

Title: Joseph's Group

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URL: <http://pirsa.org/04080006>

Abstract:

# Chaos

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Student Participants:

Calista Preusser (Germany)  
Chen Li (Canada)  
Ivan Dimitrov (Bulgaria)  
Thulani Maka (South Africa)  
Jackie (USA)

# Outline

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- Determinism

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- Pendulum
  - ◆ Regular motion
  - ◆ Irregular motion

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- What's Chaos?
  - ◆ Conceptual
  - ◆ Quantitatively

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- Determinism
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  - ◆ Irregular motion
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  - ◆ Conceptual
  - ◆ Quantitatively
- Examples of Chaos (maps)

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# Determinism

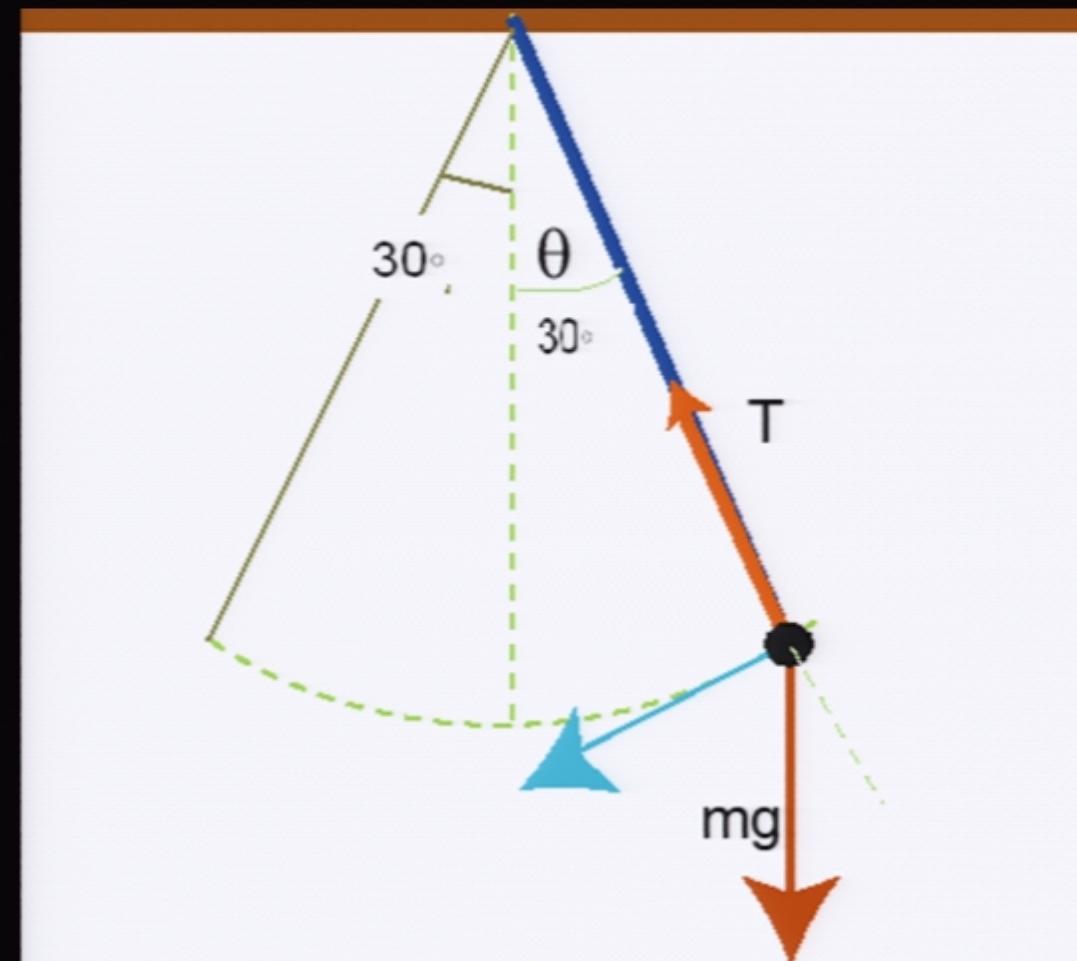
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- If you know all of the dynamical law (equation).



You can predict the configuration at later time

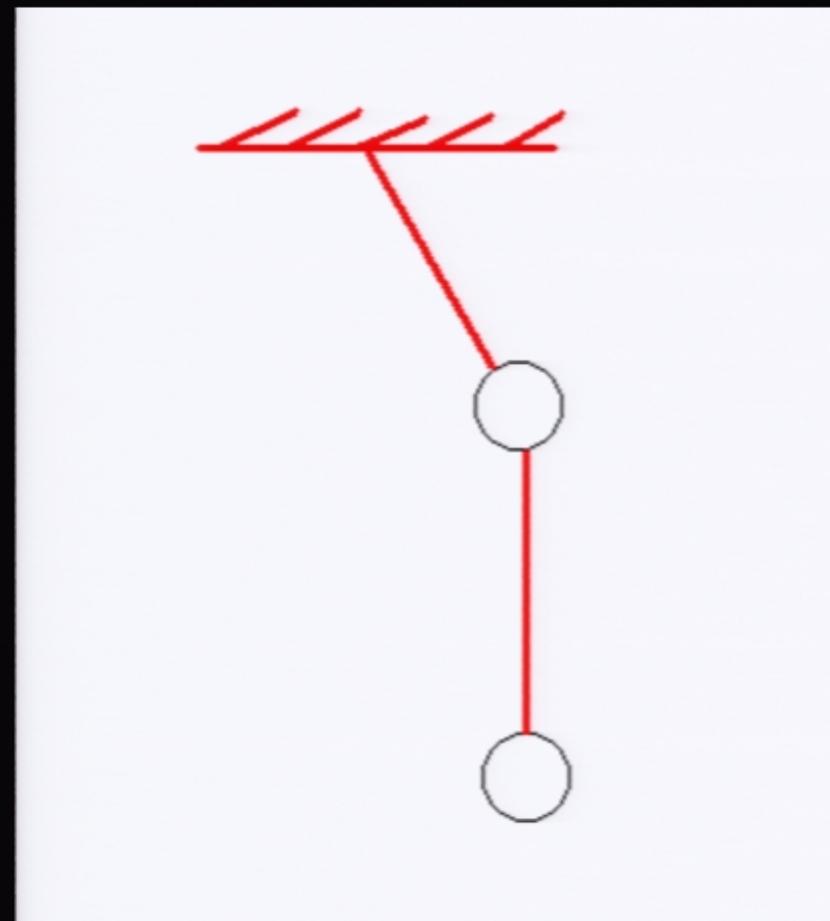
# Pendulum- regular

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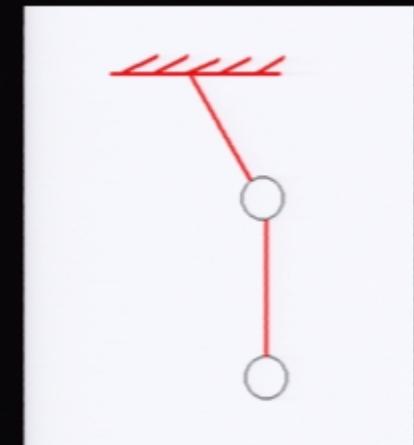
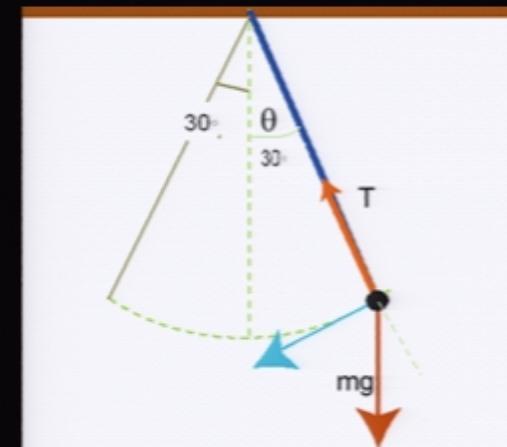


# Double pendulum - chaotic

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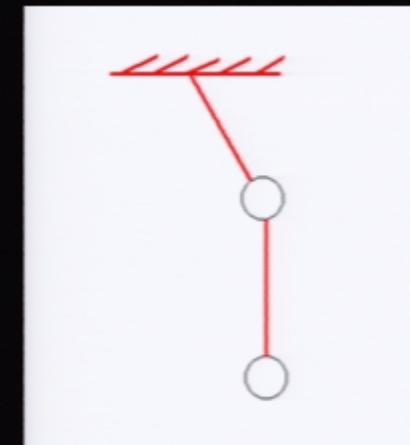
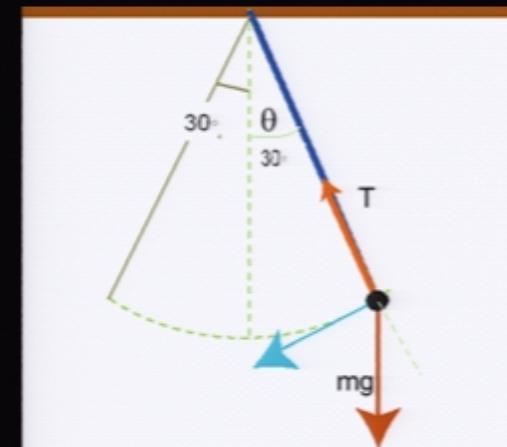


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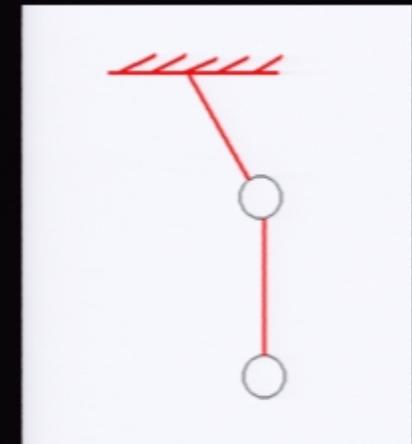
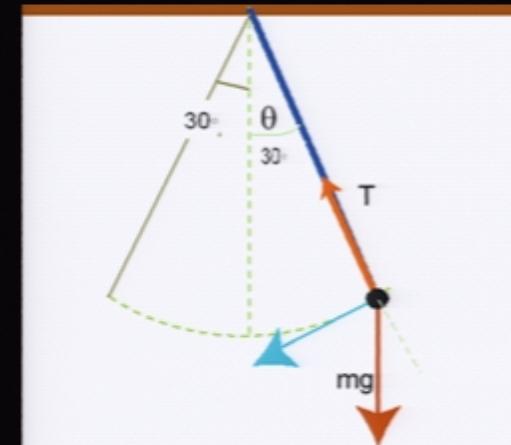
Qualitative definition



# What Is Chaos?!!

Qualitative definition

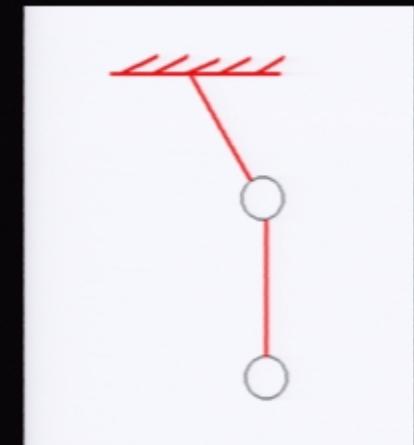
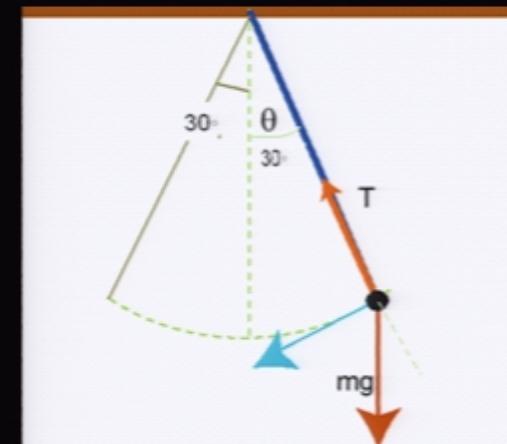
- The dynamical time revolution is very complex.



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Qualitative definition

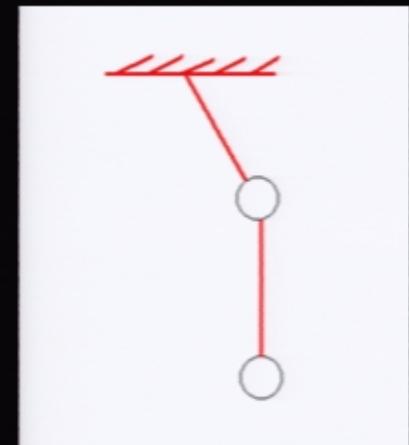
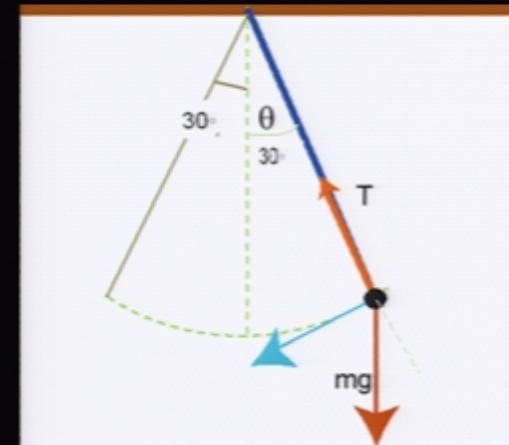
- The dynamical time revolution is very complex.
- The motion is aperiodic.



# What Is Chaos?!!

Qualitative definition

- The dynamical time revolution is very complex.
- The motion is aperiodic.
- Uncertainty grows extremely rapidly.  
“extremely sensitive to initial condition.”



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Power growth

The larger the  $\lambda$ , the further you lost prediction about the system trajectory you are trying to predict.

# Examples of Chaos - Maps

$C: 0 \rightarrow 2$

ITER  $0 \rightarrow 30\ 000$

639. 30 000

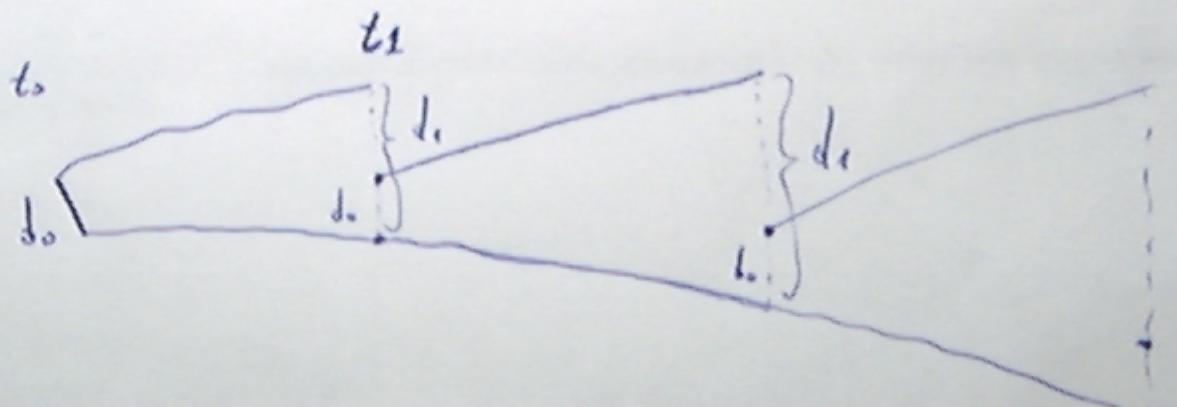
new coordinates

calculate  $\ln$

add to total

rescore  $d_i$  to  $d_0$

## Numerical Calculation of Lyapunov Exponent



$$X_{n+1} = 1 - C X_n + B X_{n+1} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad \begin{aligned} B &= 0,3 \\ C &= 0... \frac{639}{320} = 2 \end{aligned}$$

$C: 0 \rightarrow 2$

ITER 0 → 30 000      639. 30 000

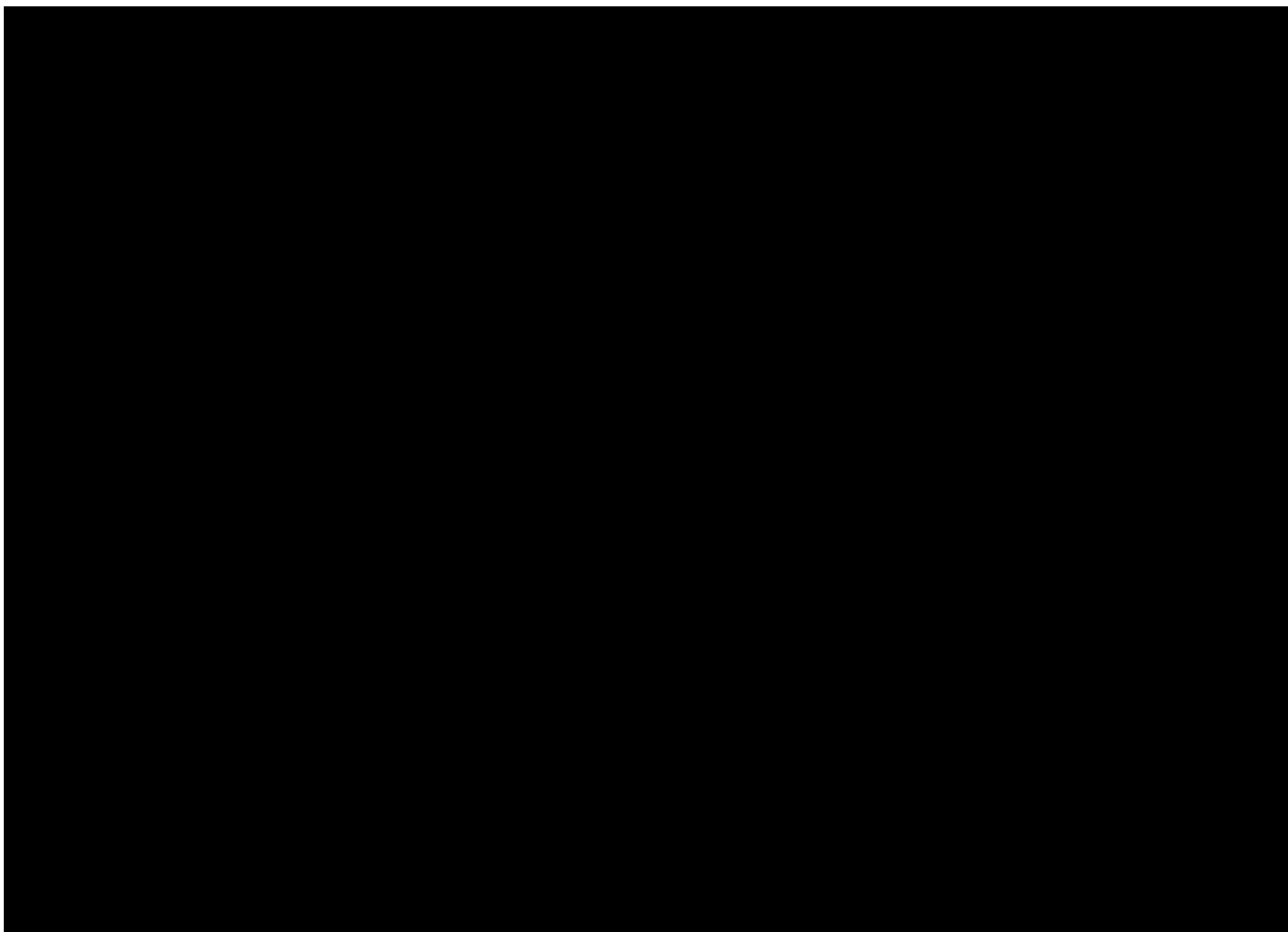
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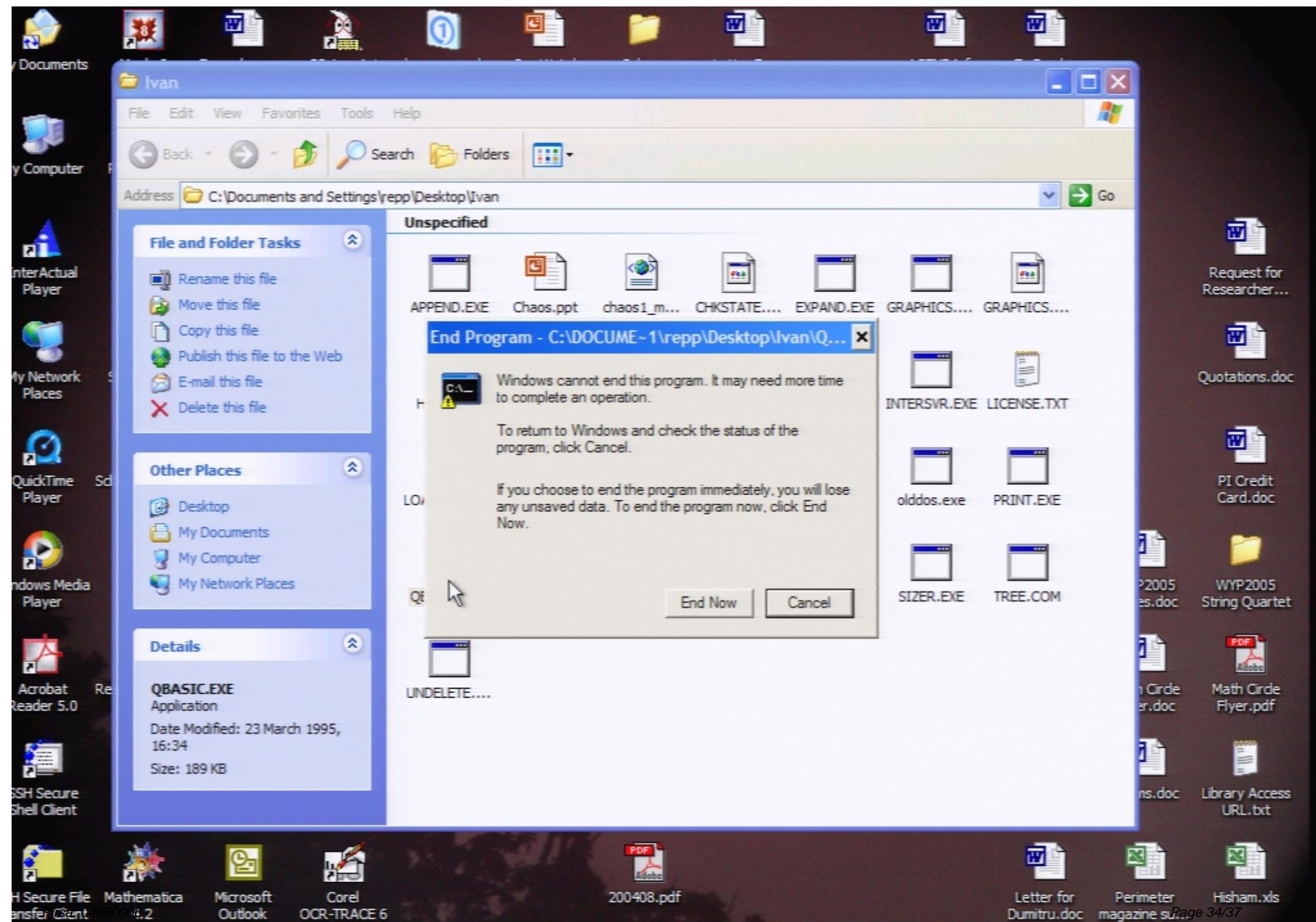
calculate  $\ln$

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rescale  $d_1$  to  $d_0$

## Numerical Calculation of Lyapunov Exponent





Documents

Computer

InterActual Player

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QuickTime Player

Windows Media Player

Acrobat Reader 5.0

SSH Secure Shell Client

H Secure File Transfer Client 408006.2

Mathematica

Microsoft Outlook

Corel OCR-TRACE 6

200408.pdf

Adobe PDF

Ivan

C:\DOCUME~1\repp\Desktop\Ivan\QBASIC.EXE

File Edit View Search Run Debug Options Help HENONLE2.BAS

```
FOR j& = 1 TO 5000
    x1new = 1 - c * x1 * x1 + .3 * y1
    y1 = x1
    x1 = x1new
    x2new = 1 - c * x2 * x2 + .3 * y2
    y2 = x2
    x2 = x2new
    d1 = SQR((x2 - x1) ^ 2 + (y2 - y1) ^ 2)
    x2 = x1 + (d0 / d1) * (x2 - x1)
    y2 = y1 + (d0 / d1) * (y2 - y1)
    IF j& > 100 THEN lsum = lsum + LOG(d1 / d0)
    IF ABS(x1) > 1000 THEN END
NEXT j&
```

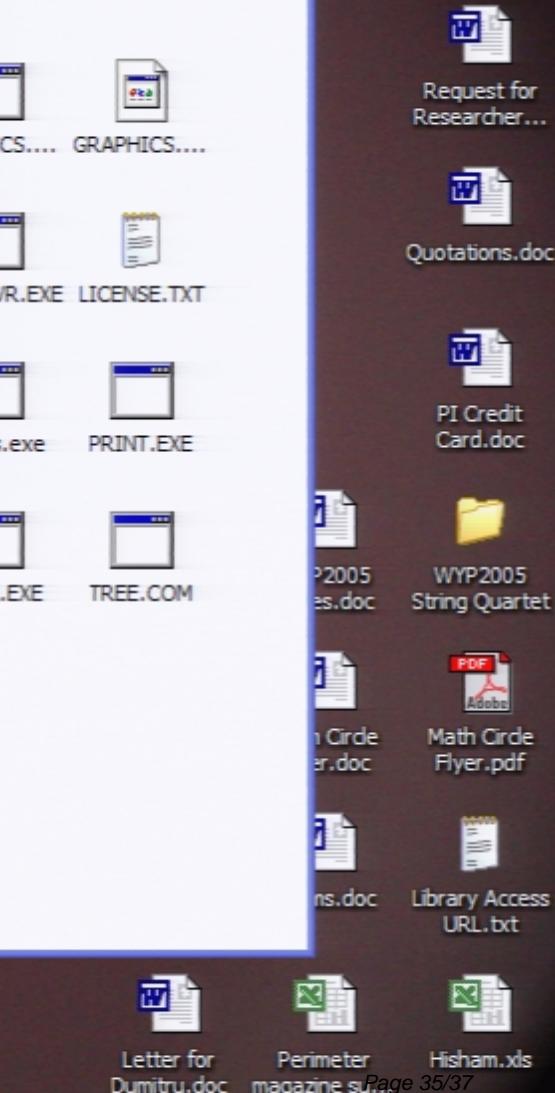
Immediate

<Shift+F1=Help> <F6=Window> <F2=Subs> <F5=Run> <F8=Step> | N 00037:001

My Computer  
My Network Places

QBASIC.EXE QBASIC.HLP README.TXT REPLACE.EXE RESTORE.EXE SIZER.EXE TREE.COM

UNDELETE....



Documents

My Computer

InterActual Player

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QuickTime Player

Windows Media Player

Acrobat Reader 5.0

SSH Secure Shell Client

H Secure File Transfer Client 4080006.2

Mathematica

Microsoft Outlook

Corel OCR-TRACE 6

200408.pdf

Adobe PDF

Ivan

File Edit View Favorites Tools Help

C:\DOCUME~1\repp\Desktop\Ivan\QBASIC.EXE

File Edit View Run Debug Options Help

HENONLE2.BAS

New Open... Save Save As... Print... Exit

```
1000 c * x1 * x1 + .3 * y1
      c * x2 * x2 + .3 * y2
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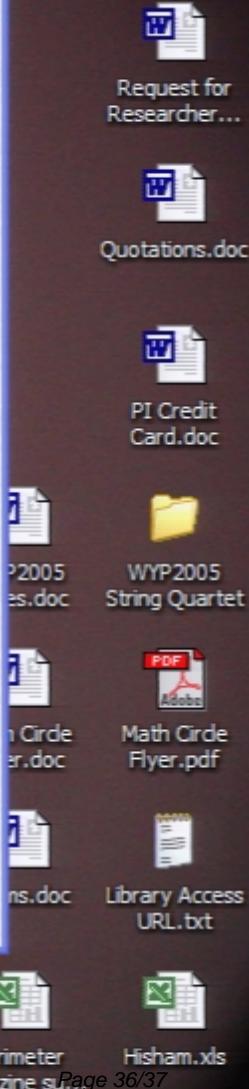
Immediate

F1=Help | Removes currently loaded file from memory | N 00039:020

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UNDELETE....



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