

Title: Invariant Quantum Information

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Abstract: Qunatum Information Theory

# Invariant quantum information



Steve Bartlett & Danny Terno

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*USING THE RESULTS OF  
NETHANEL LINDNER, TERRY RUDOLPH, ROB SPEKKENS*

# Outline

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## Introduction

Reference frames as errors, and how to overcome them

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## How to make qubits

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Massive particles  
Photons

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## Summary & omissions

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On importance of having  
a shared reference frame



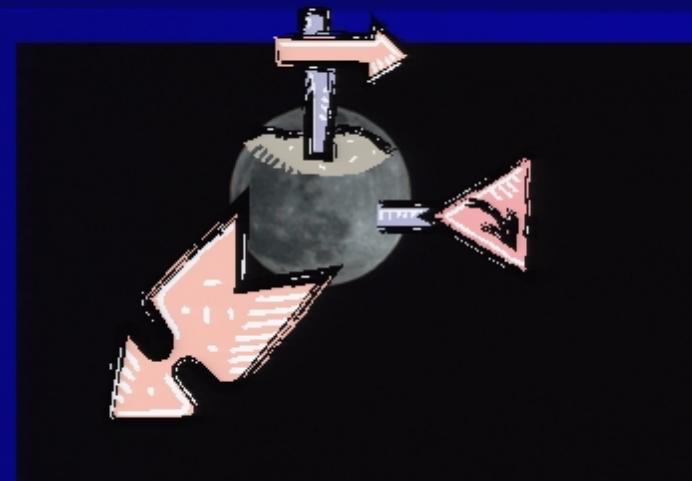
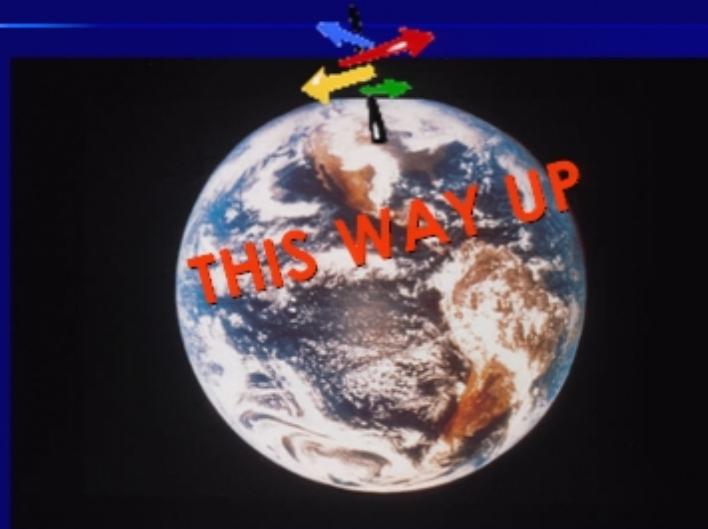
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No SRF: no info transmission in one qubit

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$$T(\rho) = \int d\Omega U(\Omega) \rho U^\dagger(\Omega) = \frac{1}{2} I$$

No SRF: no info transmission in one qubit

$T(\rho)$ : lack of SRF works as a decoherence

# Intro

On the benefits of  
reduced representations

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Bartlett, Rudolph, Spekkens,  
Phys. Rev. Lett. **91**, 027901 (2003)

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Two qubits

$$T_2(\rho_{AB}) = \int d\Omega U_A(\Omega) \otimes U_B(\Omega) \rho_{AB} U_A^\dagger(\Omega) U_B^\dagger(\Omega)$$



Invariant subspaces     $j = 0, j = 1$   
can be used to transmit one bit

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Four qubits

$$(0 \oplus 1) \otimes (0 \oplus 1) = 0 \oplus 1 \oplus 1 \oplus 0 \oplus 1 \oplus 2 = 0^{\otimes 2} \oplus 1^{\otimes 3} \oplus 2$$

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Four qubits (for dummies)

$$(0 \oplus 1) \otimes (0 \oplus 1) = 0 \oplus 1 \oplus 1 \oplus 0 \oplus 1 \oplus 2 = 0^{\otimes 2} \oplus 1^{\otimes 3} \oplus 2$$

$N$  qubits (for dummies)

Total # of invariant subspaces = # **c** bits =  $\log_2 \binom{N}{N/2}$

# Intro

On encoding of  
quantum information

Four qubits  $(0 \oplus 1) \otimes (0 \oplus 1) = 0 \oplus 1 \oplus 1 \oplus 0 \oplus 1 \oplus 2$

$$|0_L\rangle = |0;0\rangle|0;0\rangle \quad |1_L\rangle = \frac{1}{\sqrt{3}}(|1;1\rangle|1;-1\rangle - |1;0\rangle|1;0\rangle + |1;-1\rangle|1;1\rangle)$$

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Three qubits  $\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{3}{2}$

invariant subsystem  $\frac{1}{2} \oplus \frac{1}{2} = \frac{1}{2} \otimes \mathbb{C}^2$

action of  $\Omega$ :  $U(\Omega) = U_{\frac{1}{2}}(\Omega) \oplus U_{\frac{1}{2}}(\Omega) = U_{\frac{1}{2}}(\Omega) \otimes \mathbf{1}_2$

$$\sigma \otimes \rho \rightarrow \frac{1}{2}\mathbf{1}_2 \otimes \rho$$

# Intro

On invariant  
subsystems

basis states  $|\frac{1}{2}; \pm \frac{1}{2}\rangle |\bar{1}\rangle, |\frac{1}{2}; \pm \frac{1}{2}\rangle |\bar{2}\rangle$

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$$|0_L\rangle = |\frac{1}{2}, \frac{1}{2}\rangle |\bar{1}\rangle = \frac{1}{\sqrt{2}} \left( |\frac{1}{2}\rangle |-\frac{1}{2}\rangle - |-\frac{1}{2}\rangle |\frac{1}{2}\rangle \right) |\frac{1}{2}\rangle \quad \text{logical basis}$$

$$|1_L\rangle = |\frac{1}{2}, \frac{1}{2}\rangle |\bar{2}\rangle = -\frac{1}{\sqrt{6}} \left( |\frac{1}{2}\rangle |-\frac{1}{2}\rangle + |-\frac{1}{2}\rangle |\frac{1}{2}\rangle \right) |\frac{1}{2}\rangle + \sqrt{\frac{2}{3}} |\frac{1}{2}\rangle |\frac{1}{2}\rangle |-\frac{1}{2}\rangle$$

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$$\begin{pmatrix} * & * \\ * & * \end{pmatrix}_{\frac{1}{2}} \otimes \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}_{\mathbb{C}^2} \oplus (*)_{\frac{3}{2}} \rightarrow$$

**Ignorance is power!**

$$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}_{\frac{1}{2}} \otimes \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}_{\mathbb{C}^2} \oplus \frac{1}{4} \mathbf{1}_{\frac{3}{2}}$$

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Unitary Lorentz  
transformations

$$U(\Lambda)|p, \sigma\rangle = \sum_{\xi} D_{\xi\sigma}[W(\Lambda, p)]|\Lambda p, \xi\rangle$$

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Step 1:  
standard states &  
standard Lorentz transformations

massive particles:  $k_s = (m, 0, 0, 0)$

massless particles:  $k_s = (1, 0, 0, 1)$

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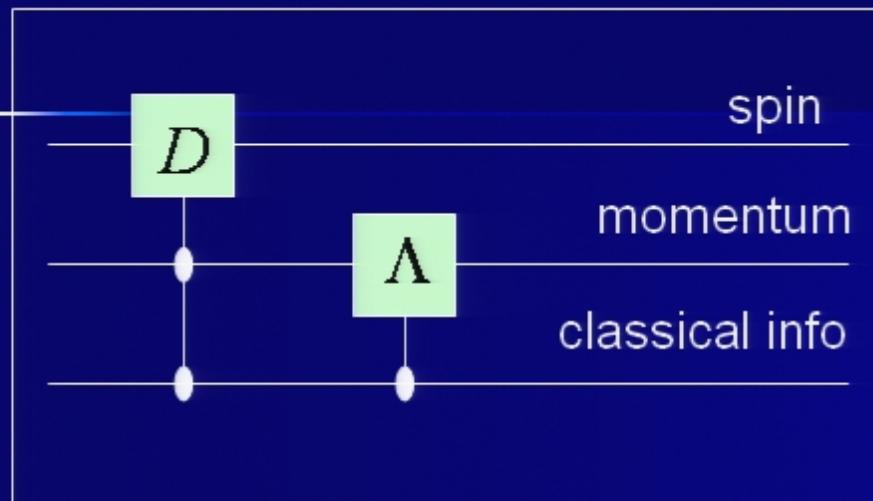
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Step 2: state conventions

$$|p, \sigma\rangle \doteq U(L_p)|k_s, \sigma\rangle$$

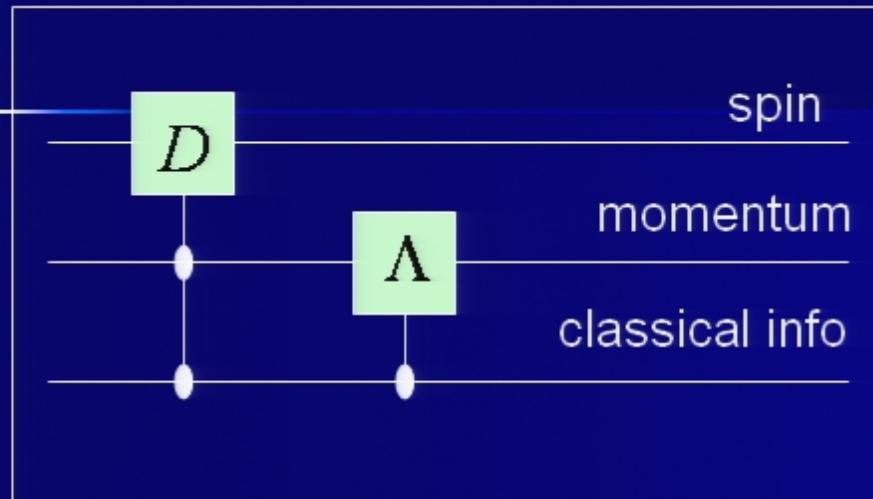
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Step 3: arbitrary transformation

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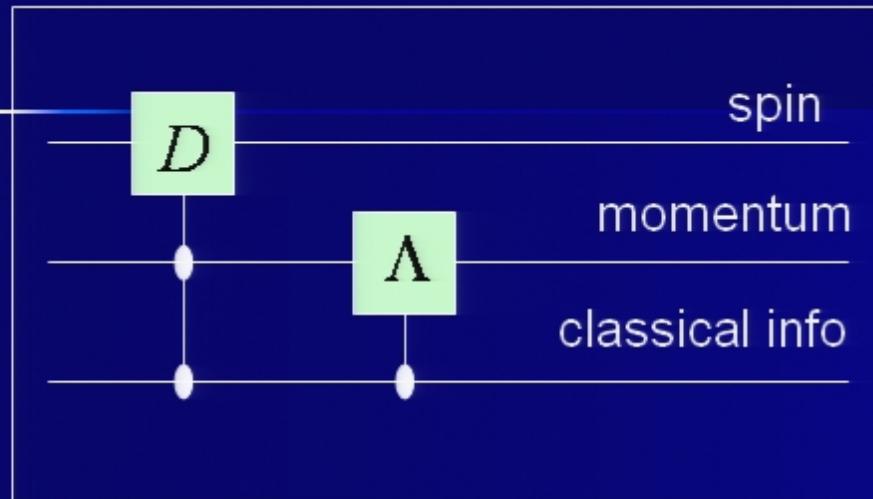


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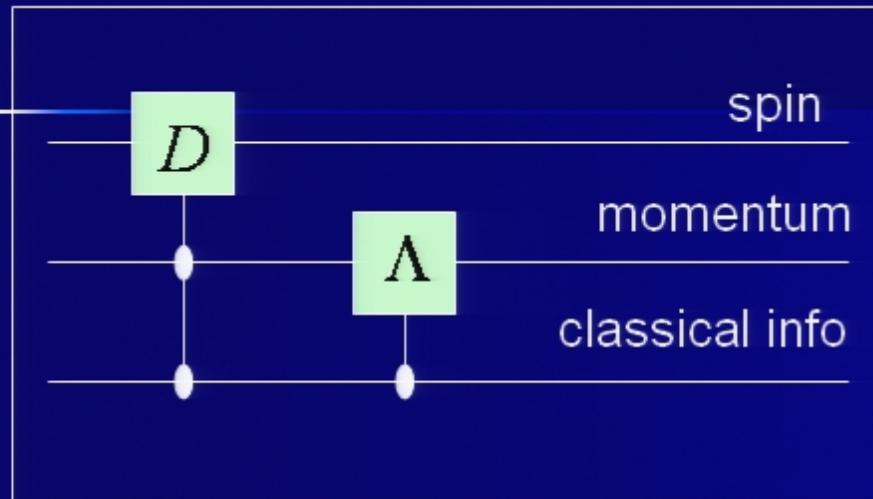


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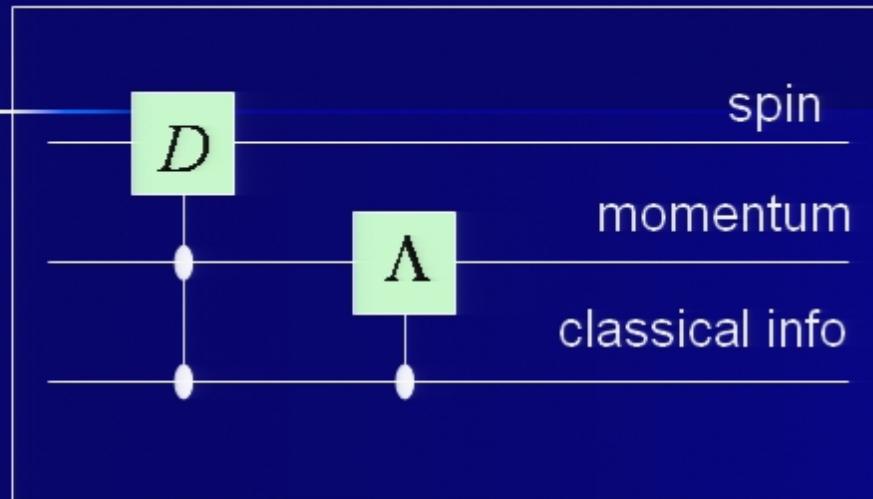


$$U(\Lambda)|p, \sigma\rangle = \sum_{\xi} D_{\xi\sigma}[W(\Lambda, p)]|\Lambda p, \xi\rangle$$

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$$U(\Lambda)|p, \sigma\rangle = U(L_{\Lambda p} L_{\Lambda p}^{-1}) U(\Lambda L_p)|k_s, \sigma\rangle$$

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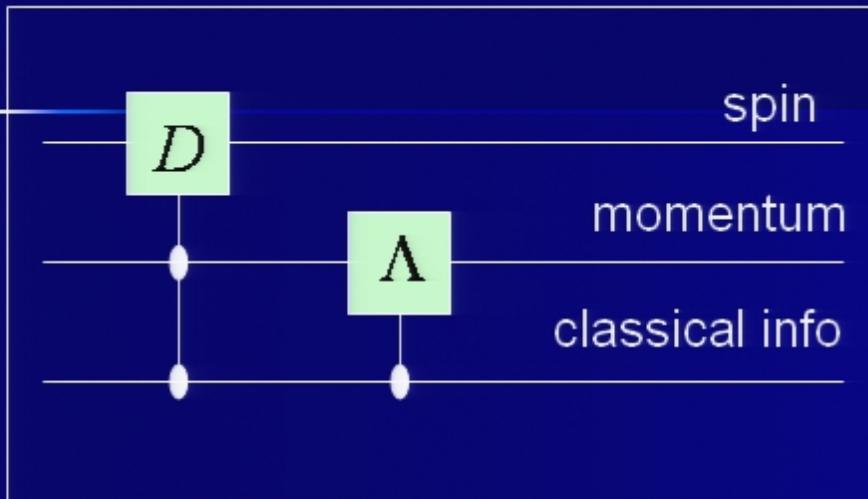


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$$U(\Lambda)|p, \sigma\rangle = U(L_{\Lambda p})U(L_{\Lambda p}^{-1}\Lambda L_p)|k_s, \sigma\rangle$$

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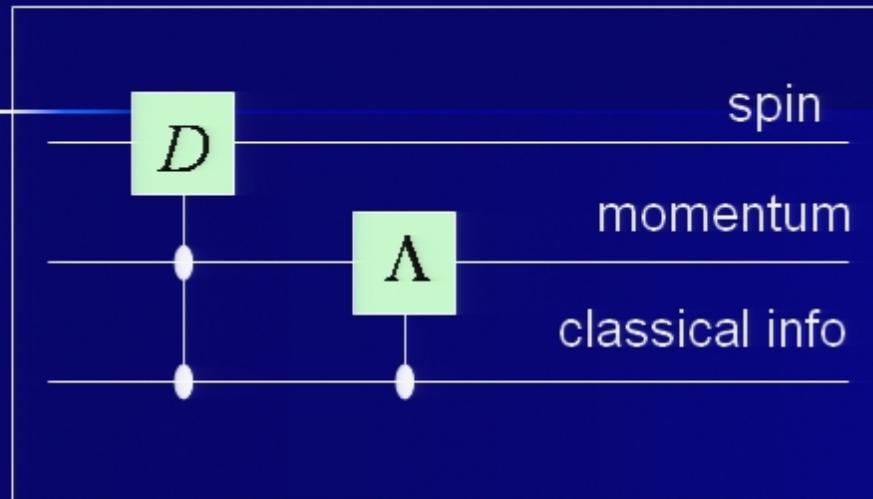


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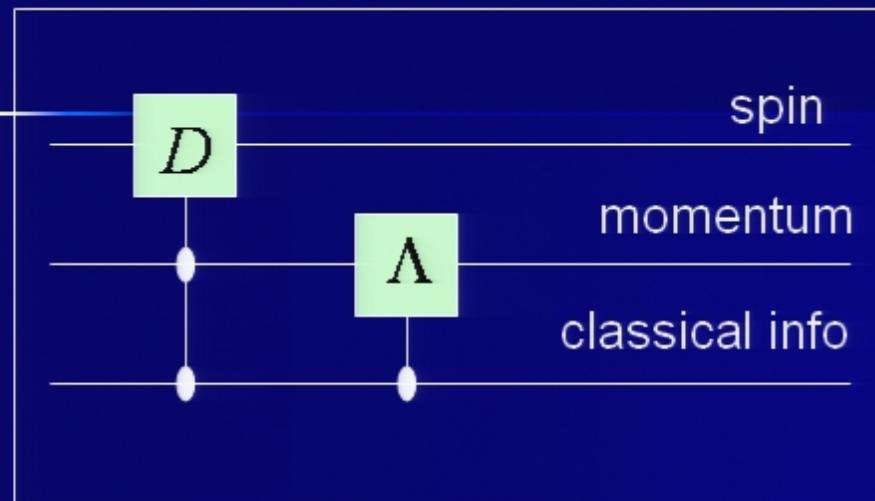


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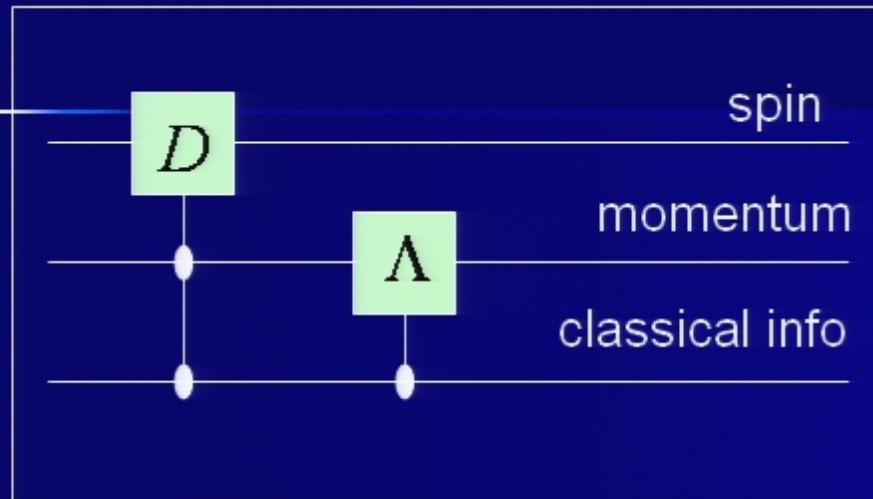
$$U(\Lambda)|p, \sigma\rangle = U(L_{\Lambda p})U(L_{\Lambda p}^{-1}\Lambda L_p)|k_s, \sigma\rangle$$

Step 4: little group  $W(\Lambda, p) \doteq L_{\Lambda p}^{-1}\Lambda L_p$

$$W k_s$$

$$k_s$$

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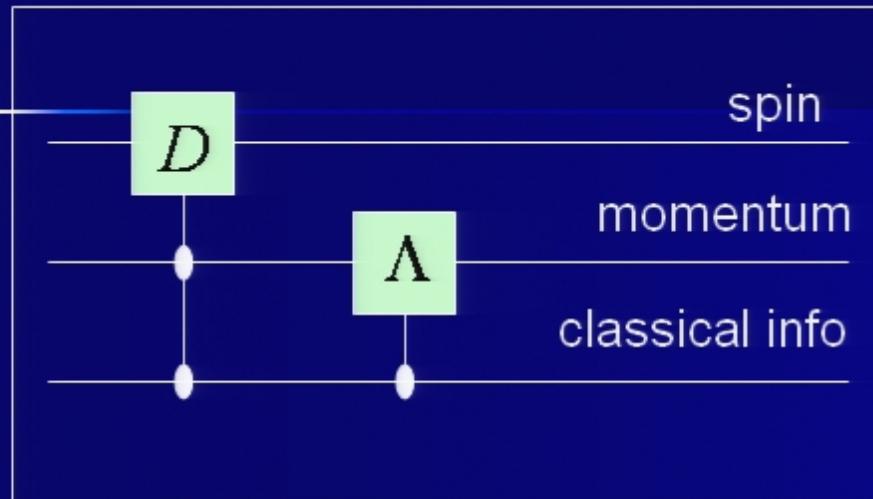
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$$k_s \xrightarrow{L_p} p$$

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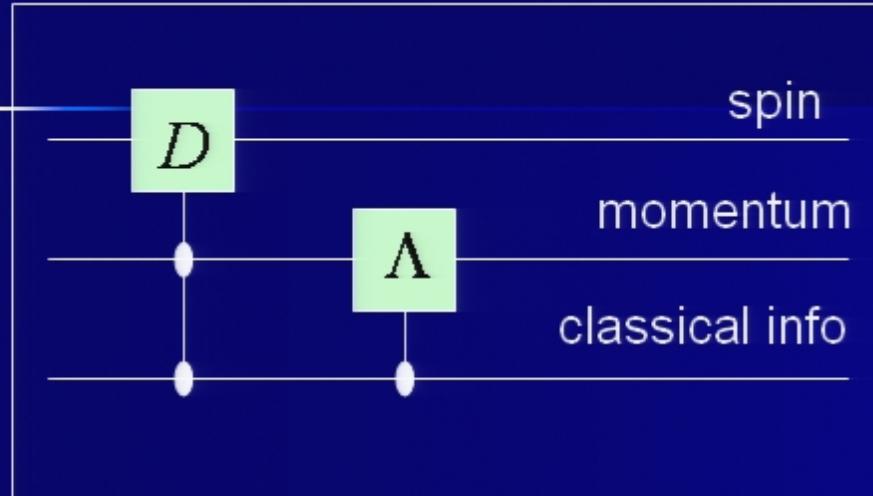
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$W k_s$

$$k_s \xrightarrow{L_p} \textcolor{blue}{p} \xrightarrow{\Lambda} \textcolor{red}{\Lambda} p$$

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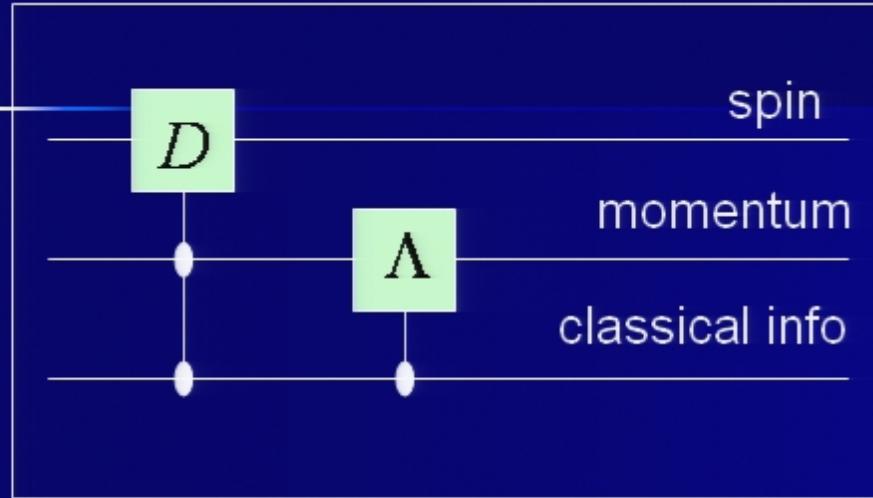
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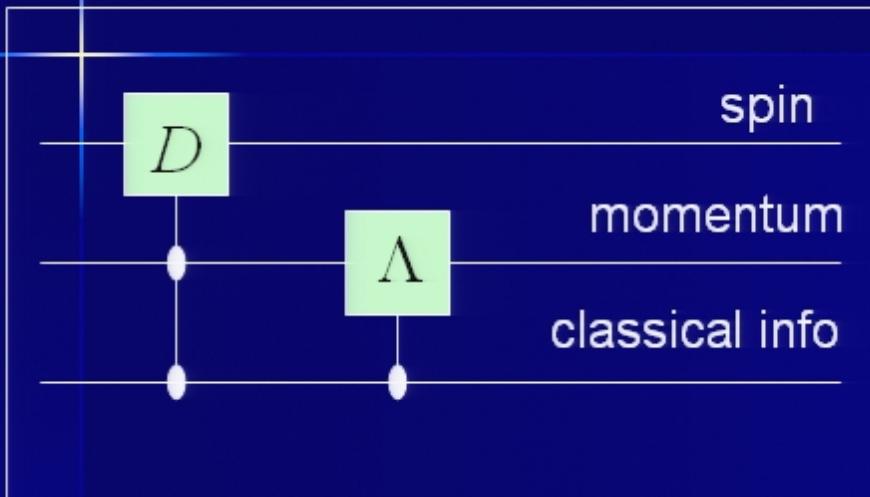
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Step 5:  $U(\Lambda)|p, \sigma\rangle = U(L_{\Lambda p}^{-1}\Lambda L_p)|\Lambda p, \sigma\rangle$

# Intro

On two varieties  
of a little group



## Massive particles

$$k_s = (m, 0, 0, 0)$$

$$\sigma = \text{spin} = \pm \frac{1}{2}$$

$$W \in SO(3)$$

$$D = e^{i\sigma \hat{n} \theta_w / 2}$$

## Photons

$$k_s = (1, 0, 0, 1)$$

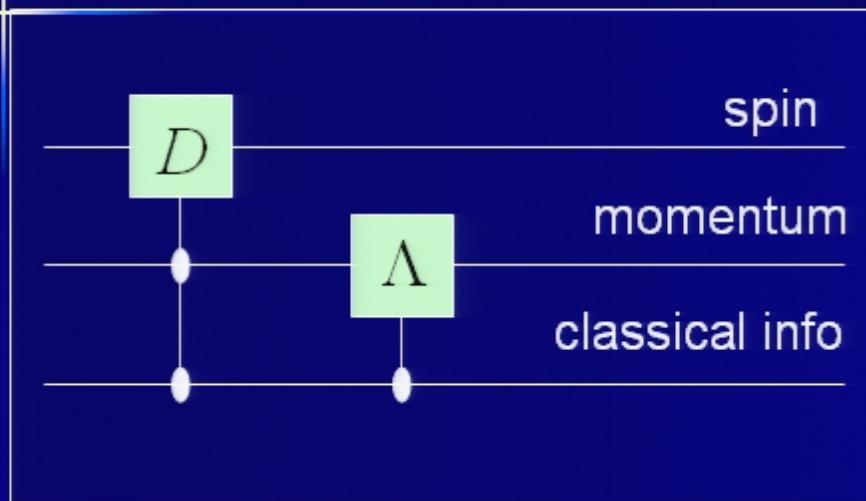
$$\sigma = \text{helicity} = \pm 1$$

$$W = S(\alpha, \beta)R_z(\varphi) \in E(2)$$

$$D_{\xi\sigma} = e^{i\sigma\varphi}\delta_{\xi\sigma}$$

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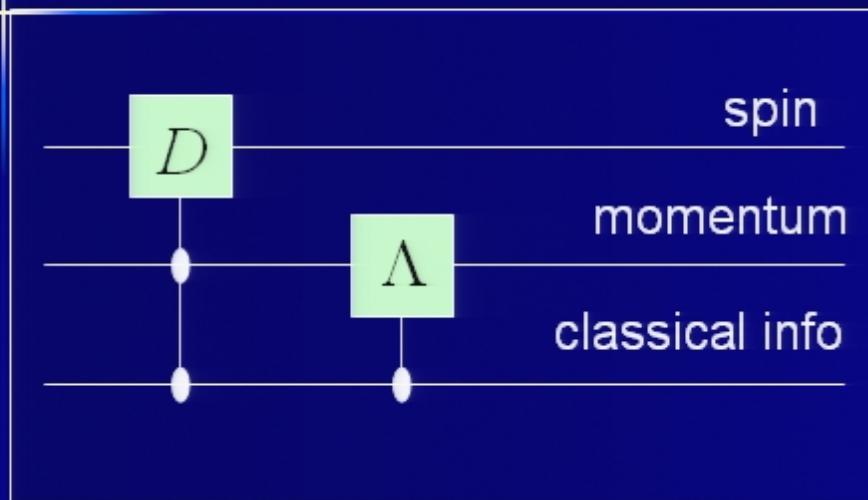
On spoiling of qubits by  
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$$U(\Lambda)|p, \sigma\rangle = \sum_{\xi} D_{\xi\sigma}[W(\Lambda, p)]|\Lambda p, \xi\rangle$$

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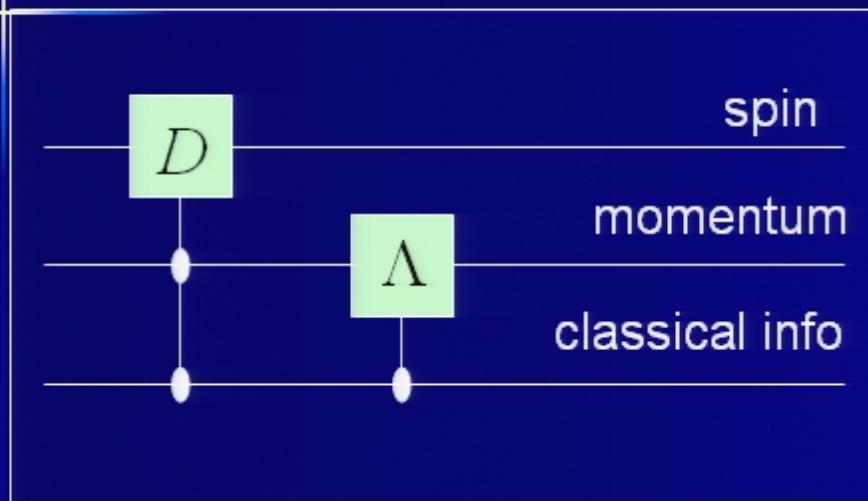
## On spoiling of qubits by Lorentz transformations


$$U(\Lambda)|p, \sigma\rangle = \sum_{\xi} D_{\xi\sigma}[W(\Lambda, p)]|\Lambda p, \xi\rangle$$

- Wigner rotation for momentum eigenstates
- Decoherence from the lack of knowledge of  $\Lambda$

# Intro

## On spoiling of qubits by Lorentz transformations



$$|\Psi\rangle = \int d\mu(p) \begin{pmatrix} \psi_+(p) \\ \psi_-(p) \end{pmatrix} |p\rangle$$

$$U(\Lambda)|p, \sigma\rangle =$$

$$\sum_{\xi} D_{\xi\sigma}[W(\Lambda, p)] |\Lambda p, \xi\rangle$$

- Wigner rotation for momentum eigenstates
- Decoherence from the lack of knowledge of  $\Lambda$
- Decoherence from spin-momentum entanglement

Peres and Terno, Rev. Mod. Phys. **76**, 93 (2004)

# How to make qubits

Objective: sharp momentum and position

$$|\Psi_a\rangle = e^{-iaP_z} |\Psi\rangle = \begin{pmatrix} \xi \\ \eta \end{pmatrix} \int d\mu(p) e^{-iap_z} \psi(p) |p\rangle$$

$$\psi = Ne^{-\mathbf{p}^2/2\Delta^2} \quad \Delta = \epsilon mc \ll mc$$

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$$\langle \Psi | \Psi_a \rangle \sim \exp(-a^2 \Delta^2 / 4\hbar^2) \quad a \gg \lambda/\varepsilon$$

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$$|\Psi\rangle = \chi |\mathbf{p}\rangle = \begin{pmatrix} \xi \\ \eta \end{pmatrix} |\mathbf{p}\rangle$$

# How to encode

massive  
particles

$$|\Psi\rangle_N \simeq \bigotimes_{n=1}^N e^{-i\alpha_z P_z} |\Psi\rangle$$

$$U_N(\Lambda, p) = \bigotimes_{n=1}^N U_n(\Lambda, p)$$

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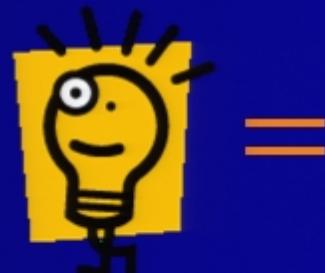
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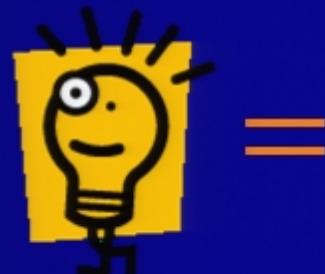
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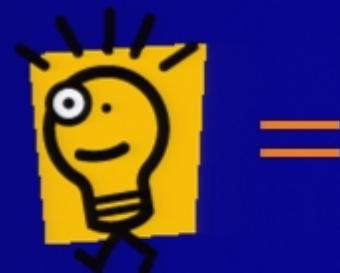
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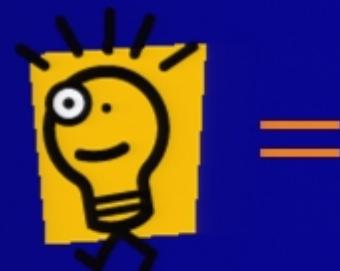
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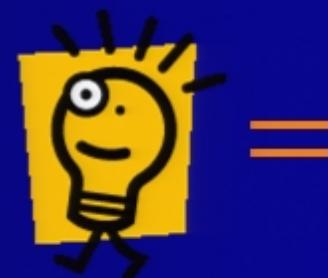
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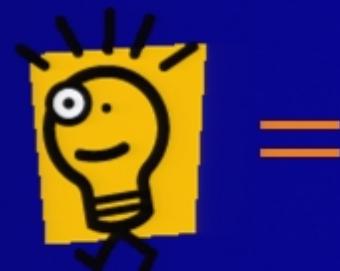
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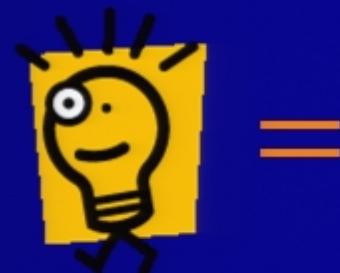
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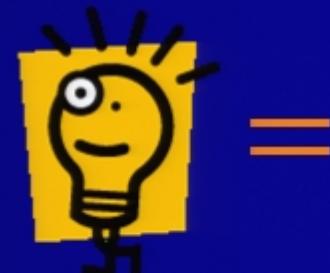
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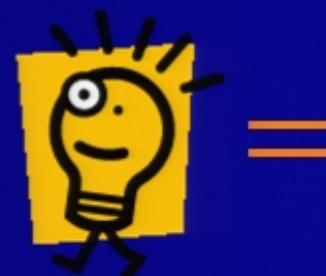
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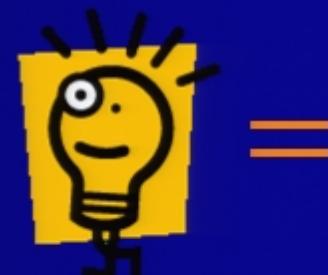
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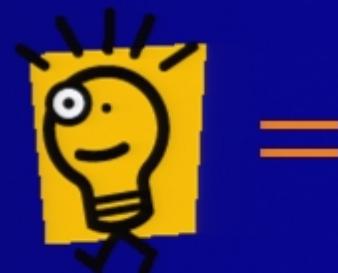
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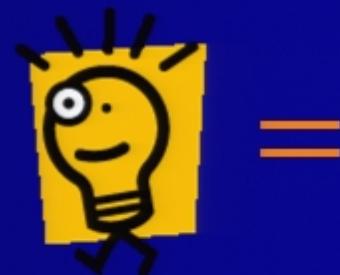
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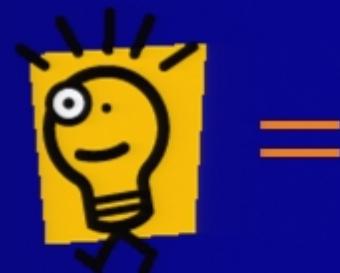
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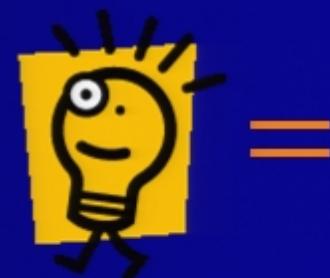
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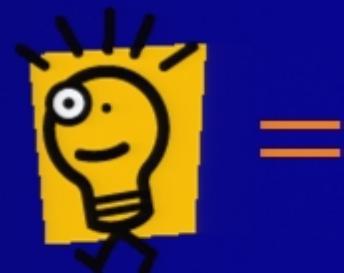
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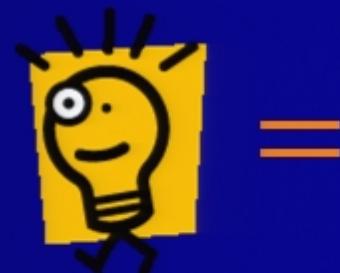
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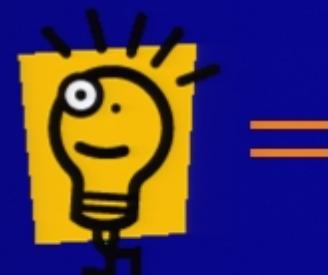
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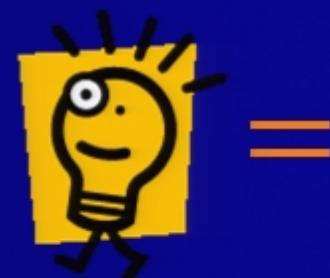
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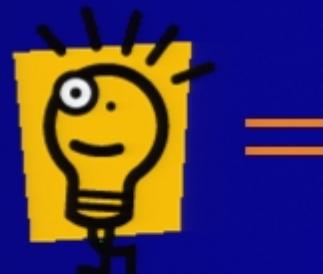
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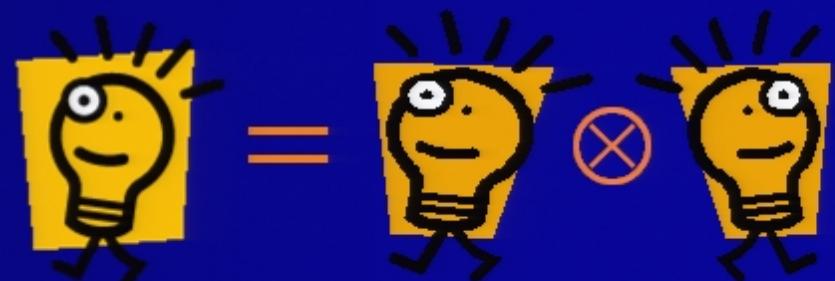


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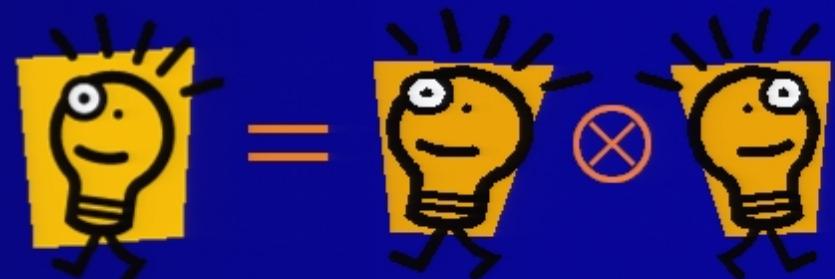


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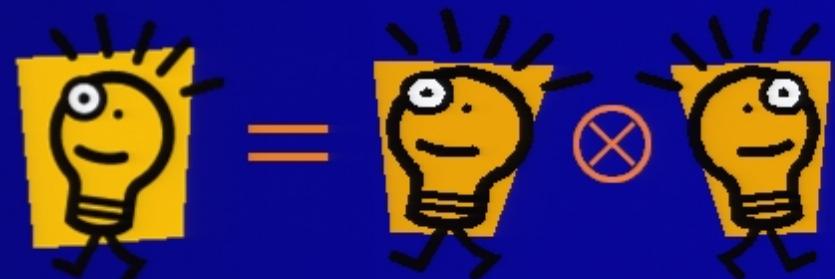


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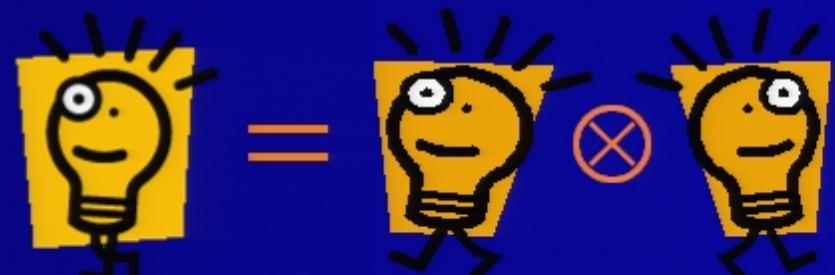


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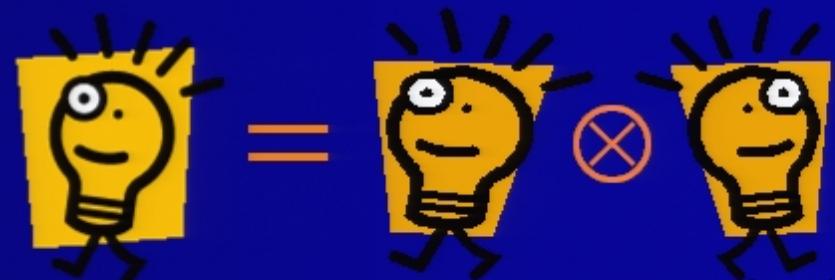


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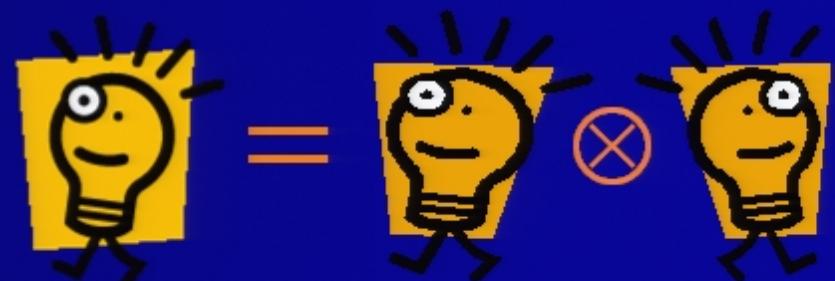


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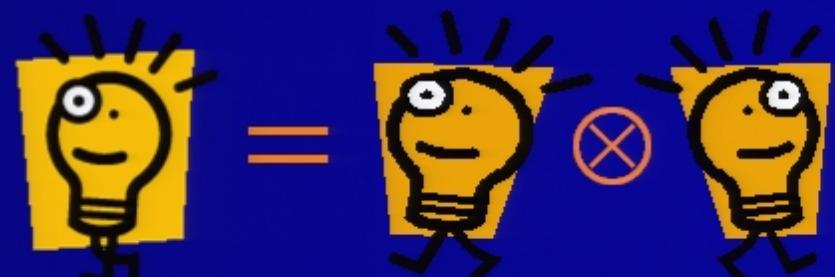


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$$T(\rho_A) \approx \rho_A \left( 1 - \frac{\Gamma^2}{4} \right) + (\sigma_x \rho_A \sigma_x + \sigma_y \rho_A \sigma_y) \frac{\Gamma^2}{8}$$

$$\Gamma = \frac{1 - \sqrt{1 - \beta^2}}{\beta} \frac{\Delta}{m}$$

Recoverable, but info on both  $\Lambda$  and  $\psi$  is required

# Conclusions

Bartlett and Terno  
quant-ph/0403014

