Title: Aspects of Dark Energy

Date: Sep 22, 2004 02:00 PM

URL: http://pirsa.org/04090002

Abstract:

ASPECTS OF DARK ENERGY

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Sept. 22, 2004

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OUTLINE

1) THE FATE OF DARK ENERGY

2) STABILITY ISSUES FOR $W < -1$

3) ENHANCED BIG RIP WITHOUT DE.

2. THE FATE OF DARK ENERGY

The concordance of CMB, LSS and SNe1a data lead to the values $\Omega_X \sim 0.7$ and $\Omega_M \sim 0.3$.

One question is: to what extent can precision cosmological data allow us to discriminate between possible future fates of the Universe?

If one assumes that Ω_X corresponds to a cosmological constant with equation of state given by $w = p/\rho = -1$ then the future evolution of the universe follows from the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} \qquad (4)$$

in which $a(t)$ is the scale factor normalized at the present time as $a(t_0) = 1$, $\rho(t) = \rho(0)a^{-3}$ is

In such a simple, and still viable, case the behavior of $a(t)$ asymptotically for large $t \to \infty$ is

$$a(t) \sim \exp\left(\sqrt{\frac{\Lambda}{3}}t\right) \tag{5}$$

so that dark energy asymptotically dominates and the Universe is blown apart in an infinite time $a(t) \to \infty$ as $t \to \infty$.

Even assuming a constant equation of state $w = p/\rho$ there is a wide spread in the range of allowed w with an upper limit of $w \sim -0.8$ and a lower limit conservatively $w = -2$.

Note that the earliest WMAP analysis in `astro-ph/0302207` used a prior that $w \geq -1$. The relaxation of this prior is awaited to find their lower limit on w.

The future fate of the dark energy ranges from a diverging scale factor at a finite future time to a disappearing dark energy with reversion to domination by "ordinary" matter $a \sim t^{2/3}$.

Constant equation of state

Keeping only the dark energy term:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_X a^{-\beta} \qquad (6)$$

where $\beta = 3(1+w)$. When $w < -1$, $\beta < 0$ and the solution of Eq.(6) diverges at a finite time $t = t^*$. Integrating

$$\int_{a(t_0)}^{\infty} a^{\beta/2-1} = H_0\sqrt{\Omega_X} \int_{t_0}^{t^*} dt \qquad (7)$$

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one finds the remaining time $t_r = (t^* - t_0)$ is given by:

the remaining time before the scale factor diverges which is:

$$t_r = \frac{2}{3H_0} \frac{1}{\sqrt{\Omega_X}(-w-1)} = \frac{11Gyr}{(-w-1)} \qquad (8)$$

In Eq.(8), putting in $\Omega_X = 0.7$ and $2/(3H_0) = 9.2Gyr$ one finds for $w = -1.5, -2$ respectively $t_r = 22Gyr, 11Gyr$. In the more extreme case $w = -2.5$, one finds $t_r = 7Gyr$.

Note that the Sun will transform into a Red Giant, and swallow the Earth, approximately 5 Gyr from now.

Gravitationally-bound systems could survive longer than t_r given by Eq.(8) but such systems the remaining time before the scale factor diverges which is:

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Equation of State Varying Linearly with Red Shift

As a more general ansatz, we consider the model for the EoS depending linearly on red shift:

$$w(z) = w(0) + Cz\theta(z_c - z) + Cz_c\theta(z - z_c) \quad (9)$$

where the modification is cut off arbitrarily at some $z = z_c > 0$. We assume $C \geq 0$ and consider the two-dimensional parameter space spanned by the two variables $w(0)$ and C.

As input data we use the CMB spectrum and CMB/LSS). Also we simultaneously make χ^2

Equation of State Varying Linearly with Red-Shift.

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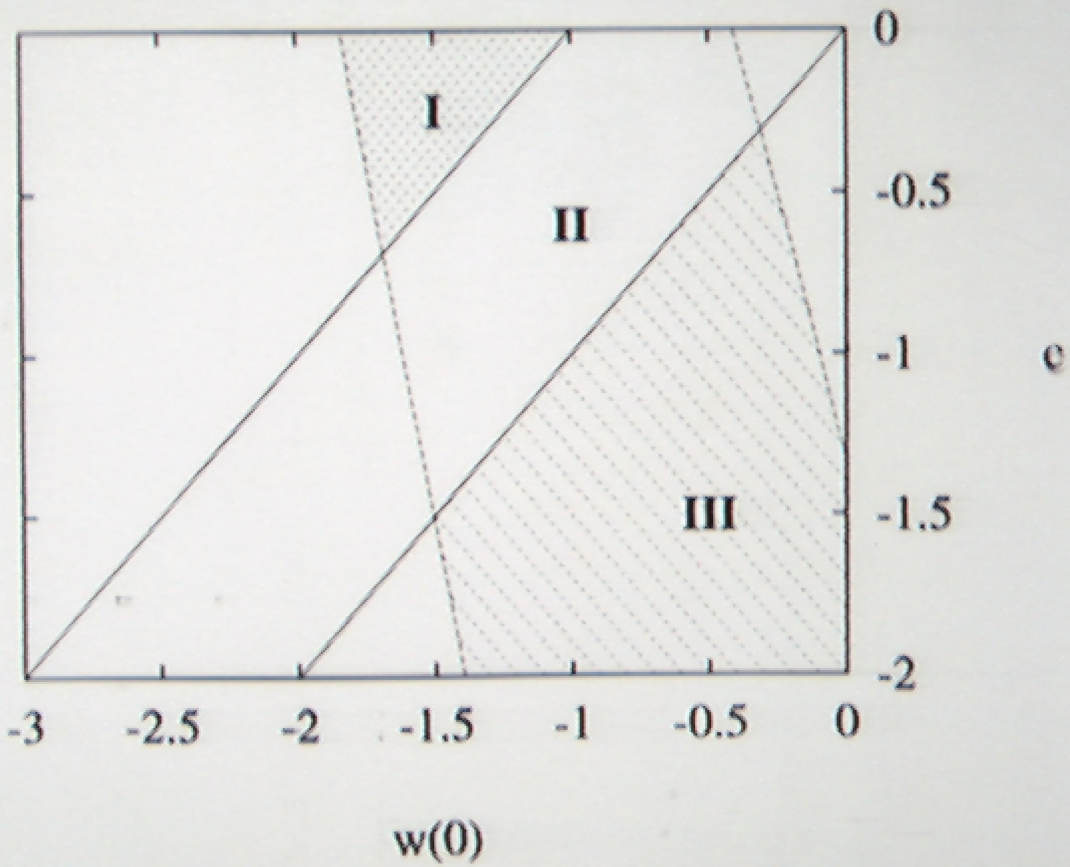
where the modification is cut off arbitrarily at some $Z = \zeta > 0$. We assume $C \leq 0$ and consider the two-dimensional parameter space spanned by the two variables w(0) and C.

As input data we use the CMB spectrum and CMBFAST. Also we simultaneously make ξ^2 fits to the SNe1A data.

To set the stage, let us first use only the SNe1A data to constrain the parameters w(0) and C. The result is shown for $\zeta = 2$ in Figure

(I) $w(0) < (C-1)$. In this case there is an end of time, at a finite future time.
(II) $(C-1) \leq w(0) < C$. Here the lifetime of the universe is infinite. The dark energy dominates over matter, as now, at all future times.
(III) $C \leq w(0)$. The lifetime of the universe is again infinite but after a finite time the dark energy will disappear relative to the dark matter and matter-domination will be re-established with $a(t) \sim t^{2/3}$.

Fig. 1



When we add the constraints imposed by the CMB data, the allowed region is smaller as shown in Figure 2, plotted for $\zeta = 0.5$. Such a small ζ still allows all three future possibilities (I), (II) and (III). For somewhat larger ζ only possibilities (I) and (II) are allowed in this particular parametrization.

The case $\zeta = 2$ is exhibited in more detail for different values of $w(0)$ and C in Figures 3 and 4.

Figure 3 shows the variation of the transition red-shift Z_{trans} where deceleration changes to accelerated cosmic expansion defined by $q(Z_{trans}) = 0$.

Figure 4 shows the corresponding fits to the

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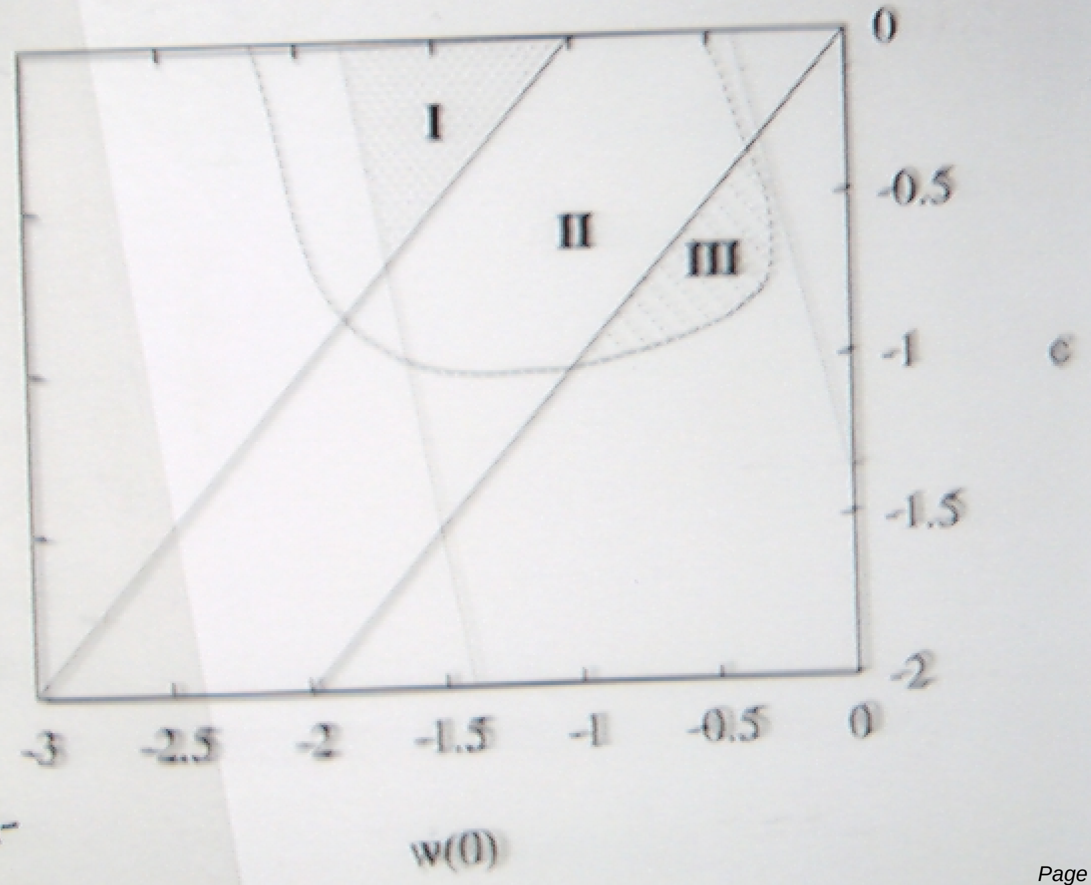
Fig. 2



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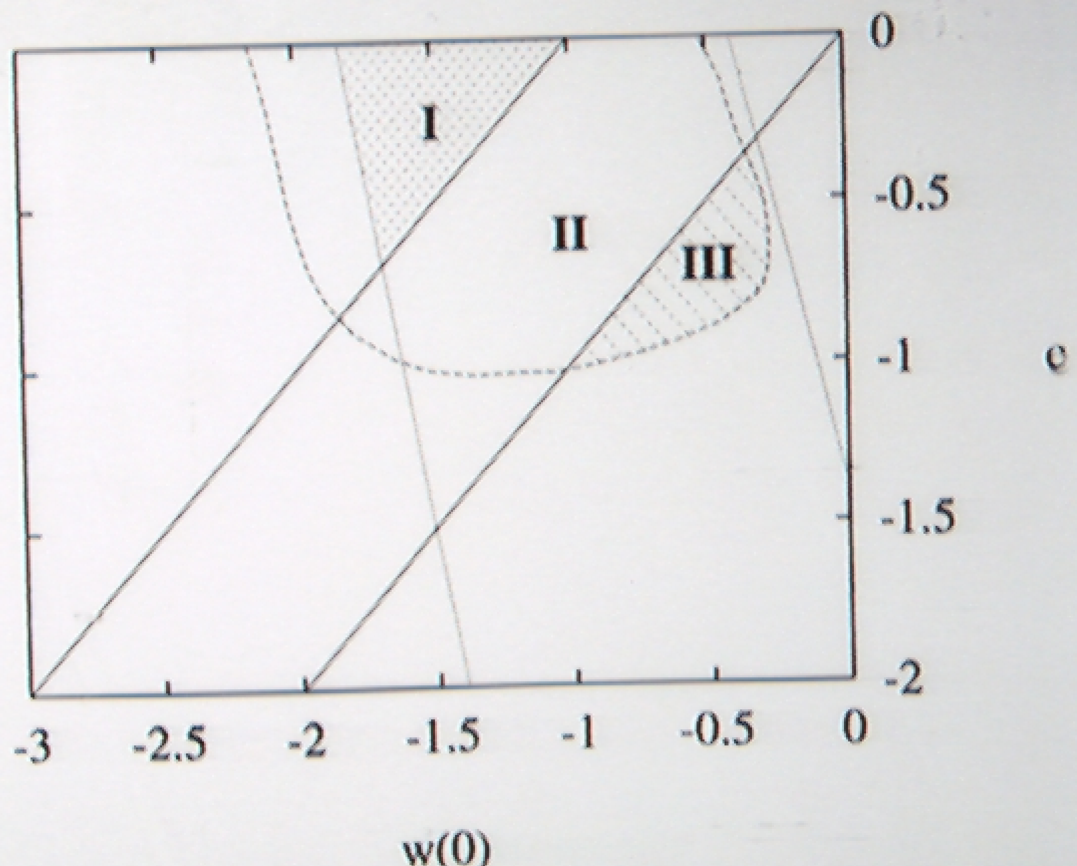
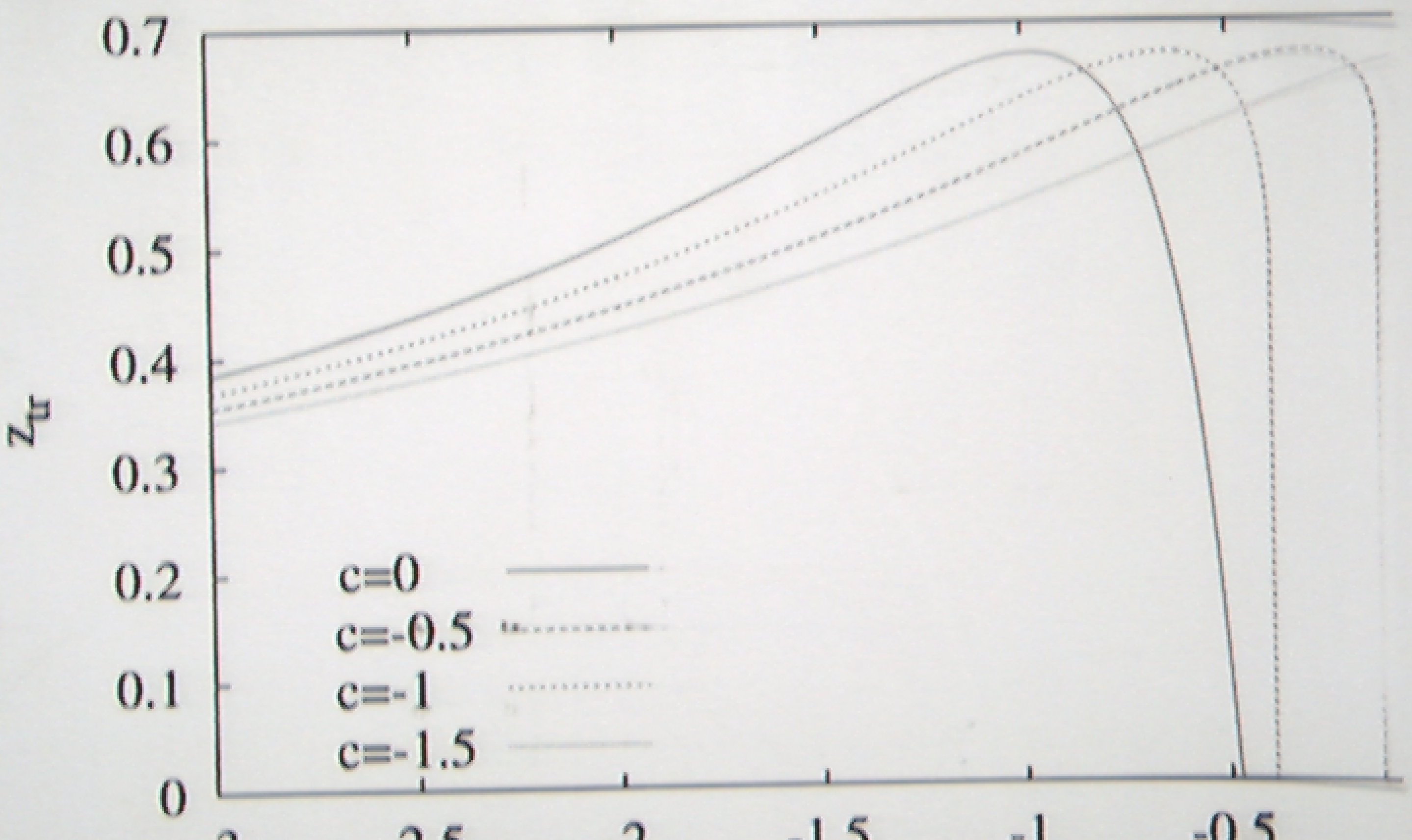


Fig. 3



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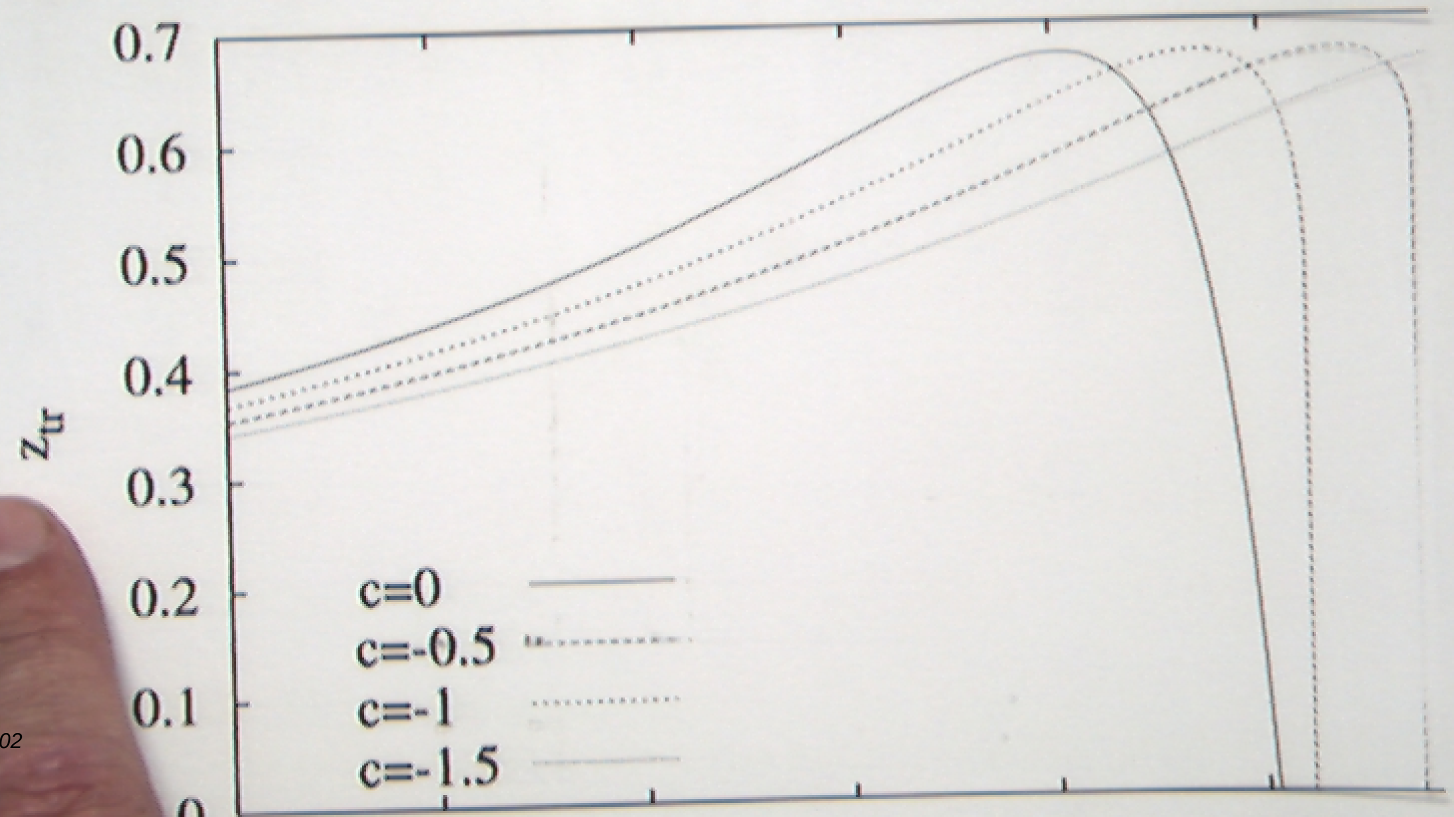

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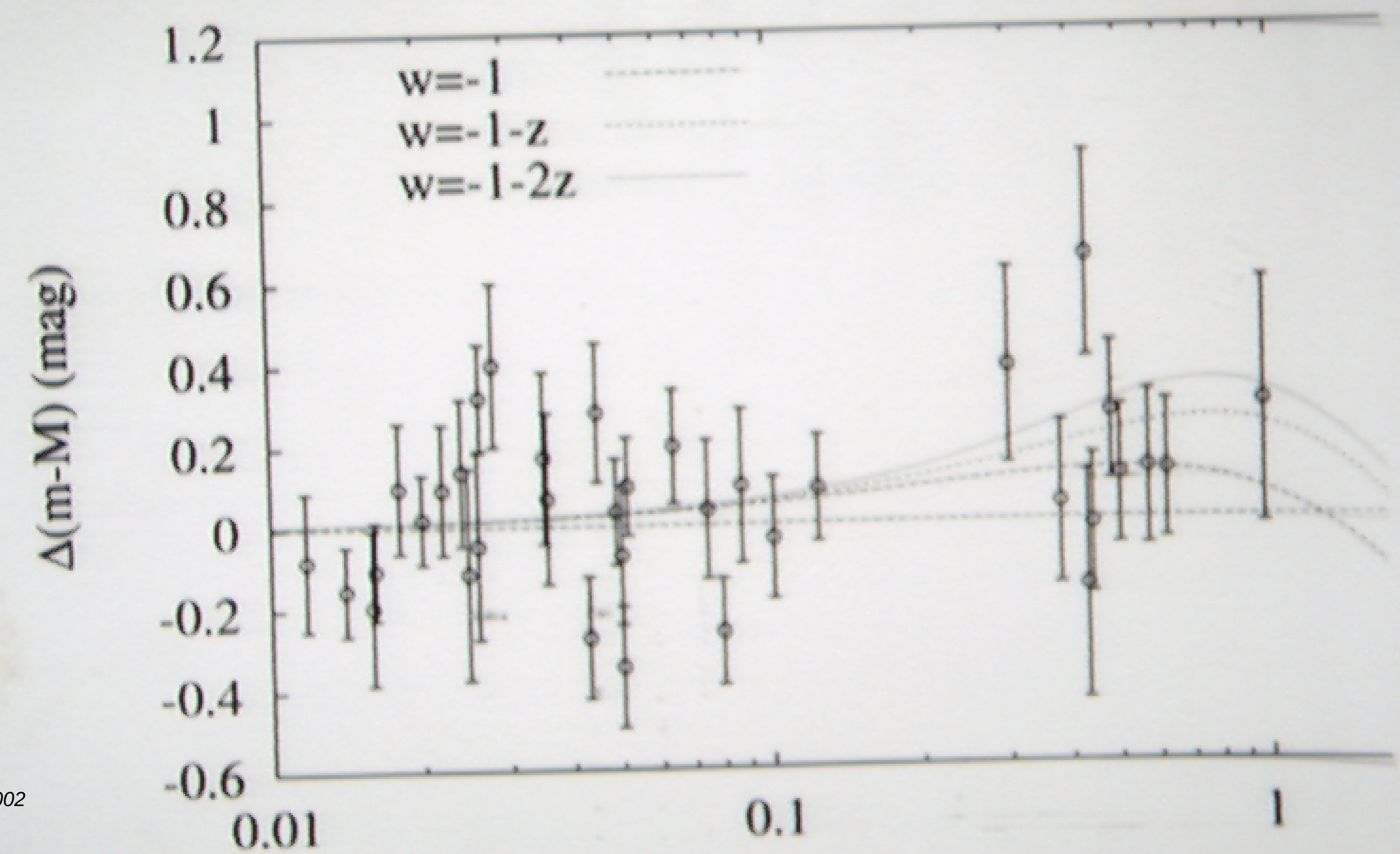

To return to our main point, let us assume that more precise cosmological data will allow an approximate determination of $w(Z) = f(Z)$ as a function of Z for positive $Z > 0$. Then to illustrate the possible future evolutions write:

$$w(Z) = f(Z)\theta(Z) + (f(0) + \alpha Z)\theta(-Z) \quad (10)$$

In this case, the future scenarios (I), (II) and (III) occur respectively for $\alpha > -f(0) > 0$, $-f(0) > \alpha > -f(0) - 1$ and $\alpha < -f(0) - 1$.

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3. Stability Issues for $w < -1$ Dark Energy

Precision cosmological data hint that a dark energy with equation of state $w = P/\rho < -1$ and hence dubious stability is viable. Here we discuss for any w nucleation from $\Lambda > 0$ to $\Lambda = 0$ in a first-order phase transition. The critical radius is argued to be at least of galactic size and the corresponding nucleation rate glacial, thus underwriting the dark energy's stability and rendering remote any microscopic effect.

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The equation of state for the dark energy component in cosmology has been the subject of much recent discussion Present data are consistent with a constant $w(Z) = -1$ corresponding to a cosmological constant. But the data allow a present value for $w(Z = 0)$ in the range $-1.38 < w(Z = 0) < -0.82$.

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Interpetation as a limiting velocity

Consider making a Lorentz boost along the 1-direction with velocity V (put c = 1). Then the stress-energy tensor which in the dark energy rest frame has the form:

$$T_{\mu\nu} = \Lambda \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & w & 0 & 0 \\ 0 & 0 & w & 0 \\ 0 & 0 & 0 & w \end{pmatrix}$$

is boosted to $T^{'}_{\mu\nu}$ given by

$$T'_{\mu\nu} = \Lambda \begin{pmatrix} 1 & V & 0 & 0 \\ V & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & w & 0 & 0 \\ 0 & 0 & w & 0 \\ 0 & 0 & 0 & w \end{pmatrix} \begin{pmatrix} 1 & V & 0 & 0 \\ V & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \Lambda \begin{pmatrix} 1 + V^2 w & V(1+w) & 0 & 0 \\ V(1+w) & V^2 + w & 0 & 0 \\ 0 & 0 & w & 0 \\ 0 & 0 & 0 & w \end{pmatrix}$$

We learn several things by studying this.

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We learn several things by studying this.

First, consider the energy component $T'_{00} = 1 + V^2 w$. Since $V < 1$ we see that for $w > -1$ this is positive $T'_{00} > 0$ and the WEC is respected. For $w = -1$, $T'_{00} \to 0$ as $V \to 1$ and is still never negative. For $w < -1$, however, we see that $T'_{00} < 0$ if $V^2 > -(1/w)$ and this is the first sign that the case $w < -1$ must be studied with great care. Looking at the pressure component T'_{11} we see the special role of the case $w = -1$ because $w = T'_{11}/T'_{00}$ remains Lorentz invariant as expected for a cosmological constant. Similarly the off-diagonal components T'_{01} vanish only in this case.

One alternative is that it is impossible for $V^2 > -(1/w)$. The highest velocities known are those for the highest-energy cosmic rays which are protons with energy $\sim 10^{20} eV$. These have $\gamma = (1 - V^2)^{-1/2} \sim 10^{11}$ corresponding to $V \sim 1 - 10^{-22}$. This would imply that:

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Interpretation as Vacuum Instability.

But let us suppose, that more precise cosmological data reveals a dark matter which has w significantly below -1. Then, by boosting to an inertial frame with $V^2 > -(1/w)$, one arrives at $T'_{00} < 0$ and this would be a signal for vacuum instability. If the cosmological background is a Friedmann-Robertson-Walker (FRW) metric the physics is Lorentz invariant and so one should be able to see evidence for the instability already in the preferred frame where $T_{\mu\nu}$ has $T_{00} > 0$.

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This goes back to work in the 1960's and 1970's where one compares the unstable vacuum to a superheated liquid. At one atmospheric pressure water can be heated carefully to above 100^0 C without boiling. The superheated water is metastable and attempts to nucleate bubbles containing steam. However, there is an energy balance for a three-dimensional bubble between the positive surface energy $\sim R^2$ and the negative latent heat energy of the interior $\sim R^3$ which leads to a critical radius below which the bubble shrinks away and above which the bubble expands and precipitates boiling.

For the vacuum the first idea in 1976 was to treat the spacetime vacuum as a four-dimensional material medium just like super-heated water. The second idea was to notice that a hyperspherical bubble expanding at the speed of light is the same to all inertial observers. This Lorentz invariance provided the mathematical relationship between the lifetime for unstable vacuum decay and the critical radius of the four-dimensional bubble or instanton. In the rest frame, the energy density is

$$T_{00} = \Lambda \sim (10^{-3} eV)^4 \sim (\mathrm{mm})^{-4}$$

In order to make an estimate of the dark energy decay lifetime in the absence of a known potential, we can proceed by assuming it is the same Lorentz invariant process of a hyperspherical bubble expanding at the speed of light, the same for all inertial observers.

Let the radius of this hypersphere be R, its energy density be ϵ and its surface tension be S_1. Then the relevant instanton action is

$$A = -\frac{1}{2}\pi^2 R^4 \epsilon + 2\pi^2 R^3 S_1$$

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The stationary value of this action is

$$A_m = \frac{27}{2}\pi^2 S_1^4/\epsilon^3$$

corresponding to the critical radius

$$R_m = 3S_1/\epsilon$$

We shall assume that the wall thickness is negligble compared to the bubble radius. The number of vacuum nucleations in the past lightcone is estimated as

$$N = (V_u \Delta^4) exp(-A_m)$$

where V_u is the 4-volume of the past and Δ is the mass scale relevant to the problem.

This vacuum decay picture led to the proposals of inflation, for solving the horizon, flatness and monopole problems (only the horizon problem was generally known in 1976). None of that work addressed why the true vacuum has zero energy. Now that the observed vacuum has non-zero energy density $+\epsilon \sim (10^{-3}eV)^4$ we may interpret it as a "false vacuum" lying above the "true vacuum" with $\epsilon = 0$.

In order to use the full power of the instanton equations we need to estimate the three mass-dimension parameters $\epsilon^{1/4}$, $S_1^{1/3}$ and Δ therein. Let us discuss these three scales in turn.

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The easiest of the three to select is ϵ. If we imagine a tunneling through a barrier between a false vacuum with energy density ϵ to a true vacuum at energy density zero then the energy density inside the bubble will be $\epsilon = \Lambda = (10^{-3} eV)^4$. No other choice is reasonable.

Next we discuss the typical [illegible] scale Δ involved in the prefactor. The va[illegible] Δ does not matter very much because i[illegible]rs as a power rather than an e[illegible]tia[illegible]us put [illegible]factor $\Delta = \epsilon^{1/4} = (1mm)^{-1}$ w[illegible] the in N is $\sim 10^{116}$. [illegible] conclusions do not [illegible]

The third and last [illegible] face tension, S[illegible]

Spontaneous dark energy decay brings us to the question of whether such decay can be initiated in an environment existing within our Universe. The question is analogous to one of electroweak phase transition in high energy particle collision. This was first raised in 1976 and revisited for cosmic-ray collisions. That was in the context of the standard-model Higgs vacuum and the conclusion is that high-energy colliders are safe at all present and planned foreseeable energies because much more severe conditions have already occurred (without disaster) in cosmic-ray collisions within our galaxy. More recently, this issue has been addressed in connection with fears that the Relativistic Heavy Ion Collider (RHIC) might initiate a di-

The dark energy density is some 58 orders of magnitude smaller $[(10^{-3}eV)^4$ compared to $(300GeV)^4]$ than for the electroweak case and so the nucleation scales are completely different. One is here led away from microscopic towards astronomical size scales.

We see that the critical radius cannot be microsopic. Think first of a macroscopic scale *e.g.* 1 meter and consider a magnetic field practically-attainable in bulk on Earth such as 10 Tesla. Its energy density is given by

$$\rho_{mag} = \frac{1}{\mu_0}\mathcal{B}^2$$

Using the value $\mu_0 = 4\pi \times 10^{-7} N A^{-2}$ and $1T = 6.2 \times 10^{12}(MeV.s.m^{-2})$ leads to an energy density $\rho_{mag} = 2.5 \times 10^{17} eV/(mm)^3$, over 20 orders above the value of Eq.() for the interior of nucleation. Magnetic fields in bulk exist in galaxies with strength $\sim 1\mu G$ and the rescaling by $\mathcal{B}^2$ then would give $\rho_{mag} \sim (2.5 \times 10^{-5} eV)(mm)^{-3}$, slightly below the dark

Assuming the dark energy can exchange energy with magnetic energy density the observed absence of stimulated decay would then imply a critical radius of at least galactic size, say, $\sim 10kpc$. Using Eq() then gives for the surface tension $S_1 > 10^{23}(mm)^{-3}$ and number of nucleations in Eq.() $N < exp(-10^{92})$. The spontaneous decay is thus glacial. Note that the dark energy has appeared only recently in cosmological time and has never interacted with background radiation of comparable energy density. Also, this nucleation argument does not require $w < -1$.

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One may speculate how such stability arguments may evolve. One may expect most conservatively that the value $w = -1$ will eventually be established empirically in which case both quintessence and the "phantom menace" will be irrelevant. In that case, indeed for any w, we may still hope that dark energy will provide the first connection between string theory and the real world as in *e.g.* the BFM stringy dark energy. Even if precise data do establish $w < -1$, as in the "phantom menace" scenario, the dark energy stability issue is still under control.

Discussion

As a first remark, since the critical radius R_m for nucleation is astronomical, it appears that the instability cannot be triggered by any microscopic process. While it may be comforting to know that the dark energy is not such a doomsday phenomenon, it also implies at the same time the dreadful conclusion that dark energy may have no microscopic effect. If any such microscopic effect in a terrestrial experiment could be found, it would be crucial in investigating the dark energy phenomenon. We note that the present arguments are less model-dependent than those given elsewhere in the literature.

BIGGER RIP WITH NO DARK ENERGY

There are two ways to accommodate the accelerating expansion in the Friedmann equation (we will assume $k=0$, a flat universe):

(1) Keep GR and add a DE term to the RHS

(2) Modify GR and omit DE.

The more conservative approach is (1) based on the spectacular success of GR at Solar System scales.

However, there are two reasons to entertain seriously possibility (2):

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However, there are two reasons to entertain seriously possibility (2):

a) The Universe has size 10^{28} cm compared to the Solar System 10^{14} cm. There is no independent evidence that GR survives this huge extrapolation.

b) DE has bizarre properties so its avoidance might be considered.

DGP gravity suggested by

G. Dvali, G. Gabadadze and M. Porrati
PL B485, 208 (2000). hep-th/0005016.

is one modification of GR.

A 3-brane is embedded in Minkowski M_5

with an intrinsic curvature term:

$$S = M(t)^3 \int d^5X \sqrt{G}\, R^{(5)} + M_{Pl}^2 \int d^4x \sqrt{g}\, R$$

$R^{(5)}$ = 5-dimensional scalar curvature
$M(t)$ = time-dependent 5-dim. Planck mass.
G = determinant of 5-dim metric

At small distances, Newton's law is recovered on the brane.

At large distances $F \sim \frac{1}{r^3}$ in 5-dim.

The length scale where the two regimes cross is

$$L(t) = \frac{M_{Pl}^2}{M(t)^3}$$

The t-dependence represents a generalization of the original DGP approach.

4-dimensional coordinates are labeled by

$i, k = 0, 1, 2, 3$

Einstein's equations in empty space are modified to

$$\left(R^{ik} - \frac{1}{2} R g^{ik}\right) + \frac{2\sqrt{G}}{L(t)\sqrt{g}} \left[\left[\left(R^{(\alpha)ik} - \frac{1}{2} G^{ik} R^{(\alpha)}\right)\right]\right] = 0$$

where the notation is

$$\int d\alpha \, [[F(\alpha)]] \equiv f'(\alpha) S(\alpha)$$

Generalising the Schwarzschild solution leads to a modified potential V(r):

$$V(r) = -\frac{GM}{r} - \frac{2\sqrt{2}\sqrt{r_g r}}{L}$$

$L \gg r \gg r_g$
weak gravity regime

where

$$r_g = 2GM$$

is the Schwarzschild radius

Fractional change in V is

$$\left|\frac{\Delta V}{V}\right| = \sqrt{\frac{8r^3}{L^2 r_g}}$$

5

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Fractional change

$$\left|\frac{\Delta V}{V}\right.$$

To explore this we assume a power-law time dependence

$$L(t) = L(t_0)\, T(t)^p .$$

where $T(t) = \left(\frac{t_{rip} - t}{t_{rip} - t_0}\right)$

We assume $p > 0$ so that $L(t)$ decreases with increasing t. (for $p < 0$ it increases)

A gravitationally-bound system will become unbound at $t = t_u$ estimated by $\left|\frac{\Delta V}{V}\right| = 1$

$$\frac{8 r^3}{L(r_u)^2 r_g} = 1$$

$$t_{rip} - t_u = \frac{1}{8} \left(\frac{8 \ell_0^3}{L_0^2 r_g} \right)^{\frac{1}{2p}}$$

where $\gamma = (t_{rip} - t_0)^{-1}$ and ℓ_0 = characteristic scale of bound system

At a later time $t = t_{caus}$ the previously bound system became causally disconnected until t_{rip}:

$$t_{rip} - t_{caus} = \frac{\ell_0}{c} \left(\frac{a(t_{caus})}{a(t_u)} \right)$$

With $p = 1$ $\quad L(t_0) = H_0^{-1} = 1.3 \times 10^{28}$ cm

$\gamma = (20\,Gy)^{-1}$ $\quad$ (time $t_{rip} - t_0 = 20\,Gy$)

	l_0 (cm)	r_g (cm)	$t_{rip} - t_u$	$t_{rip} - t_{cons}$
GALAXY	5×10^{22}	3×10^{16}	100 My	4 My
SUN-EARTH	1.5×10^{13}	3×10^{5}	2 mos	31 hr
EARTH-MOON	3.5×10^{10}	0.86	2 wks	1 hr

As another example

$p = 1$ $\quad L(t_0) = 1.3 \times 10^{28}$ cm

$\gamma = (50\,Gy)^{-1}$ $\quad t_{rip} - t_0 = 50\,Gy$

	l_0 (cm)	r_g (cm)	$t_{rip} - t_u$	$t_{rip} - t_{cons}$
GALAXY	5×10^{22}	3×10^{16}	250 My	7 My
[illegible]	1.5×10^{13}	3×10^{5}	5 mos	2 days

MOON 3.5x10 0.86 2 [illegible]

...s another example

$p = 1$

$\gamma = (50\,Gy)^{-1}$

$L(t_0) = 1.3 \times 10^{28}$ cm

$t_{rip} - t_0 = 50\,Gy$

	l_0(cm)	r_g(cm)	$t_{rip} - t_u$	$t_{rip} - t_{caus}$
...AXY	5×10^{22}	3×10^{16}	250My	7My
...EARTH	1.5×10^{13}	3×10^{5}	5mos	2 days
MOON	3.5×10^{10}	0.86	1mo	2hrs

With a longer wait until trip disintegration of structure and causal disconnection occur correspondingly earlier before eventual Rip.

$p=1$ resembles the Big Rip, so now we investigate general $p>1$ which gives a Bigger Rip: a more singular $a(t)$ at $t \to t_{r.i.p.}$

As a specific example we will look at $p=2$ but develop the formalism for general p.

The modified Friedmann equation for DGP gravity is

$$H^2 - \frac{H}{L(t)} = 0$$

leading to

$$\frac{\dot{a}}{a} = H = H(t_0)\,\frac{1}{\quad}$$

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The modified Friedmann equation for DGP gravity is

$$H^2 - \frac{H}{L(t)} = 0$$

leading to

$$\frac{\dot{a}}{a} = H = H(t_0)\frac{1}{T^p}$$

Define as before ... then

$$\ln a(t) = -\int_1^T \frac{dT}{\gamma L_0 T^p}$$

For $p=1$ $a(t) = T^{-\frac{1}{\gamma L_0}}$ like $w<-1$ DE

but for $p>1$ there is the Bigger Rip

$$a(t) = a(t_0) \exp\left[\left(\frac{1}{T^{p-1}} - 1\right) \frac{1}{(p-1)\gamma L_0}\right]$$

which diverges more "essentially" as $T \to 1$.

Inversion gives:

$$T = \left[1 + (p-1)\gamma L_0 \ln a(t)\right]^{-\frac{1}{(p-1)}}$$

We should regard this as the dark energy although any function can be re-expressed as some $w_{eff}(t)$

$$w_{eff}(t) = -1 - \frac{2}{3}\left(\frac{p\gamma L_0}{1+(p-1)\gamma L_0 \ln a(t)}\right)$$

So that

$$w_{eff}(t_0) = -1 - \frac{2}{3} p \gamma L_0$$

and

$$w_{eff}(t \to t_{inf}) \to -1$$

e.g. for $p=1$ $\gamma = (20 G_N)^{-1}$

$w_{eff}(t_0) = -1.47$ is allowed with the

$$3\left(\frac{\ldots}{1+(p-1)\gamma L_0 \ln a(t)}\right)$$

So that

$$w_{eff}(t_0) = -1 - \frac{2}{3} p \gamma L_0$$

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g. for $p=1$ $\gamma = (20 \, Gy)^{-1}$

$$w_{eff}(t_0) = \underline{-1.47}$$

IS allowed with this parametrization (see below)

Here ...

$$\gamma = (20\ \text{Gy})^{-1} \qquad L_0 = 1.3 \times 10^{28}\ \text{cm}$$

	l_0 (cm)	r_g (cm)	$t_{rip} - t_u$	$t_{caus} - t_u$
GALAXY	5×10^{22}	3×10^{16}	2.37 Gy	1.1 Gy
SUN-EARTH	1.5×10^{13}	2.95×10^{5}	9.6×10^{4} y	7 y
EARTH-MOON	3.5×10^{10}	0.86	2.5×10^{4} y	6 MOS

Compared to the $p=1$ Big Rip the $p=2$ Bigger Rip with other parameters same leads to more rapid expansion

earlier t_u

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So that

$$W_{eff}(t_0) = -1 - \frac{2}{3} p \gamma L_0$$

and

$$W_{eff}(t \to t_{rip}) \to -1$$

e.g. for $p = 1$ $\gamma = (20\,G_{y})^{-1}$

$W_{eff}(t_0) = -1.47$ <u>is</u> allowed with this

$$\gamma = (20\,Gy)^{-1} \qquad L_0 = 1.3 \times 10^{28}\ cm$$

	l_0 (cm)	r_g (cm)	$t_{rip} - t_m$	$t_{cross} - t_m$
GALAXY	5×10^{22}	3×10^{6}	2.27 Gy	1.1 Gy
SUN-EARTH	1.5×10^{13}	2.95×10^{5}	9.6×10^{9} y	7 y
EARTH-MOON	3.5×10^{10}	0.86	3.5×10^{9} y	6 mos

Compared to the $p=1$ Big Rip the $p=2$ Bigger Rip with other parameters same leads to more rapid expansion.

earlier to

We must include (especially for the past) the DM. Including all components gives the generalised Friedmann equation

$$\left(H^2 + \frac{k}{a^2}\right) = \left(\sqrt{\frac{\rho_m}{3M_{Pl}^2} + \frac{1}{4L^2}} + \frac{1}{2L}\right)^2$$

Defining $\Omega_M = \frac{\rho_m}{\rho_c} = \rho_{m0}(1+z)^3$:

$$H^2 = H_0^2\left[\Omega_k(1+z)^2 + \left(\sqrt{\Omega_L} + \sqrt{\Omega_L + \Omega_m(1+z)^3}\right)^2\right]$$

At the present time:

$$\Omega_k + \left(\sqrt{\Omega_L} + \sqrt{\Omega_L + \Omega_m}\right)^2 = 1$$

We show the cases $\gamma = 1/(15G_N)$

and $\gamma = 1/(30G_N)$ with $p = 2$

Shown (next transparency) are in $\Omega_\Lambda - \Omega_m$ plot

95% CL (dotted)

99% CL (dashed)

Solid line is flat $k=0$ (using modified constraint!)

(The lowest plot is for Λ = constant.)

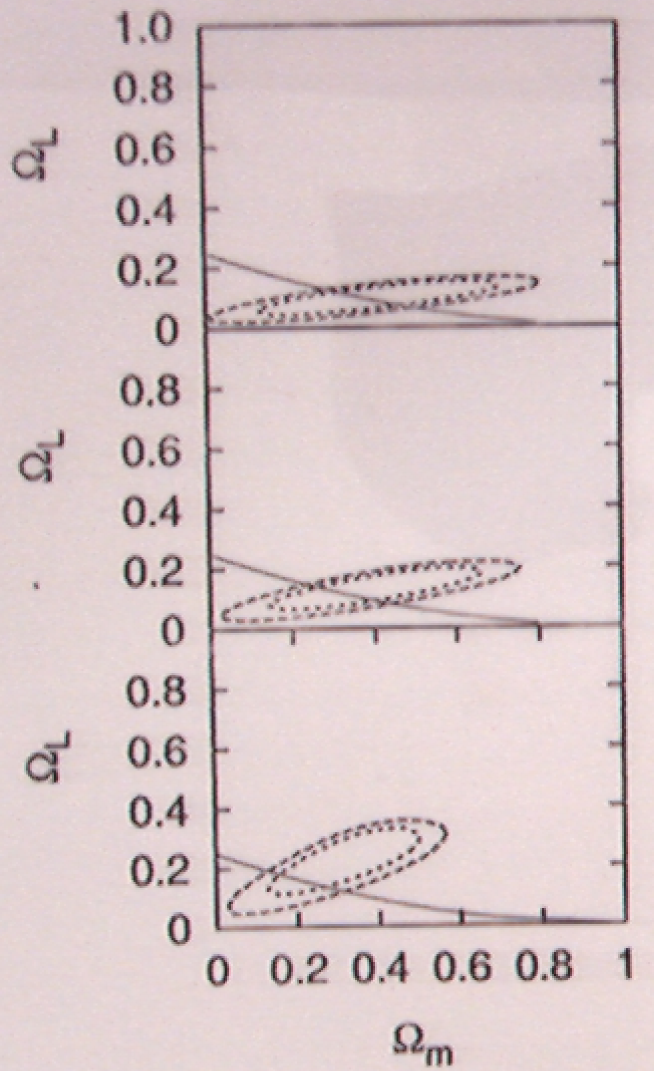

Figure 1: Constraint from the SNeIa observation [8] in the Ω_L-Ω_m plane for the case with the constant L (bottom), $\gamma = 1/15$(Gyr) (middle) and $\gamma = 1/30$(Gyr) (top). Here we take $p = 2$. Contours are for 95 % (dotted line) and 99 % (dashed line) C.L. constraints respectively. The solid line indicates parameters which give a flat universe.

The final figure shows the $\gamma - \Omega_M$

plane for a $k=0$ flat universe.

Putting $\Omega_M = 0.3$ leads to $\frac{1}{\gamma} > 14\,Gy$

Note that the effective $w_{eff}(t_0)$ can be

more negative than for $w =$ constant

e.g. $\Omega_M = 0.3$, $\frac{1}{\gamma} = 14\,Gy$ permits for $p = 2$

$w(t_0) = -1 - \frac{2}{3} p \gamma t_0 = -2.9$

(c.f. $w > -1.2$ for constant case)

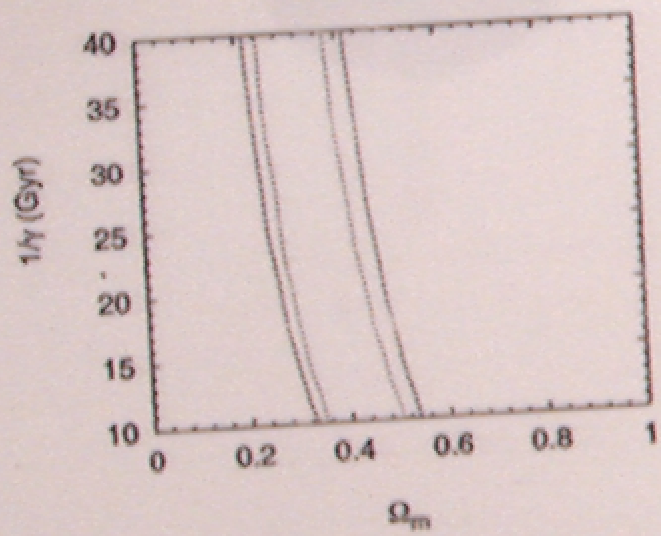

Figure 2: Constraint from SNeIa observation in the γ-Ω_m plane. Contours are for 95 % (dotted line) and 99 % (dashed line) C.L. constraints respectively. In this figure, we assume a flat universe and $p = 2$.

The values of certain parameters extracted from WMAP + LSS (2dF, SDSS) make the most conventional and conservative assumptions of GR at all length scales and DE with constant w.

Several groups* have recently pointed out that the parameters can be quite different if these assumptions are relaxed. [e.g. our $w(z_0) = -1.47$ for $p=1$ in (15)
$w(z_0) = -2.9$ for $p=2$ in (16)]

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* S.K. Srivastava astro-ph/0407048
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J. M. Virey et al astro-ph/0407452

The Big and Bigger Rip can be philosophically more attractive than the standard model. Intricately-formed structure from the past is systematically disintegrated and causally disconnected as we approach the $t \to t_{rip}$.

Here we have modified the LHS (geometry) of the Friedmann equation. Although the term can be reinterpreted on the RHS there is no simple $w = \frac{p}{\rho}$ as in a conventional dark energy.

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