Title: David's Group

Date: Aug 27, 2004 09:50 AM

URL: http://pirsa.org/04080012

Abstract:

A quantum state is defined by a vector:

If you measure spin in the ~~east~~ direction: $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\sigma_z \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$ +z direction

$\sigma_z \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -\begin{pmatrix} 0 \\ 1 \end{pmatrix} =$ -z direction

$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} =$ ~~both~~ superposition of +z and -z spins.

If you measure spin in the z direction

$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$

$\alpha^2$ probability of getting +z spin

and

$\beta^2$ probability of getting -z spin

$\otimes$ = tensor operator

Acts like an ~~add~~ AND

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$ first state: +z spin AND second state: -z spin

Also means that...

If this happens..

$\boxed{\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix}}$

What happens here

However some states cannot be

a vector: defined by

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If this happens...

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What happens here

Is independent of what happens here.

... some states cannot be ... like this: $\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix}$

Entangled states are dependent on each other, no matter how far apart they are!

Example of an entangled state:

$$\frac{1}{\sqrt{2}}\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right)$$

If we get +z spin for one...

we must get it for the other.

It turns out that, if we measure +x, -x, +y, or -y spin for one, we get the same thing for the other state.

This leads to wierd effects.

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If we get +z [illegible] for one..

we mu[illegible] for the oth[illegible]

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This [illegible] effects.

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This leads to wierd effects.

# Magic Square

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\sigma_x \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\sigma_x \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\sigma_y \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\sigma_y \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -i \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

TRICKS:

(VALID ONLY FOR PAULI-MATRICES)

$\sigma_x \sigma_x = 1$ $\qquad$ $\sigma_x \sigma_y = -\sigma_y \sigma_x$

$\sigma_y \sigma_y = 1$ $\qquad$ $\sigma_y \sigma_z = -\sigma_z \sigma_y$

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COMMUTATION PROPERTY:

IF TWO OBSERVABLES COMMUTE, THAN YOU CAN FIND AN EIGENSTATE OF BOTH.

$$A \cdot B = B \cdot A \iff A\vec{v} = \alpha\vec{v} \text{ AND } B\vec{v} = \beta\vec{v}$$

QUANTUM SYSTEM OF 3 PARTICLES: 1, 2 AND 3.

OBSERVABLES:

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$A \cdot B =$ ...

QUANTUM S...

...RVAB...

... OBSERVABLES ALL COMMUTE

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$\vec{V} = \frac{1}{\ldots}$ ... OTE

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$$\vec{V} = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] = \text{EIGENSTATE}$$

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Tricks:

$\sigma_x \sigma_x = 1$ $\qquad \sigma_x \sigma_y = -\sigma_y \sigma_x$ $\qquad$ (Valid only for Pauli-matrices)

$1$ $\qquad \sigma_y \sigma_z = -\sigma_z \sigma_y$

$\sigma_x$

...tion pro...

...observa... ...t... you can find an eigenstate of both.

$A \cdot B$ ... and $B\vec{v} = \beta\vec{v}$

... 1, 2 and 3.

... three observables all commute

... other, so we can find an

... eigenstate of all ~~...~~ the three

... observables.

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LET'S APPLY THE OBSERVABLES TO $\vec{v}$:

$$\left(\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3\right) \cdot \vec{v} = \frac{1}{\sqrt{2}}\left(\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]\right) =$$

$$= \frac{1}{\sqrt{2}}\left(i^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} - (-i)^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) =$$

$$= \frac{1}{\sqrt{2}}\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = +\vec{v}$$

THE SAME CALCULATION FOR

$$\left(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3\right) \quad \text{AND} \quad \left(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3\right)$$

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$$\left(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3\right) \cdot \vec{v} = +\vec{v}$$
$$\left(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3\right) \cdot \vec{v} = +\vec{v}$$
$$\overline{-\left(\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3\right) \cdot \vec{v} = +\vec{v}}$$

CAN PARTICLES HAVE SOME HIDDEN VARIAB

LET'S THINK THAT PARTICLES HAVE A

① $x_1, y_1$ ② $x_2, y_2$

ALL $y$ AND $x$ VALUES ARE

SO $y_1 y_1 = 1$, $y_2 y_2 = 1$, $y_3 y_3 = 1$

LET'S APPLY THE OBSERVABLES TO $\vec{\psi}$:

$$(\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3)\cdot\vec{\psi} = \frac{1}{\sqrt{2}}\left\{\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\otimes\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\otimes\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\cdot\left[\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right]\right\} =$$

$$= \frac{1}{\sqrt{2}}\left\{i^2\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}-(-i)^2\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right\} =$$

$$= \frac{1}{\sqrt{2}}\left\{\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right\} = +\vec{\psi}$$

THE SAME CALCULATION FOR

$$(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3) \quad \text{AND} \quad (\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)$$

MAGIC SQUARE:

$$(\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3)\cdot\vec{\psi} = +\vec{\psi}$$

$$(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)\cdot$$

$$(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)$$

$$-(\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3)$$

CAN PARTICLES HAV[illegible] [illegible]DDEN VA[illegible]

LET'S THINK THAT [illegible] HAVE

(1) $(x_1, y_1)$ (2) [illegible]

ALL $y$ AND $x$ VALU[illegible]

SO $y_1 y_1 = 1$, $y$[illegible]

$y_1 y_2 x_3 =$ [illegible]

LET'S APPLY THE OBSERVABLES TO $\Psi$:

$$(\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3)\cdot\Psi = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\otimes\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\otimes\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\cdot\left[\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right]\right] =$$

$$= \frac{1}{\sqrt{2}}\left[i^2\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}-(-i)^2\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right] =$$

$$= \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right] = +\Psi$$

THE SAME CALCULATION FOR

$(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)$ AND $(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)$

MAGIC SQUARE:

$$(\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3)\cdot\Psi = +\Psi$$
$$(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)\cdot\Psi = +\Psi$$
$$(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)\cdot\Psi = +\Psi$$
$$-(\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3)\cdot\Psi = +\Psi$$

CAN PARTICLES HAVE SOME HIDDEN VARIABLES?

LET'S THINK THAT PARTICLES HAVE A BRAIN...

① $(x_1, y_1)$ ② $(x_2, y_2)$ ③ $(x_3, y_3)$

ALL $y$ AND $x$ VALUES ARE $\pm 1$ (LIKE SPIN UP OR DOWN)

SO $y_1 y_1 = 1$, $y_2 y_2 = 1$, $y_3 y_3 = 1$

$y_1 y_2 x_3 = 1$

$$= \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right] = +\vec{\nu}$$

THE SAME CALCULATION FOR

$(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)$ AND $(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)$

MAGIC SQUARE:

$$(\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3)\cdot\vec{\nu} = +\vec{\nu}$$
$$(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)\cdot\vec{\nu} = +\vec{\nu}$$
$$(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)\cdot\vec{\nu} = +\vec{\nu}$$
$$-(\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3)\cdot\vec{\nu} = +\vec{\nu}$$

CAN PARTICLES HAVE SOME HIDDEN VARIABLES?

LET'S THINK THAT PARTICLES HAVE A BRAIN...

1 $(x_1, y_1)$ 2 $(x_2, y_2)$ 3 $(x_3, y_3)$

ALL $y$ AND $x$ VALUES ARE $\pm 1$ (LIKE SPIN UP OR DOWN)

SO $y_1 y_1 = 1$, $y_2 y_2 = 1$, $y_3 y_3 = 1$

$$y_1\, y_2\, x_3 = 1$$
$$y_1\, x_2\, y_3 = 1$$
$$x_1\, y_2\, y_3 = 1$$
$$x_1\, x_2\, x_3 = 1$$

BUT EVERYTIME YOU MEASURE $(\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3)$, YOU OBTAIN $-1$, SO THERE CAN'T BE ANY HIDDEN VARIABLES.

CONCLUSION: OBSERVATIONS NOT ONLY DISTURB WHAT HAS TO BE MEASURED, THEY PRODUCE IT... WE OURSELVES PRODUCE THE RESULTS OF MEASUREMENT. [P. JORDAN]

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