

With 3 charges, each pair expands:

Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$



$$ds^2 = ds_5^2 + X^1\left(dz_1^2 + dz_2^2\right) + X^2\left(dz_3^2 + dz_4^2\right) + X^3\left(dz_5^2 + dz_6^2\right)$$

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$$\omega(r,\Theta; Q_i, q_i, R) = \omega_\psi\, d\psi + \underbrace{\omega_\phi\, d\phi}_{\text{Extra angular momentum}}$$

$$H_3 = 1 + \underbrace{\frac{Q_3 - q_1 q_2}{\Sigma}}_{\text{Harmonic with ring source}} + q_1 q_2 \frac{r^2}{\Sigma^2}$$

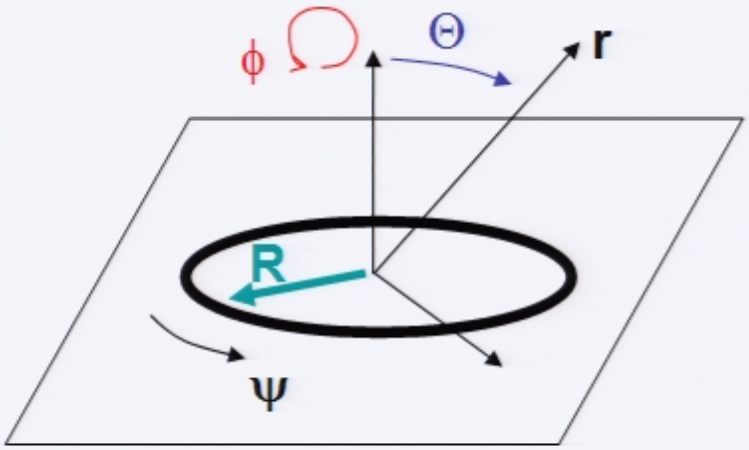
Globally well-defined (no Dirac-Misner strings) since

$$\omega_\psi(\Theta = 0) = \omega_\phi(\Theta = \pi/2) = 0$$

With 3 charges, each pair expands:

Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$



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Q_1 M2:	1	2	_	_	_	_	_	_	_	_
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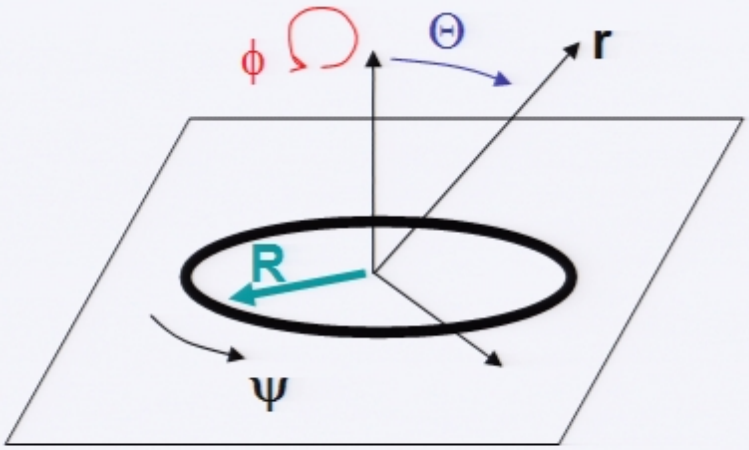
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$



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$$A^i = H_i^{-1}(dt + \omega) + \frac{q_i}{2\Sigma}\left[(r^2 + \Sigma^2 - R^2)d\psi + (r^2 + \Sigma^2 + R^2)d\phi\right]$$

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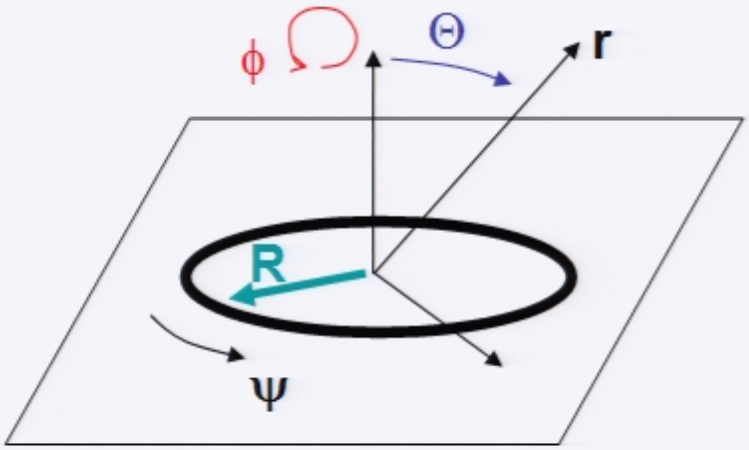
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$



$$ds^2 = ds_5^2 + X^1\left(dz_1^2 + dz_2^2\right) + X^2\left(dz_3^2 + dz_4^2\right) + X^3\left(dz_5^2 + dz_6^2\right)$$

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➡ Q_i = M2 charges

q_i = M5 dipoles

With 3 charges, each pair expands:

Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
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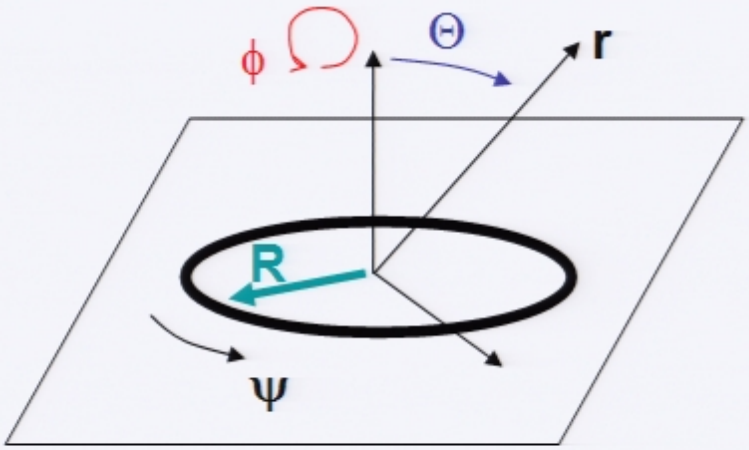
$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$



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$\longrightarrow$ Q_i = M2 charges $\qquad \mathcal{Q}_1 \equiv Q_1 - q_2 q_3\,,\ \mathcal{Q}_2 \equiv Q_2 - q_1 q_3\,,\ \mathcal{Q}_3 \equiv Q_3 - q_1 q_2$

q_i = M5 dipoles

With 3 charges, each pair expands:

Q_1 M2:	1	2	_	_	_	_	_	_	_	_
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q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
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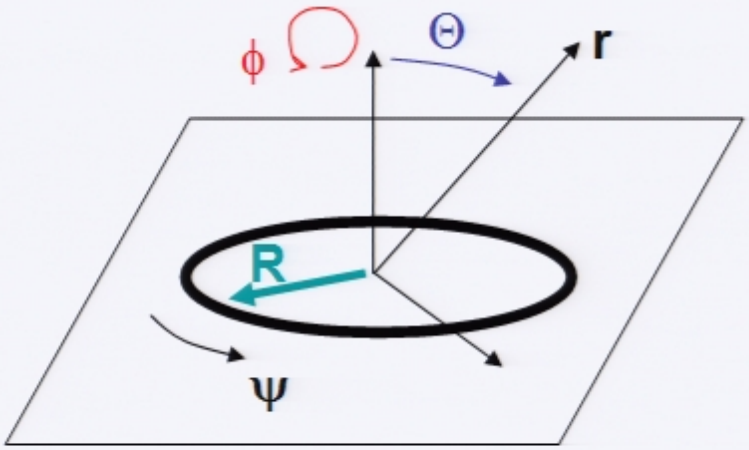
$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$



$$ds^2 = ds_5^2 + X^1\left(dz_1^2 + dz_2^2\right) + X^2\left(dz_3^2 + dz_4^2\right) + X^3\left(dz_5^2 + dz_6^2\right)$$

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$\Longrightarrow$ Q_i = M2 charges $\qquad \mathcal{Q}_1 \equiv Q_1 - q_2q_3\,,\ \mathcal{Q}_2 \equiv Q_2 - q_1q_3\,,\ \mathcal{Q}_3 \equiv Q_3 - q_1q_2$

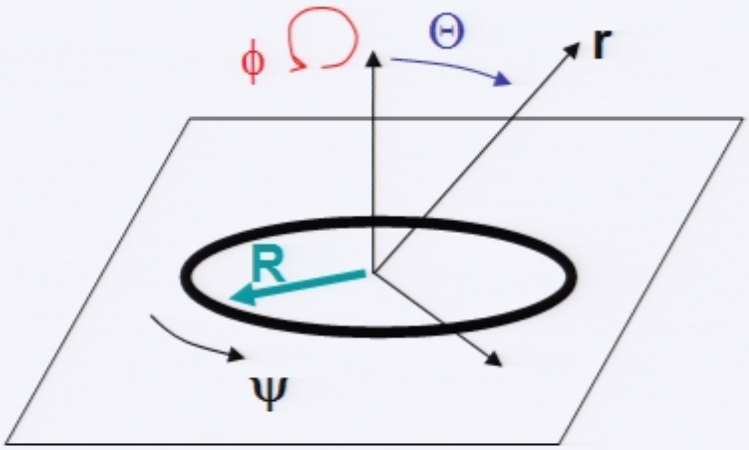
q_i = M5 dipoles

$H_i \geq 0$ and no CCCs $\rightarrow \mathcal{Q}_i \geq 0$

With 3 charges, each pair expands:

Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

$\mathbb{T}^6 = \{ z^1, \dots , z^6\}$ $\mathbb{E}^4 = \{ r, \Theta, \psi, \phi \}$



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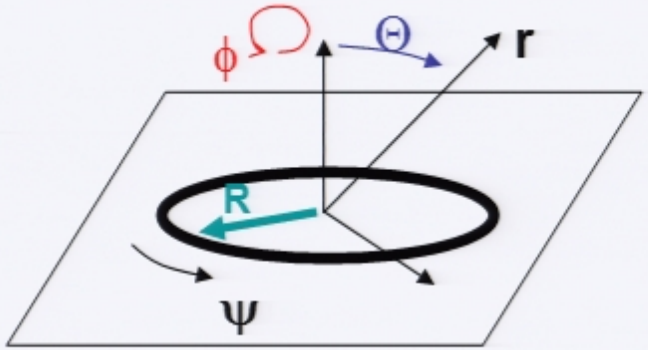
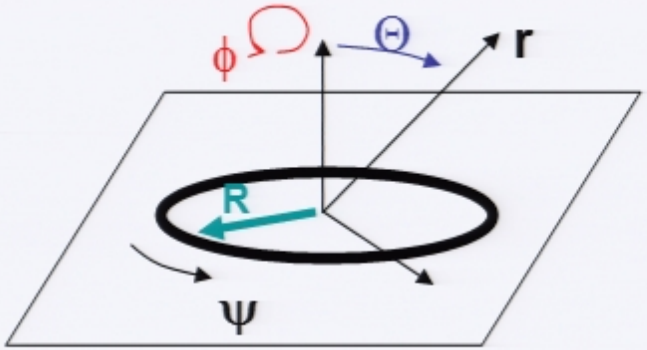
q_i = M5 dipoles $\qquad H_i \geq 0$ and no CCCs $\rightarrow \mathcal{Q}_i \geq 0 \qquad q \equiv (q_1 q_2 q_3)^{1/3}$

Physical Properties

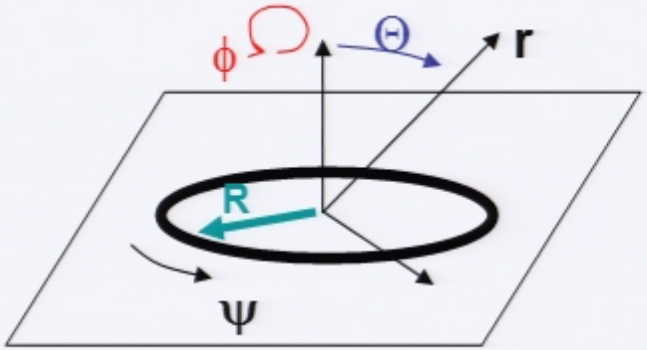


Physical Properties



Solution preserves 4 supercharges (1/8-Susy in 11D)

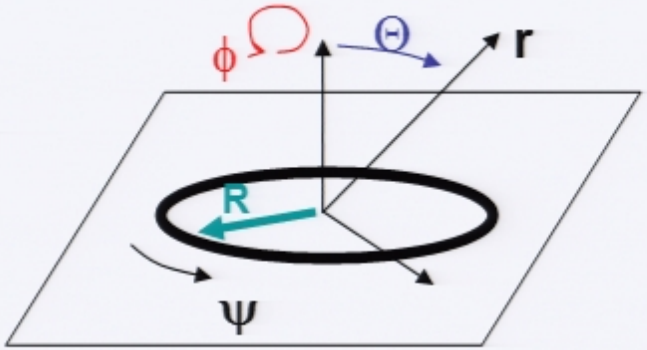
Physical Properties



Solution preserves 4 supercharges (1/8-Susy in 11D)

Stationary and asymptotically flat (in 5D)

Physical Properties



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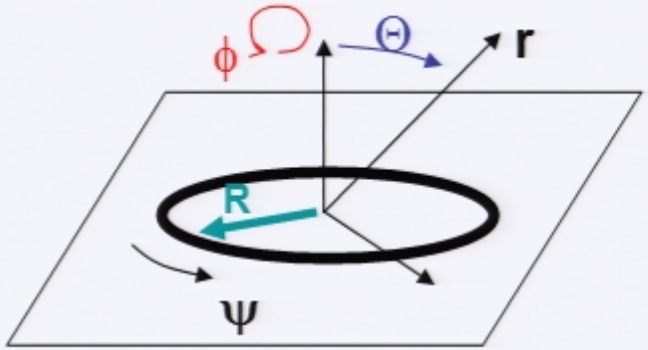
Independent Conserved Charges: Three Gauge Charges Q_i and two ang mom:

Physical Properties



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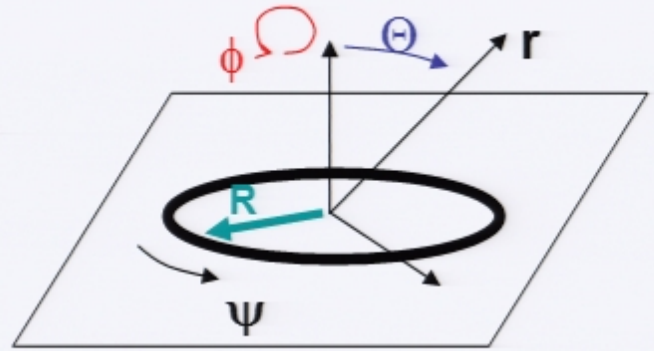
$$J_\phi = \frac{\pi}{8G_5}\left(q_1Q_1 + q_2Q_2 + q_3Q_3 - q_1q_2q_3\right)$$

$$J_\psi = \frac{\pi}{8G_5}2R^2\left(q_1 + q_2 + q_3\right) + J_\phi$$

Physical Properties



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$\Rightarrow$ $J_\psi > J_\phi$

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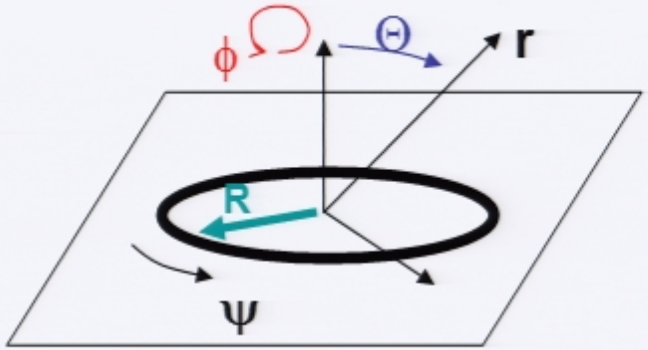

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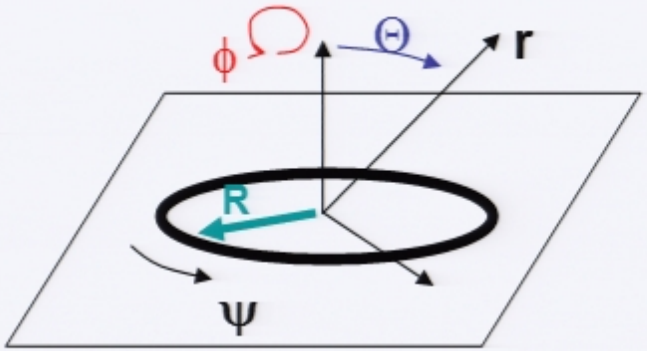
(Mass is fixed by BPS bound: $M = Q_1 + Q_2 + Q_3$)

Physical Properties



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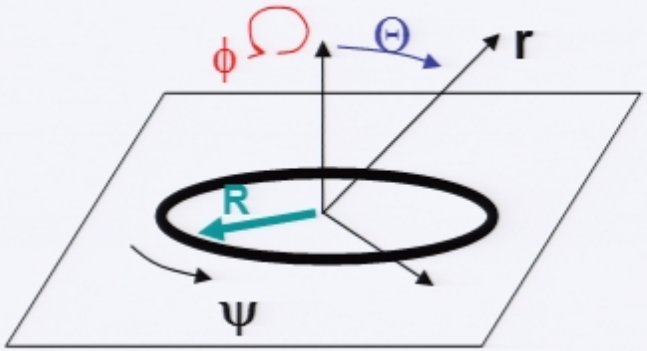
$\longrightarrow$ $J_\psi > J_\phi$

(Mass is fixed by BPS bound: $M = Q_1 + Q_2 + Q_3$)

Absence of CCCs $\longleftrightarrow$ $L^2 = \frac{1}{4q^4}\left(2\sum_{i<j}\mathcal{Q}_iq_i\mathcal{Q}_jq_j - \sum_i \mathcal{Q}_i^2q_i^2 - 4R^2q^3\sum_i q_i\right) \geq 0$

Physical Properties



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$$J_\phi = \frac{\pi}{8G_5}\left(q_1Q_1 + q_2Q_2 + q_3Q_3 - q_1q_2q_3\right)$$

$$J_\psi = \frac{\pi}{8G_5}2R^2\left(q_1 + q_2 + q_3\right) + J_\phi$$

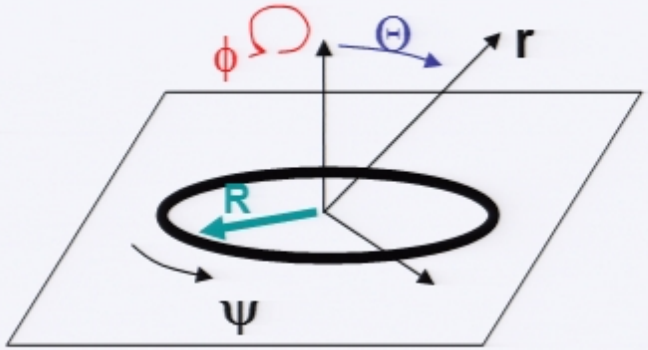
$\longrightarrow$ $J_\psi > J_\phi$

(Mass is fixed by BPS bound: $M = Q_1 + Q_2 + Q_3$)

Absence of CCCs $\longleftrightarrow$ $L^2 = \frac{1}{4q^4}\left(2\sum_{i<j}\mathcal{Q}_i q_i \mathcal{Q}_j q_j - \sum_i \mathcal{Q}_i^2 q_i^2 - 4R^2q^3\sum_i q_i\right) \geq 0$

$\longrightarrow$ Upper bound on R, or equivalently on $J_\psi - J_\phi$

Physical Properties



Solution preserves 4 supercharges (1/8-Susy in 11D)

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Independent Conserved Charges: Three Gauge Charges Q_i and two ang mom:

$$J_\phi = \frac{\pi}{8G_5}(q_1Q_1 + q_2Q_2 + q_3Q_3 - q_1q_2q_3)$$

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(Mass is fixed by BPS bound: $M = Q_1 + Q_2 + Q_3$)

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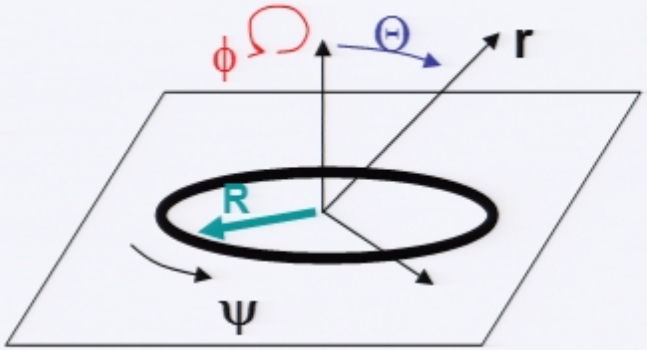

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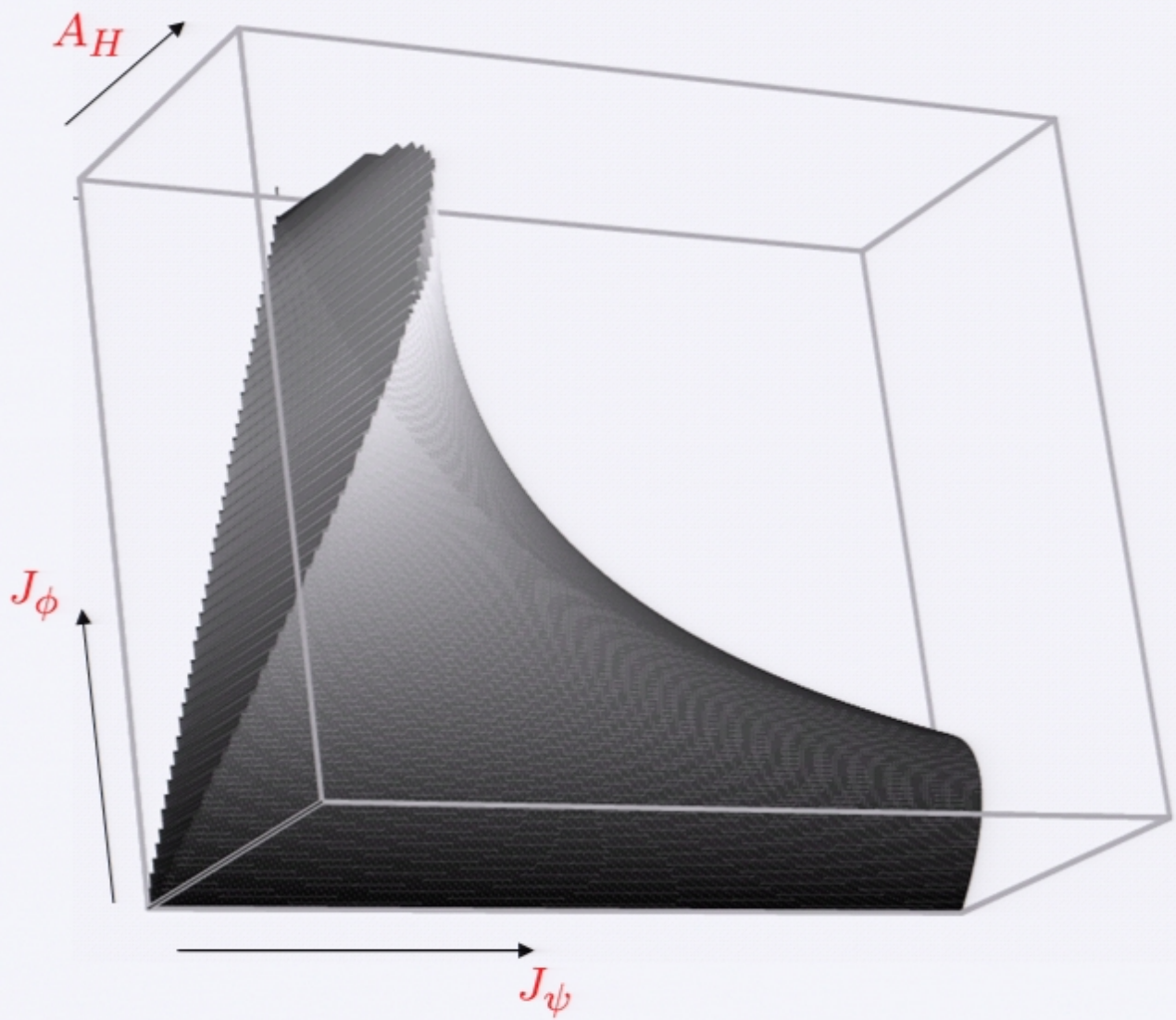

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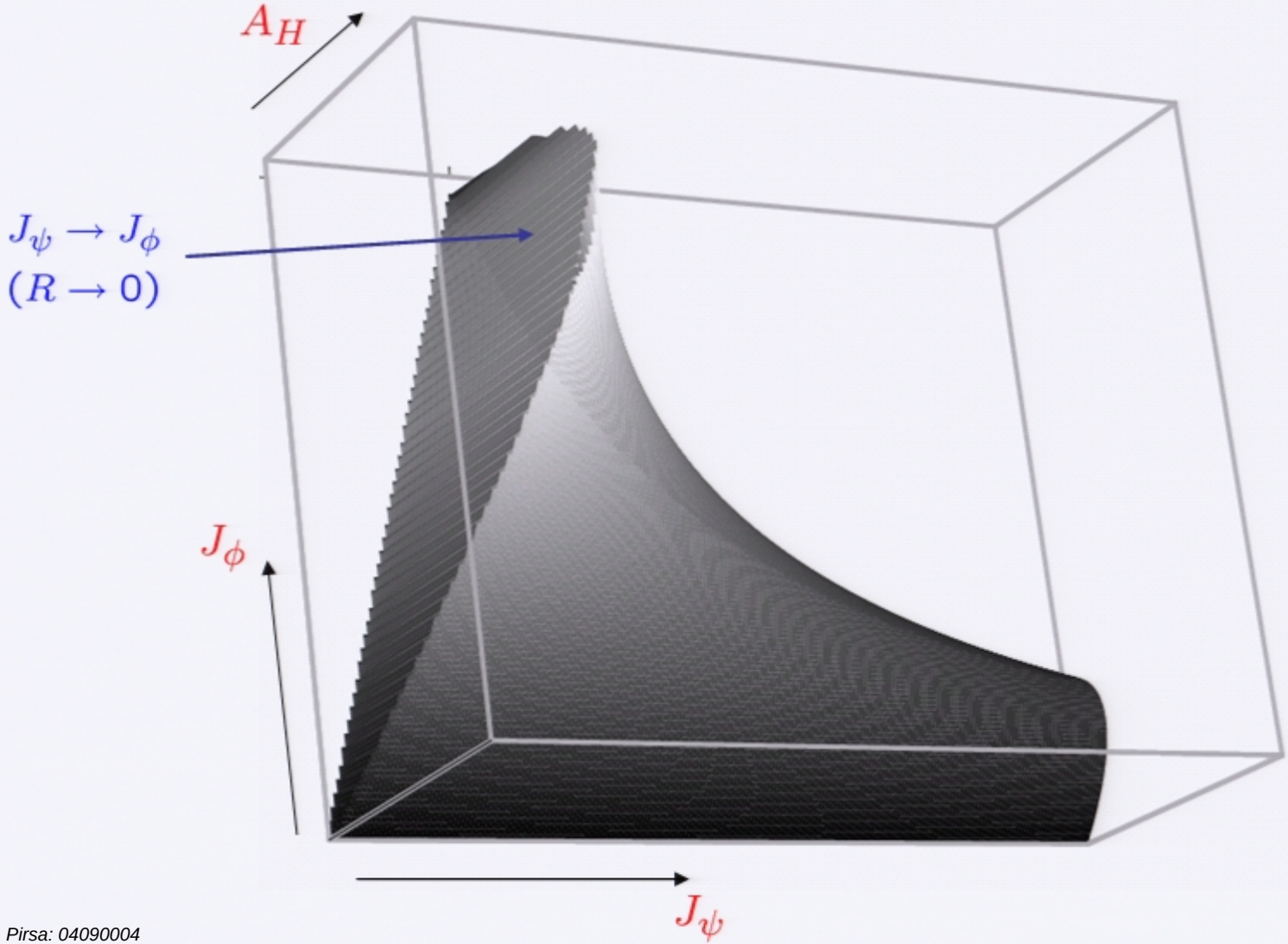

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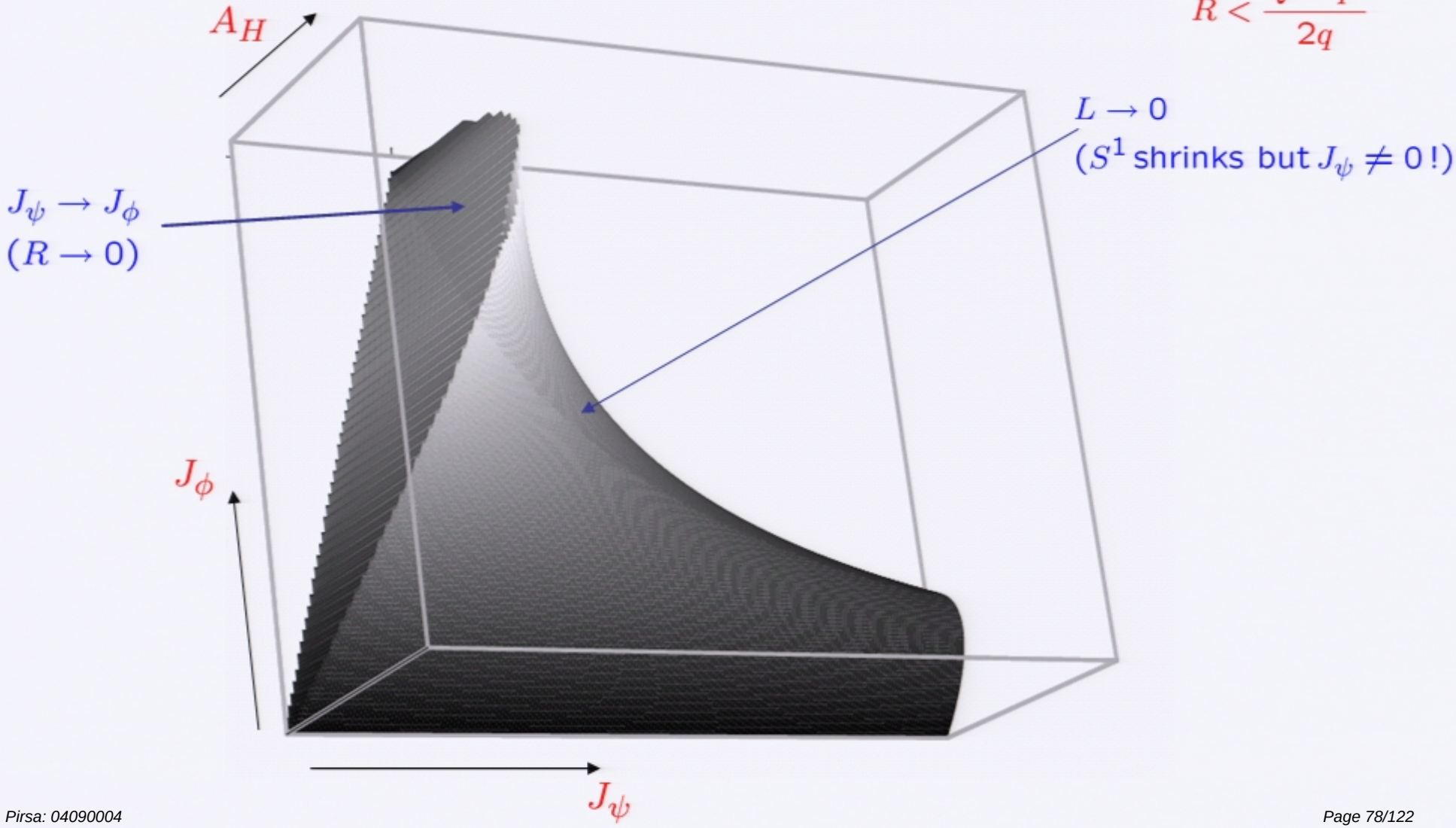

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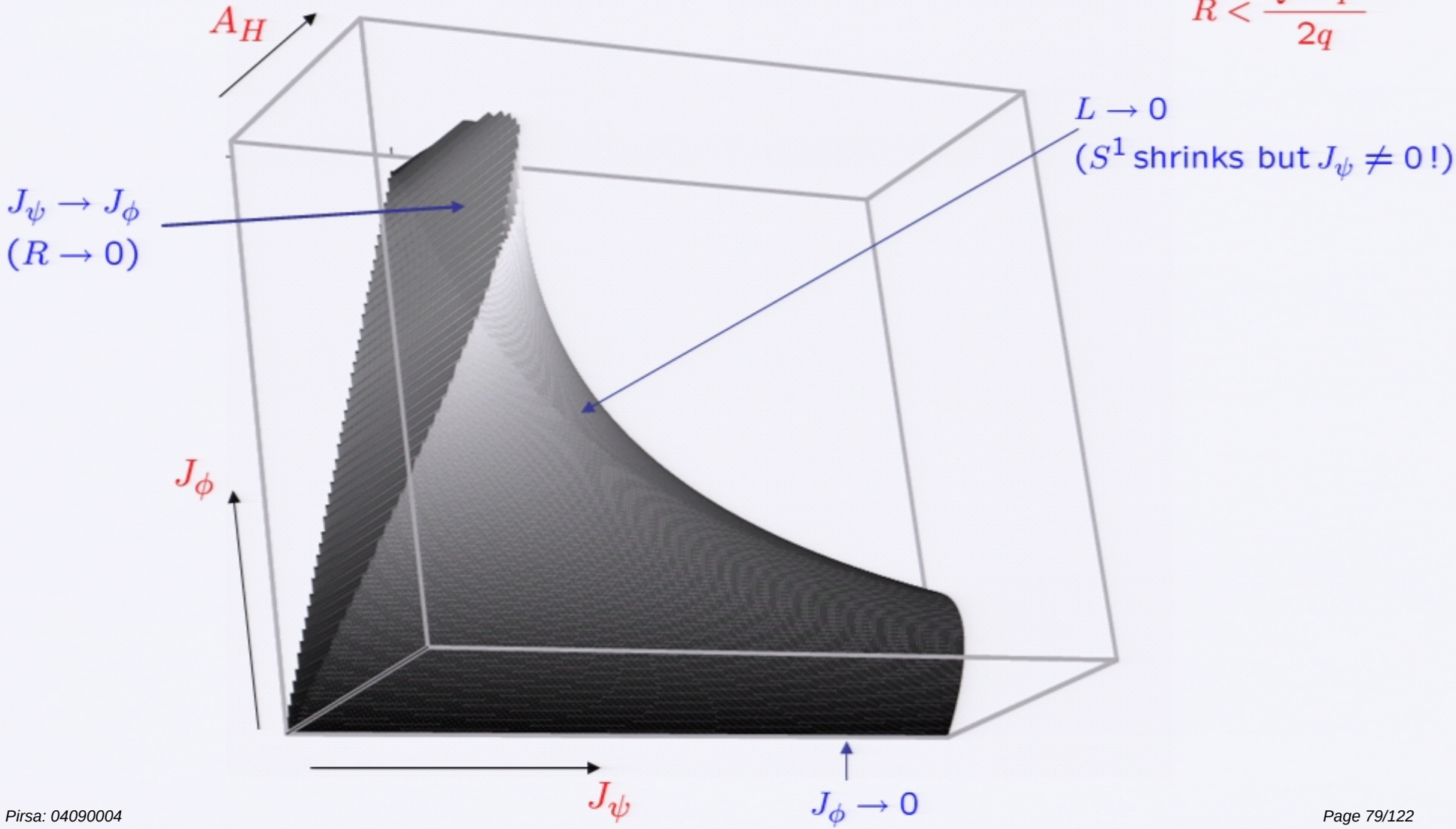

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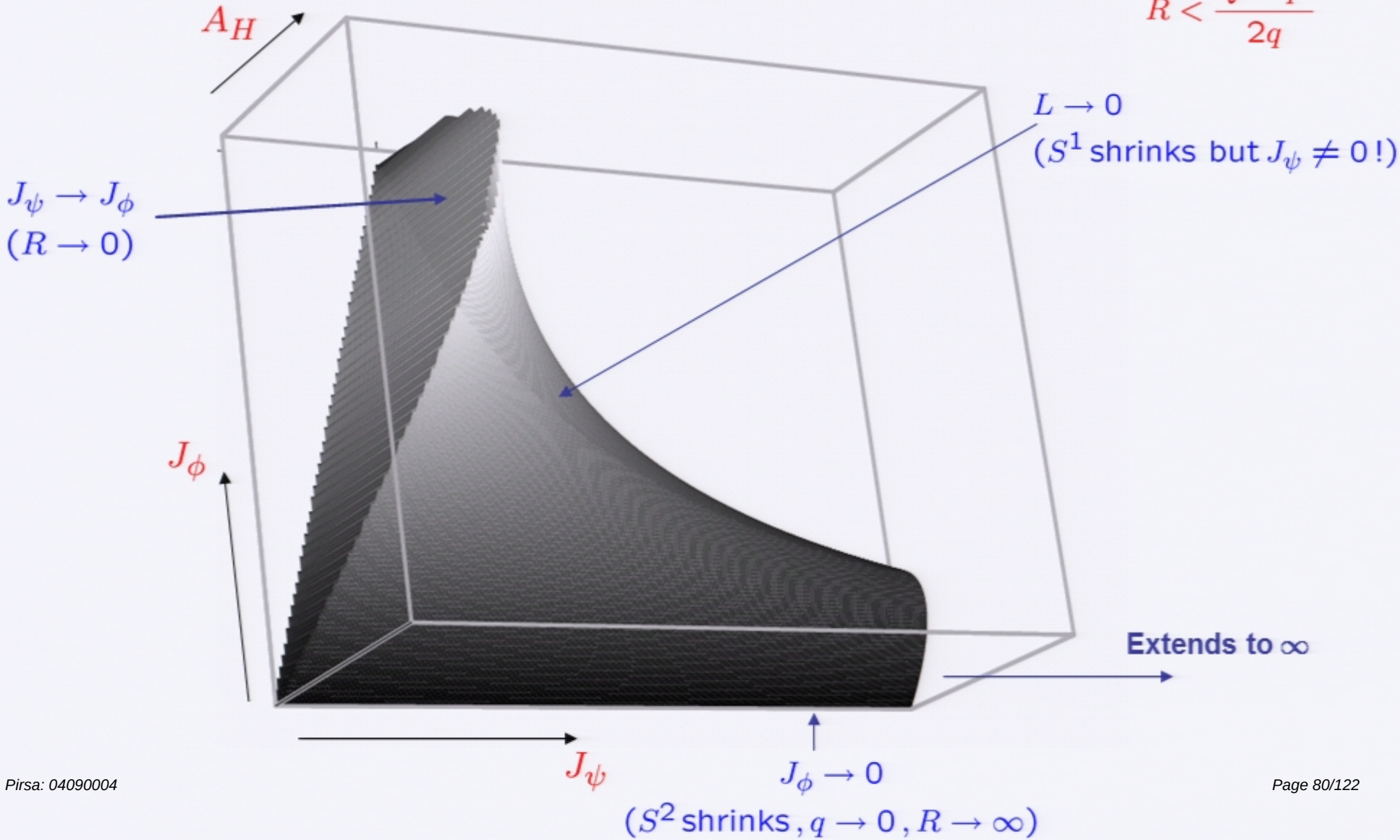

Non-uniqueness

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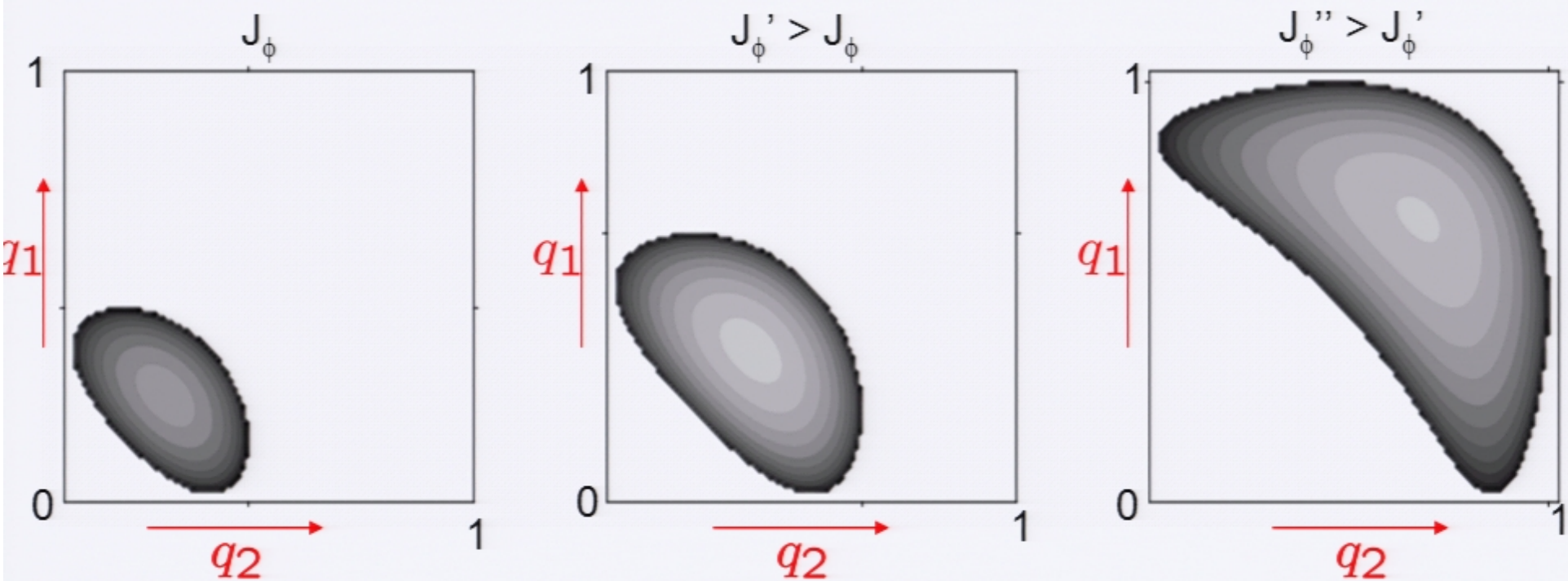

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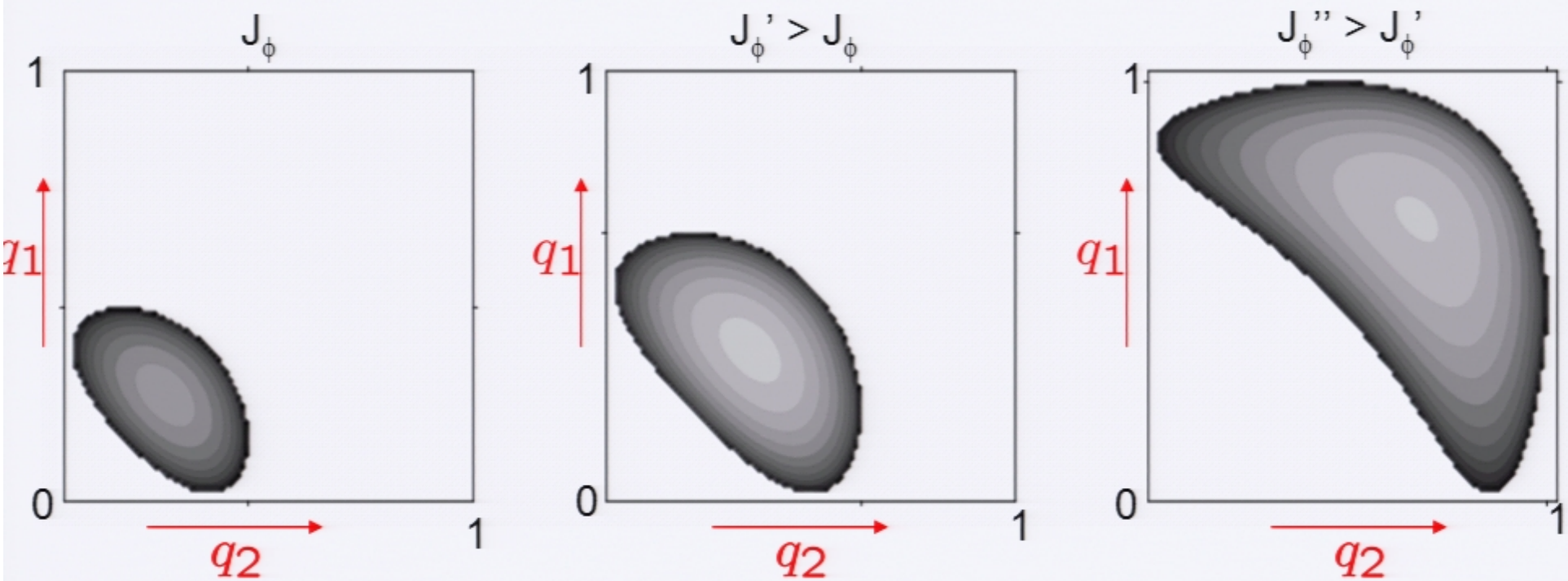

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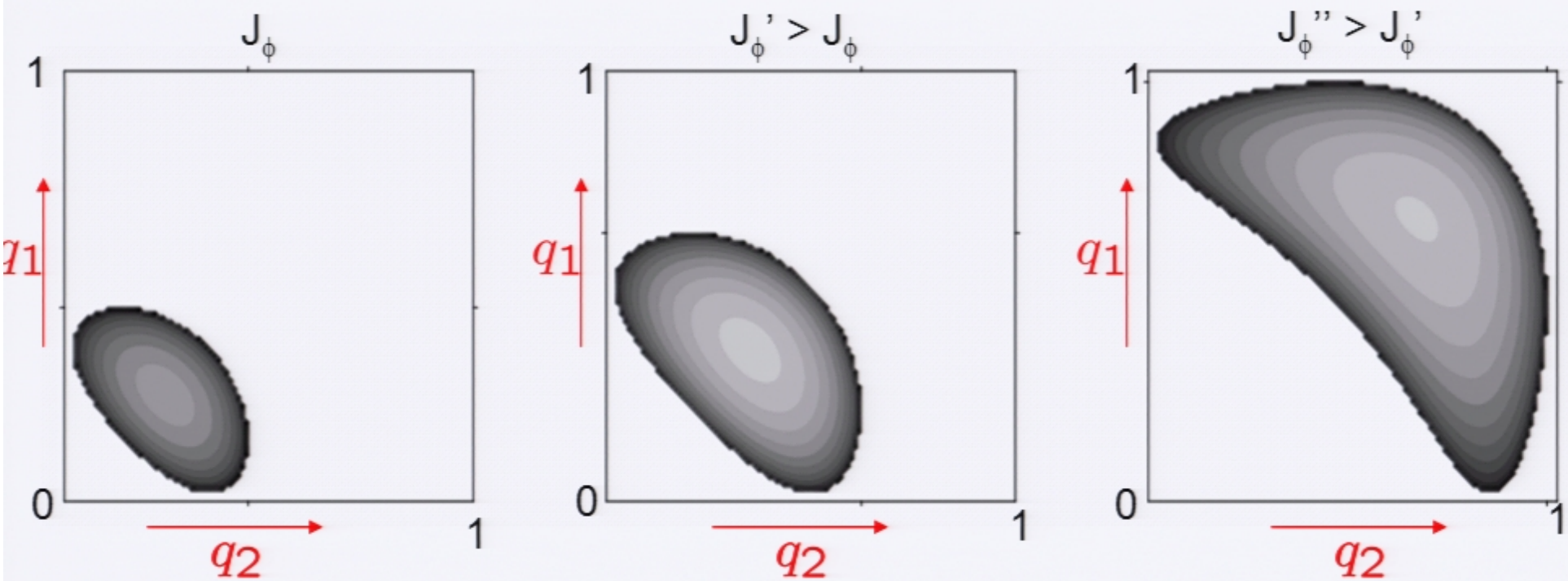

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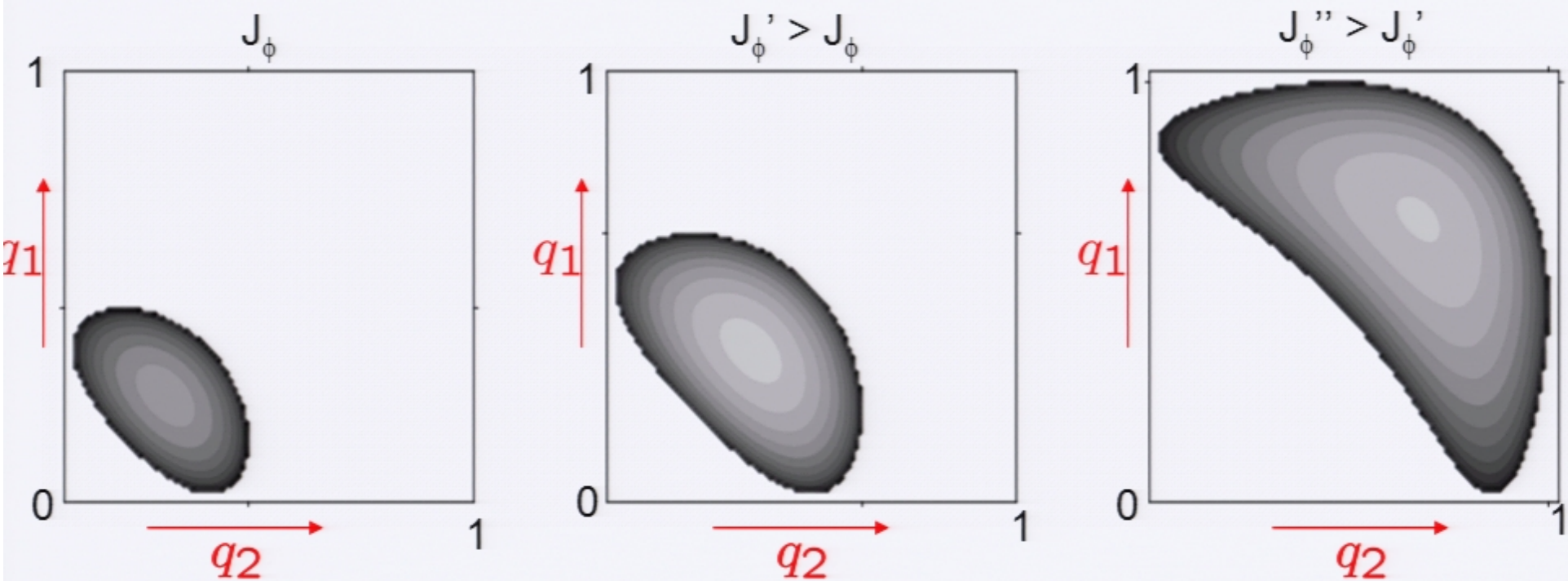

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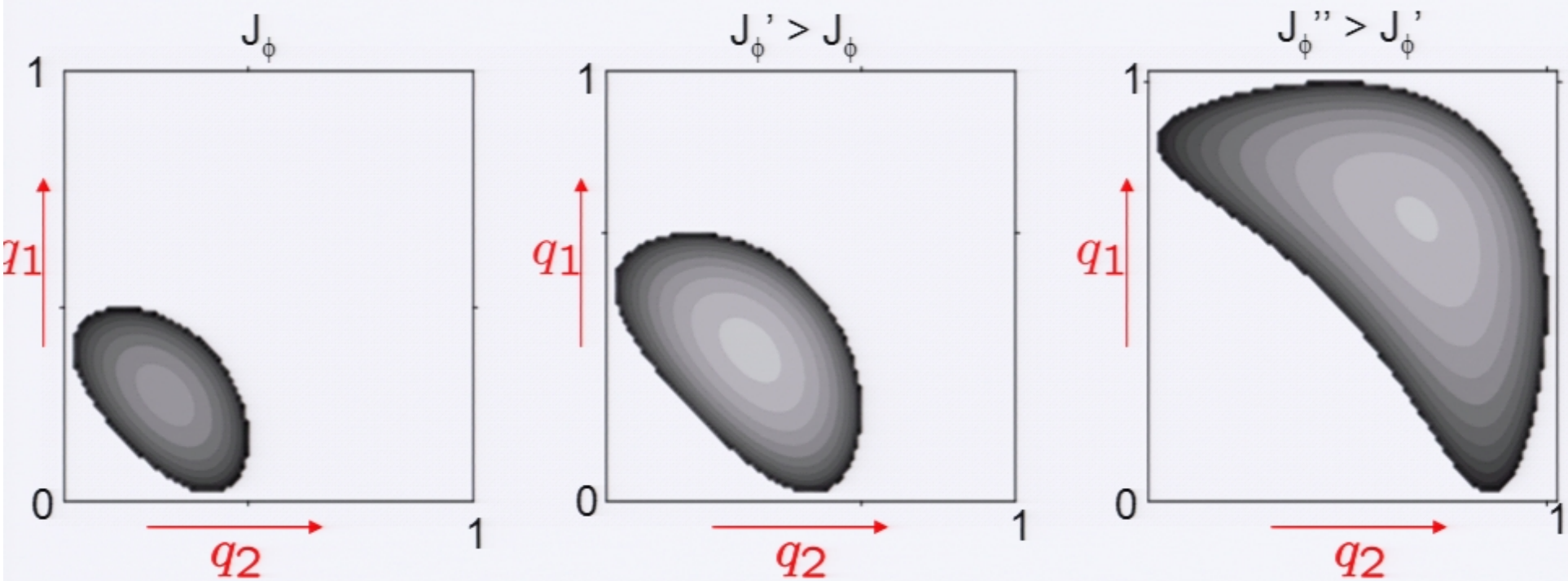

Even with extra requirement of constant area → Infinitely-many black rings (except for maximum area)

In quantum theory Q_i and q_i are quantised, $Q_1 \geq q_2 q_3$, etc → Finite non-uniqueness

Particular cases and limits: R=0

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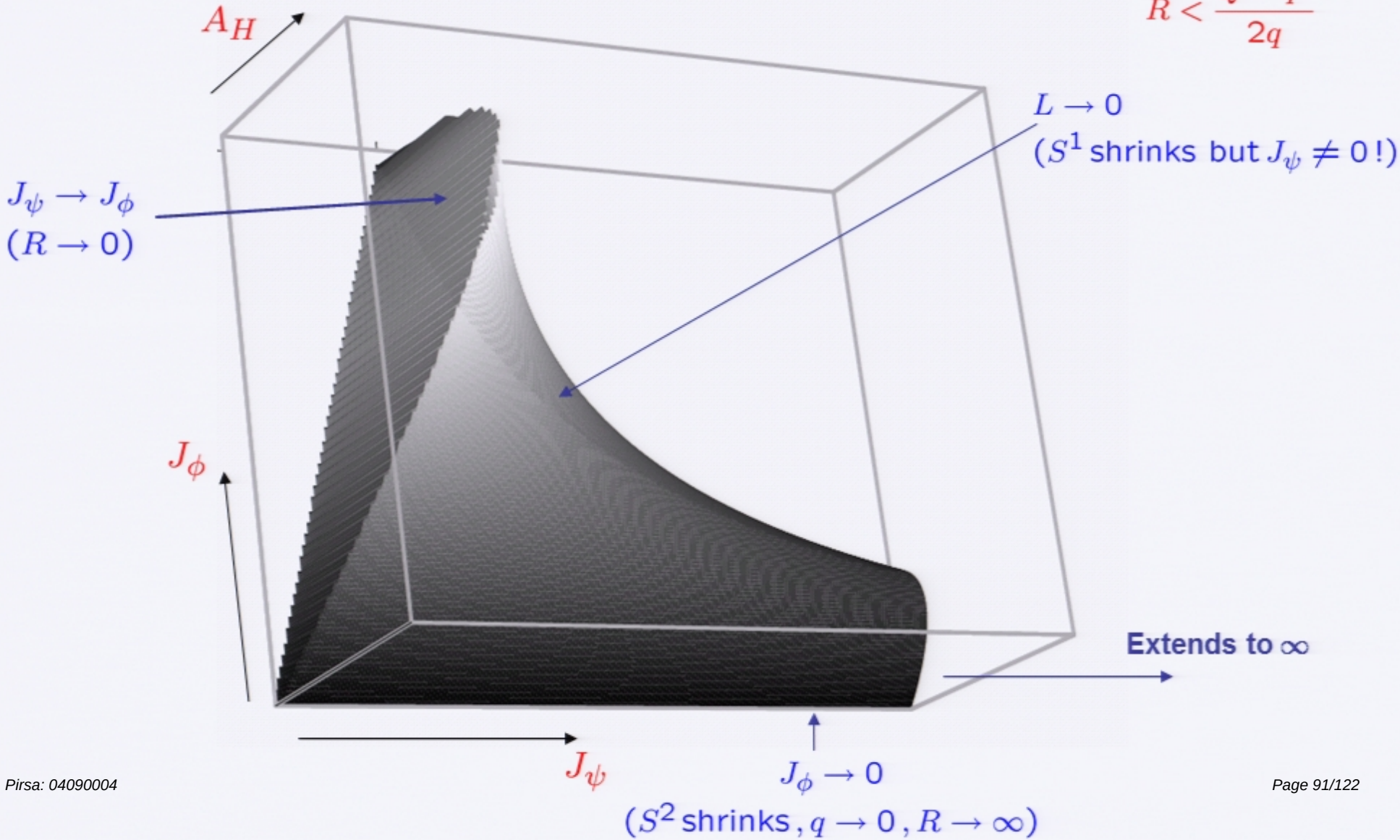
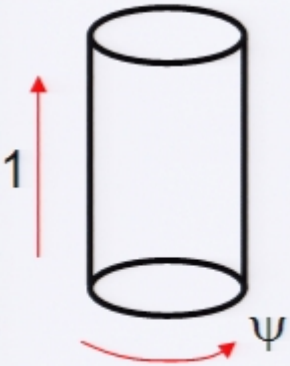


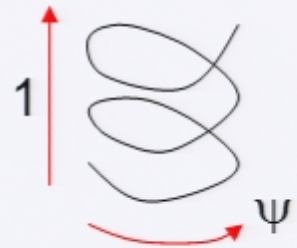
2-charge Supertubes → D1/D5 System

Lunin&Mathur
Maoz&Maldacena
Lunin,Maldacena&Maoz
.....



F1:	_	1
D0:	_	_
d2:	ψ	1

T1 →



P:	_	1
D1:	_	1
d1:	ψ	_

T2345 →

P:	_	1	_	_	_	_
D5:	_	1	2	3	4	5
d5:	ψ	_	2	3	4	5

S →

P:	_	1	_	_	_	_
NS5:	_	1	2	3	4	5
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(Trivial T) **S** →

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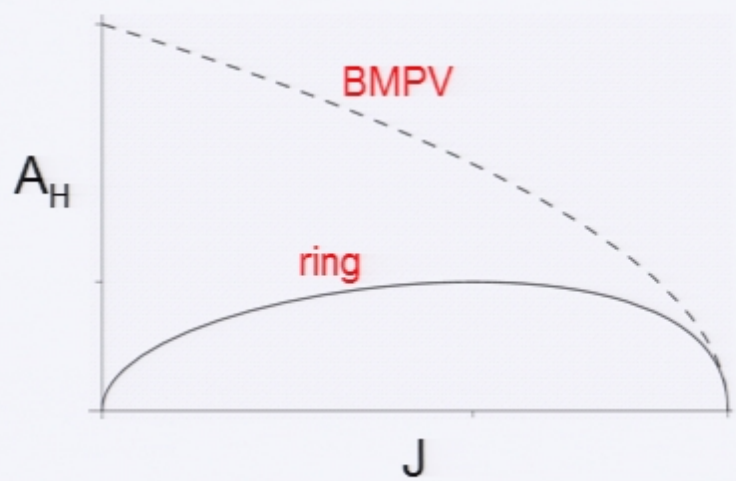

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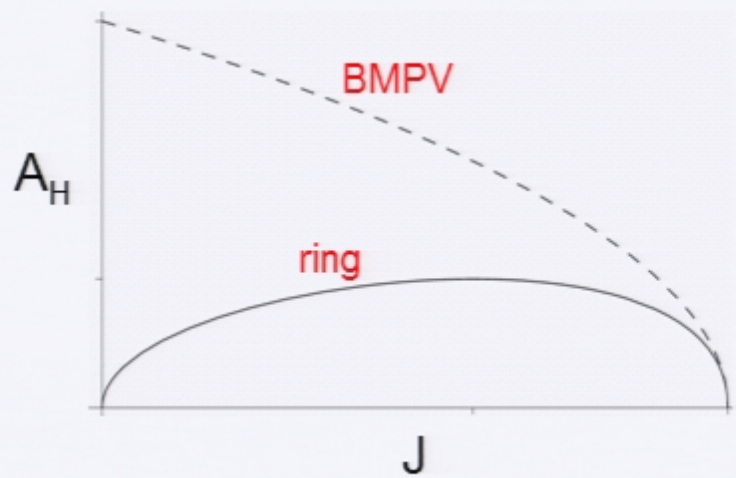
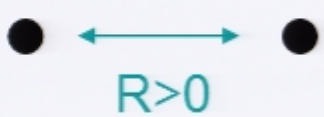


Analogous to the merging of two 4D RN black holes:



$A = 2 \times 4\pi Q^2$

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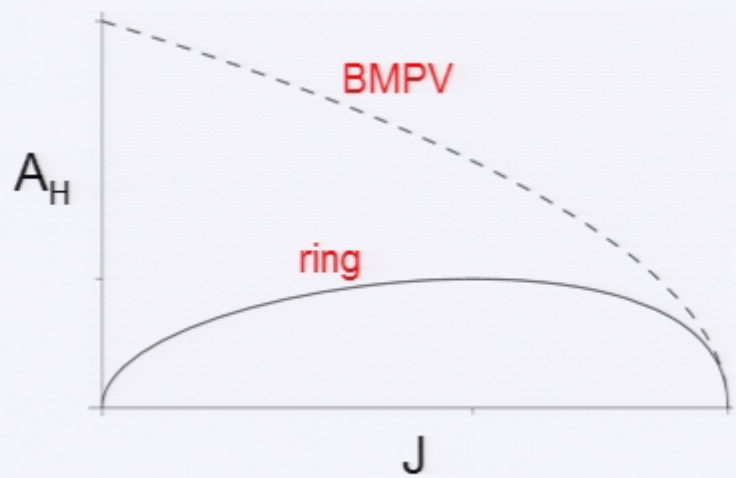

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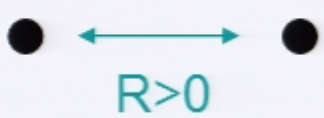

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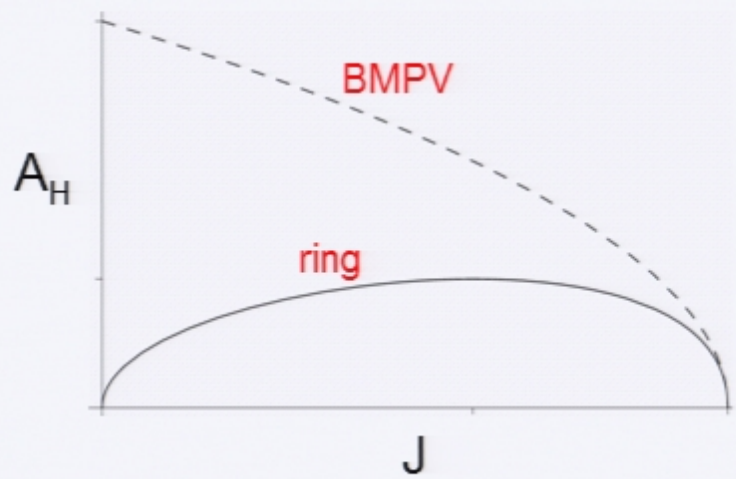

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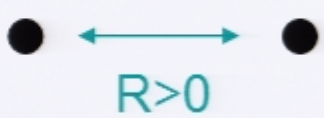

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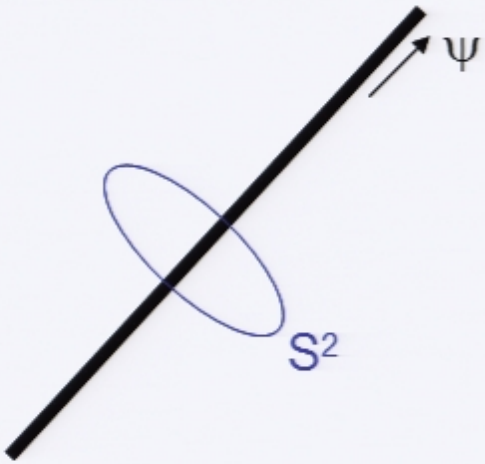

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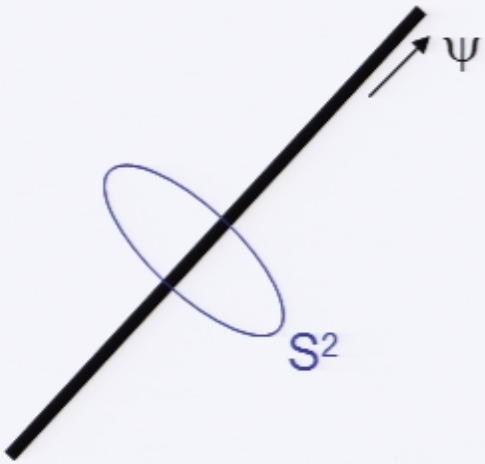

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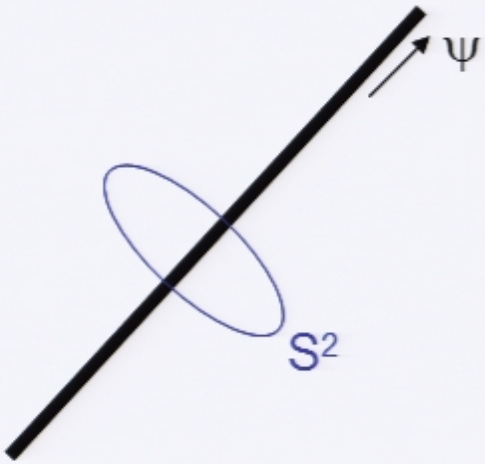

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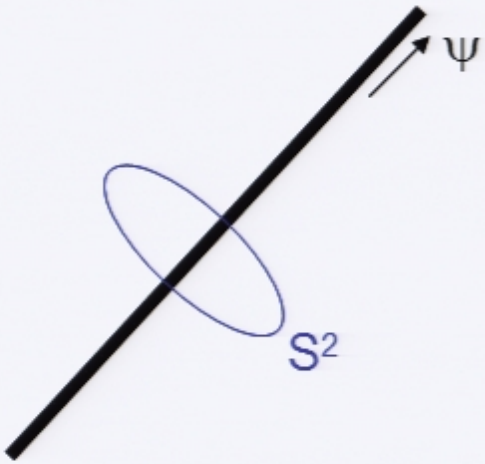
Important: $J_\psi = P_\psi \neq 0$ but $J_\phi \to 0$!

In fact, required by SO(3) invariance:



Speculation: J_ϕ Poynting-generated by SUGRA fields

Worldvolume 3-charge Supertubes = Calibrated Supertubes

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First step: F1/D4/D0 bound state with D2/D6/NS5 dipoles

Bena & Kraus

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First step: F1/D4/D0 bound state with D2/D6/NS5 dipoles

Bena & Kraus

Q_1 M2:	1	2	_	_	_	_	_	_	_	_
Q_2 M2:	_	_	3	4	_	_	_	_	_	_
Q_3 M2:	_	_	_	_	5	6	_	_	_	_
q_1 m5:	_	_	3	4	5	6	ψ	_	_	_
q_2 m5:	1	2	_	_	5	6	ψ	_	_	_
q_3 m5:	1	2	3	4	_	_	ψ	_	_	_

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q_1 m5:	_	_	3	4	5	6	7	_	_	_
q_2 m5:	1	2	_	_	5	6	7	_	_	_
q_3 m5:	1	2	3	4	_	_	7	_	_	_

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Single M5-brane = 2-dim complex surface S in $\mathbb{C}^3$

Gibbons & Papadopoulos
Gauntlett, Lambert & West

Calibrated by $h \wedge h$, with $h = dz^1 \wedge dz^2 + dz^3 \wedge dz^4 + dz^5 \wedge dz^6$ Kahler form

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Turning on $H = h \wedge d\sigma + H'$ induces M2 charges and allows arbitrary C

Elvang, Emparan, Reall & D.M.

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q_3 m5:	1	2	3	4	_	_	C	_	_	_

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(locally 2-charge $\rightarrow$ susy junction)

3-charge Supertubes → D1/D5/P System

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Reducing to 10D and dualising:

Q_1 D5:	1	2	3	4	5	_	_	_	_
Q_2 D1:	_	_	_	_	5	_	_	_	_
Q_3 P:	_	_	_	_	5	_	_	_	_
q_1 d1:	_	_	_	_	_	ψ	_	_	_
q_2 d5:	1	2	3	4	_	ψ	_	_	_
q_3 kk5:	1	2	3	4	•	ψ	_	_	_

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Q_2 D1:	_	_	_	_	5	_	_	_	_
Q_3 P:	_	_	_	_	5	_	_	_	_
q_1 d1:	_	_	_	_	_	ψ	_	_	_
q_2 d5:	1	2	3	4	_	ψ	_	_	_
q_3 kk5:	1	2	3	4	•	ψ	_	_	_

Decoupling limit: $\alpha' \to 0$, r /α' fixed, etc.

$$AdS_3 \times S^3 \times T^4 \xrightarrow{\text{RG}} AdS_3 \times S^3/Z_{q_3} \times T^4$$

$$\ell_{UV} = (Q_1 Q_2)^{1/4} \quad > \quad \ell_{IR} = \sqrt{q_3}(q_1 q_2)^{1/4}$$

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Q_3 P:	_	_	_	_	5	_	_	_	_
q_1 d1:	_	_	_	_	_	ψ	_	_	_
q_2 d5:	1	2	3	4	_	ψ	_	_	_
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Reproduction of the entropy by `splitting string':

Bena & Kraus

$$J_\phi = \frac{\pi}{8G_5}(q_1 Q_1 + q_2 Q_2 + q_3 Q_3 - q_1 q_2 q_3)$$

$$J_\psi = \frac{\pi}{8G_5} 2R^2 (q_1 + q_2 + q_3) + J_\phi$$

BMPV-like, carried by fermion momentum modes

Supertube-like, carried by fermion zero modes

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