Title: Mixed-state entanglement in the light of pure-state entanglement constrained by superselection rules

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Abstract: Quantum Information Theory

Mixed-state entanglement in the light of pure-state entanglement constrained by superselection rules

Stephen Bartlett

THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

Collaborators:

Robert Spekkens

Howard Wiseman

A generic quantum system

ρ — Density matrix

$\mathcal{E} : \rho \rightarrow \rho'$ — Trace-preserving CP maps

- Consider all states, all maps, all measurements
- Dimension (number of degrees of freedom) determines everything

A generic quantum system

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Density matrix   Trace-preserving CP maps

BORING!

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Interesting restrictions

Split the system up into two parts

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- bipartite: restrict to LOCC
- *entanglement* → Bell inequalities, quantum cryptography

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Can pure state entanglement with SSRs teach us about mixed state entanglement?

Mixed State Entanglement 101

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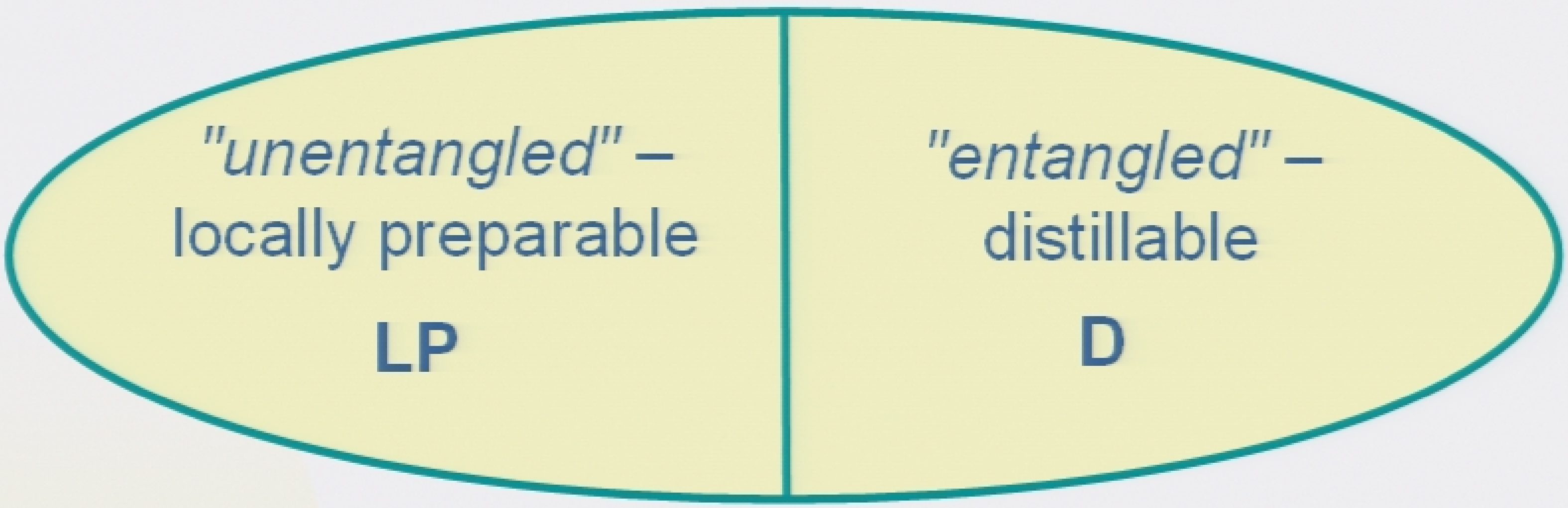
Good references:

Eggeling, Vollbrecht, Werner and Wolf, *Phys. Rev. Lett.* **87**, 257902 (2001)

Horodecki (x3), arXiv:quant-ph/0109124

Bipartite entanglement

All pure states



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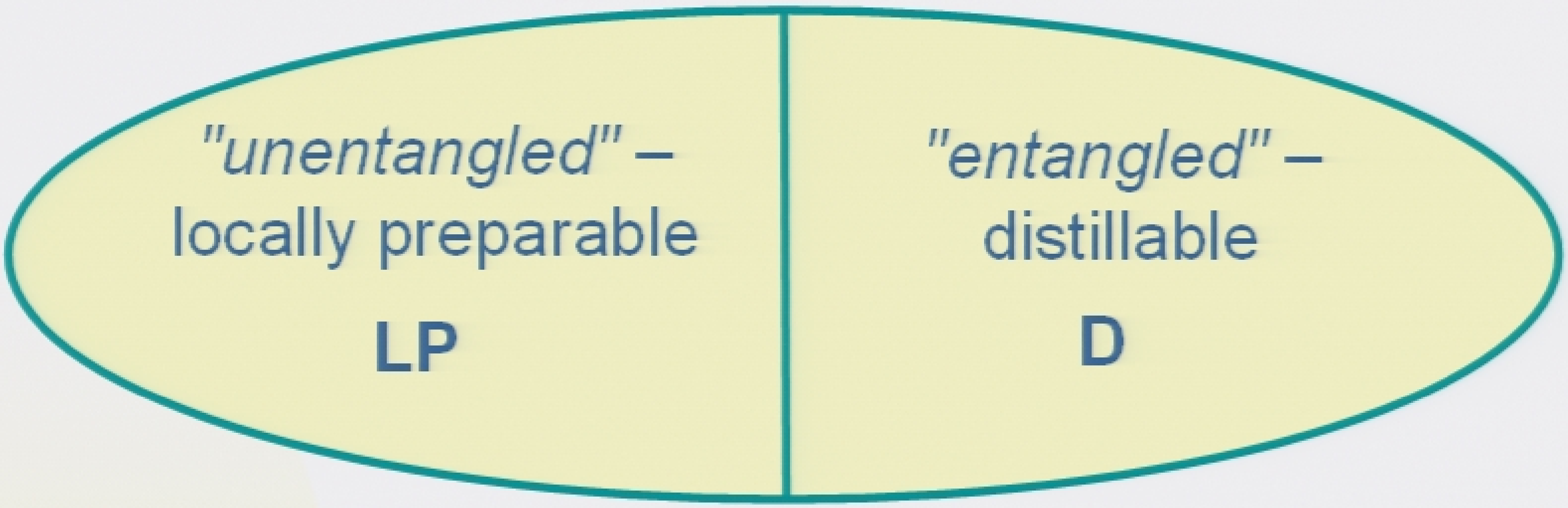

"Unentangled"

- *Physically:*
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- *Mathematically:*
  product states

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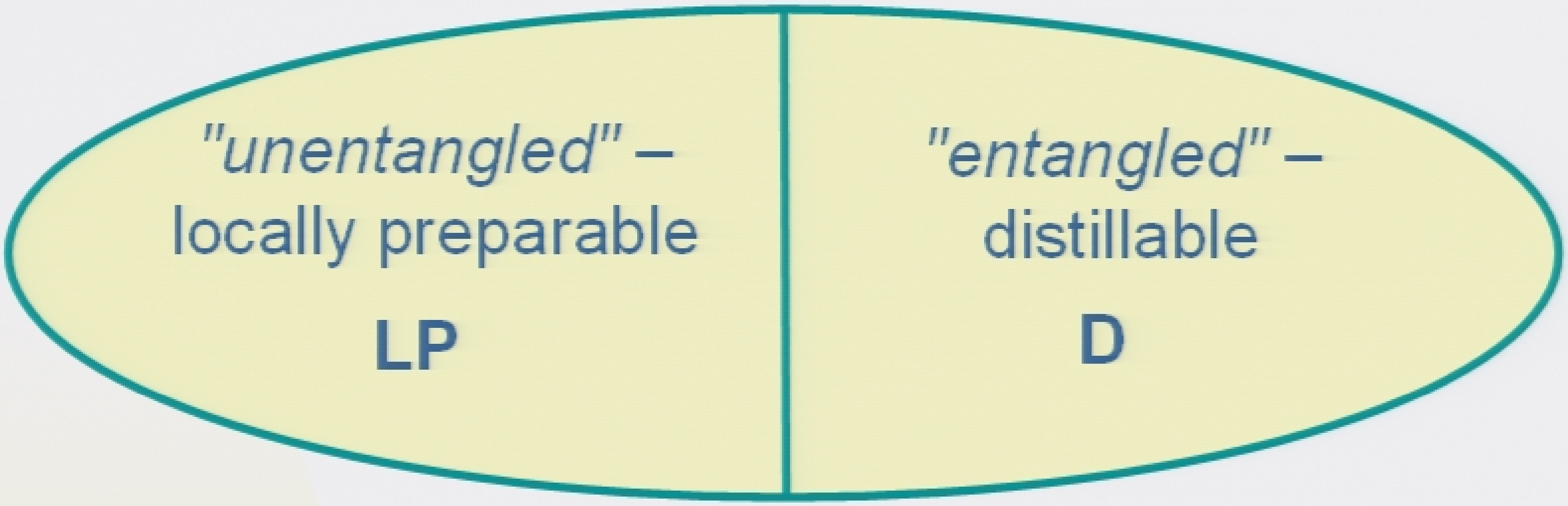

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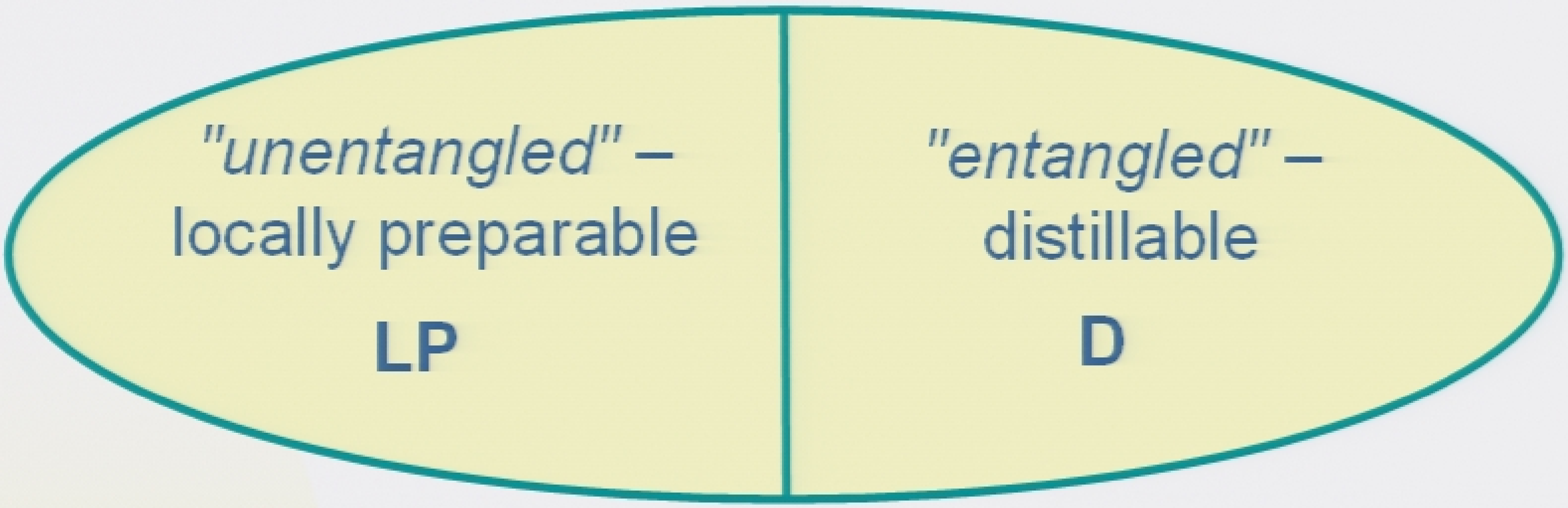

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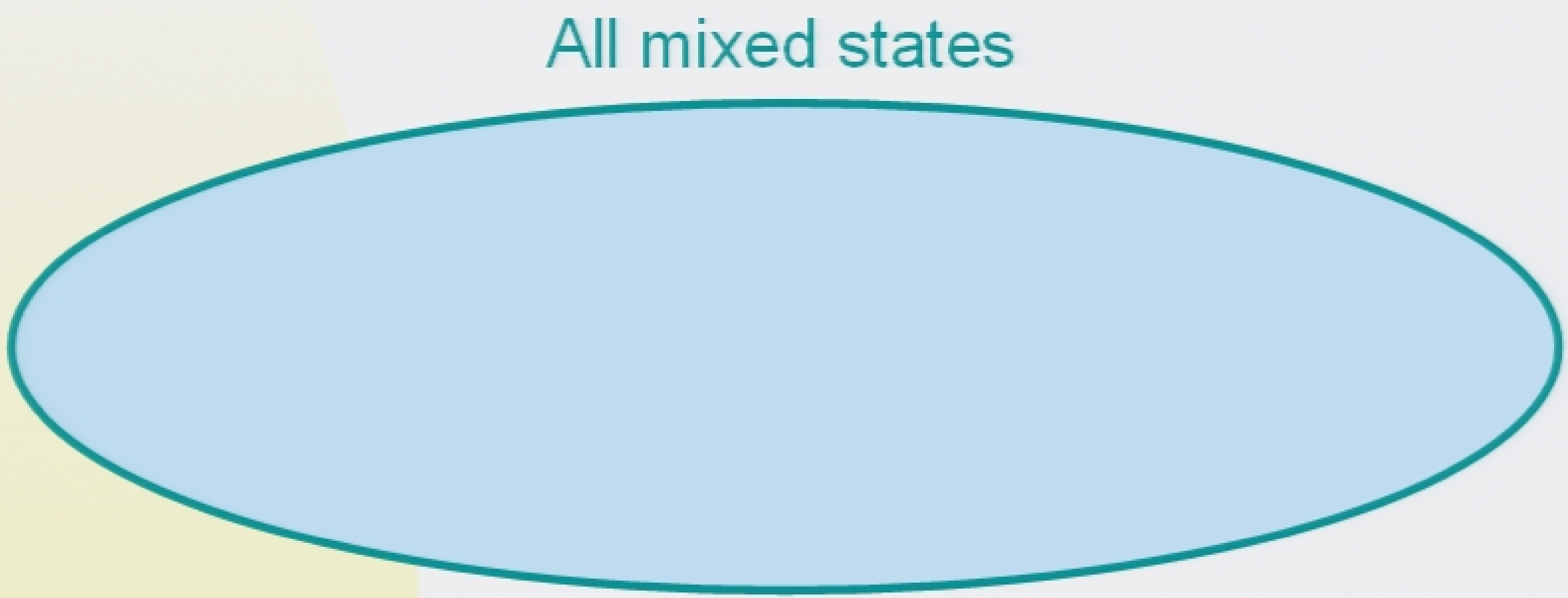
$$LP \cap D = 0$$

Entanglement as a resource

Mixed state entanglement



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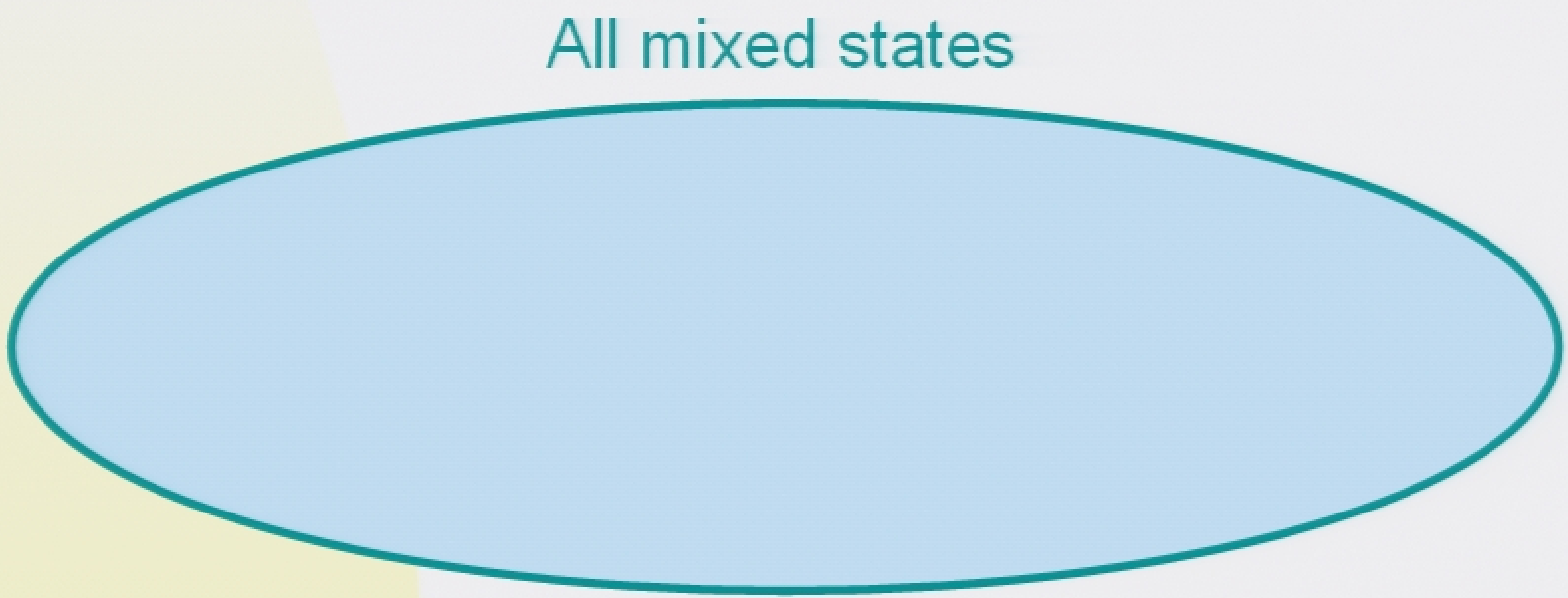
$$\rho = \sum_i p_i \, \sigma_i^A \otimes \sigma_i^B$$



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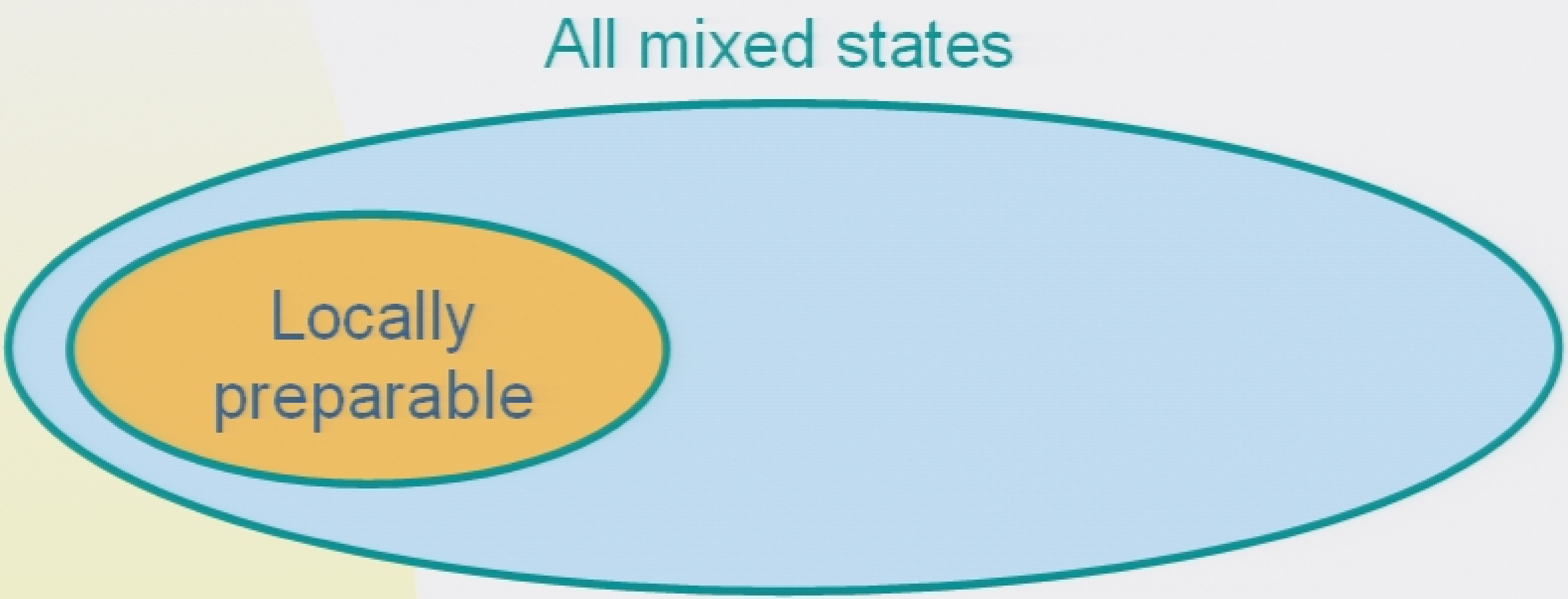

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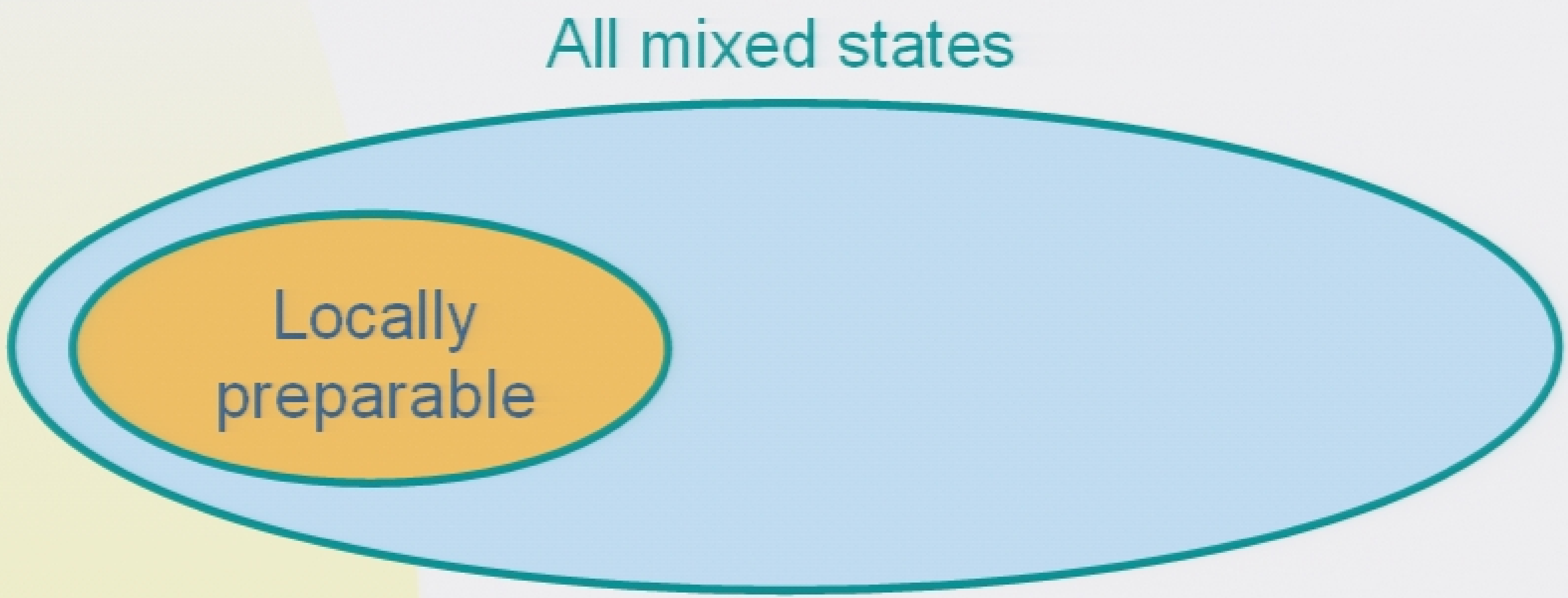
- *Mathematically:* ?????



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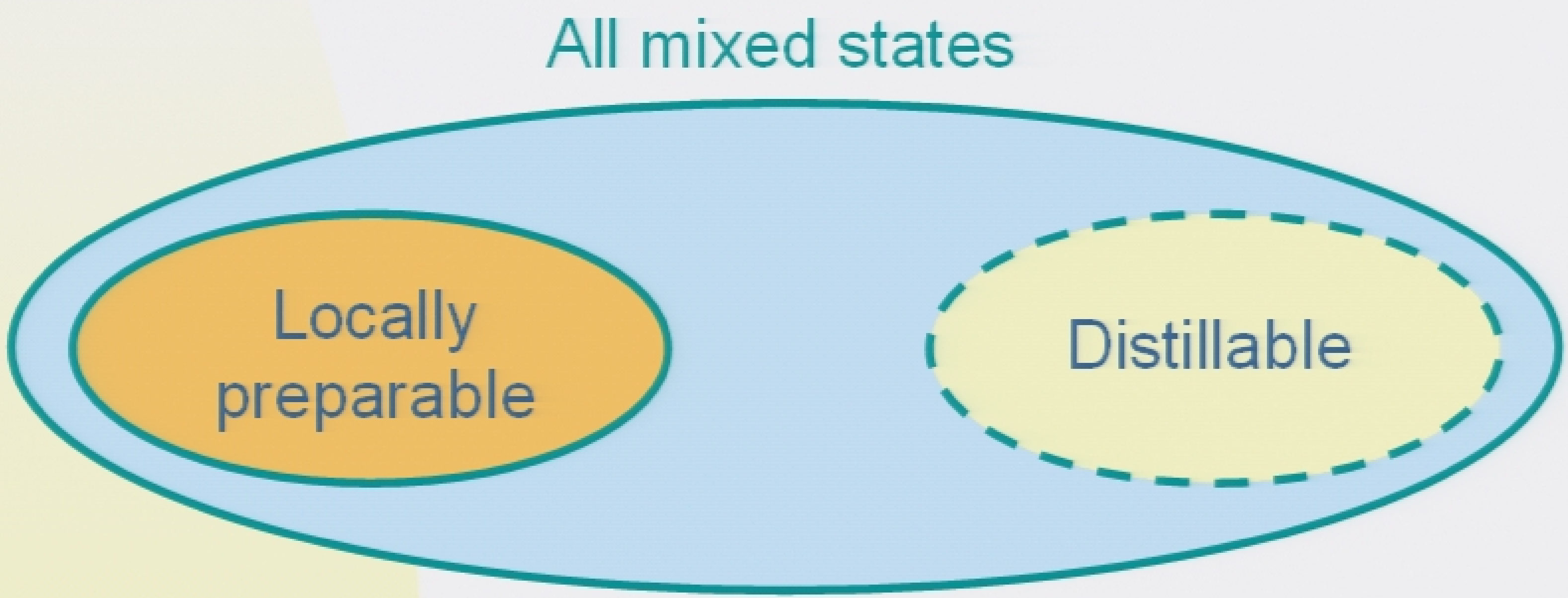

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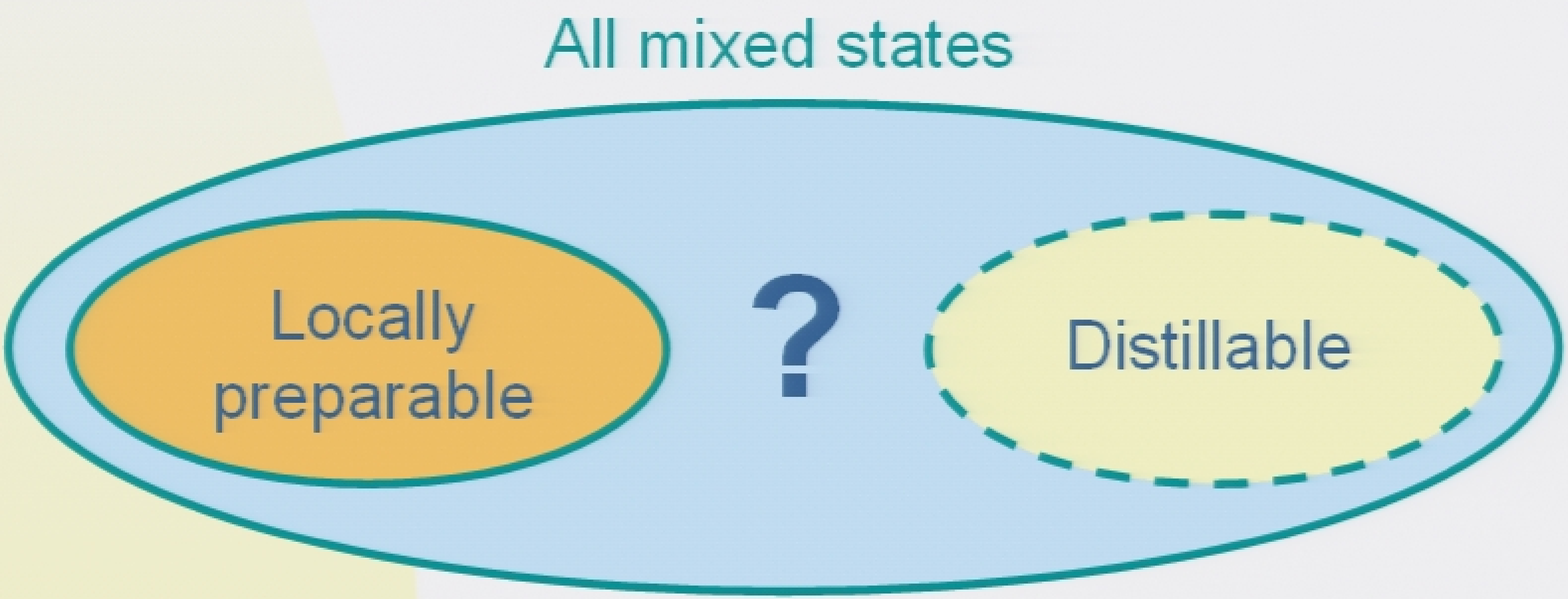
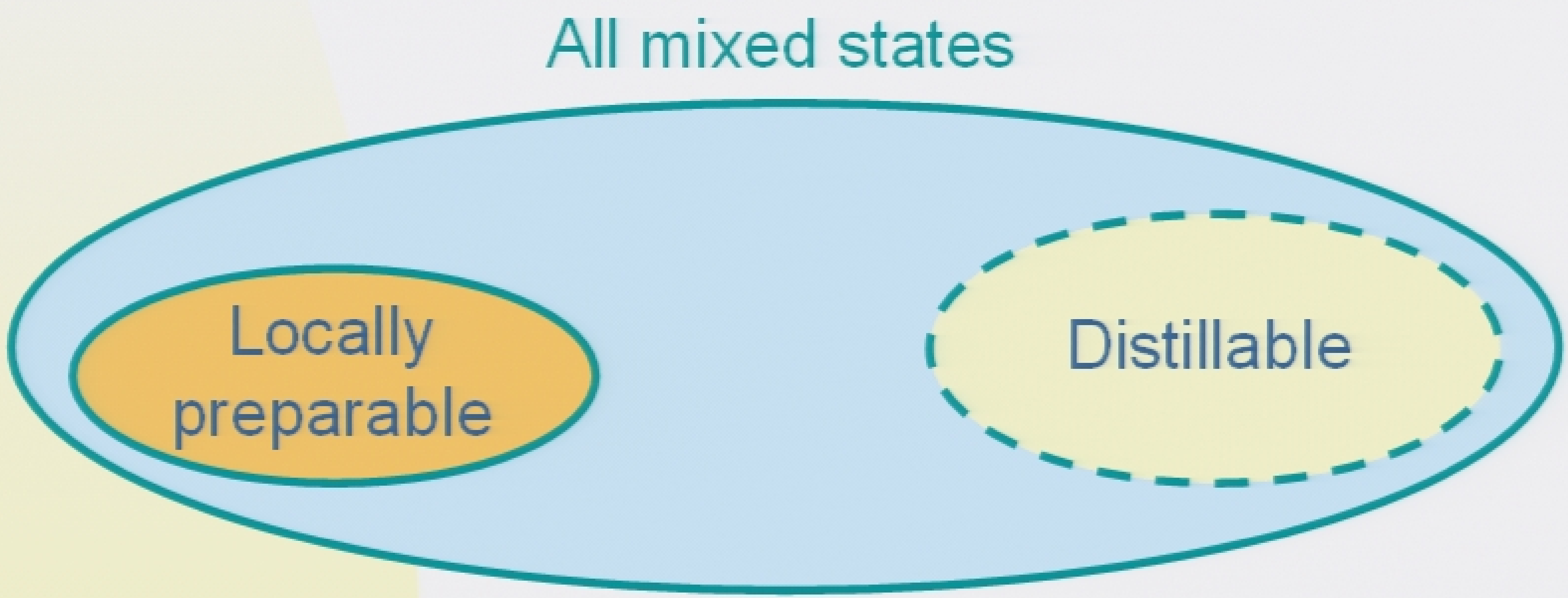


Do there exist states that are not locally preparable and not distillable?

Bound entanglement

1998: Horodecki (x3)

- LOCC operations: preserve positivity of partial transpose (PPT)
- Distillation is the creation of pure Bell states, which are NPPT (*not* PPT)
- ∴ PPT states that are not locally preparable cannot be distilled under LOCC



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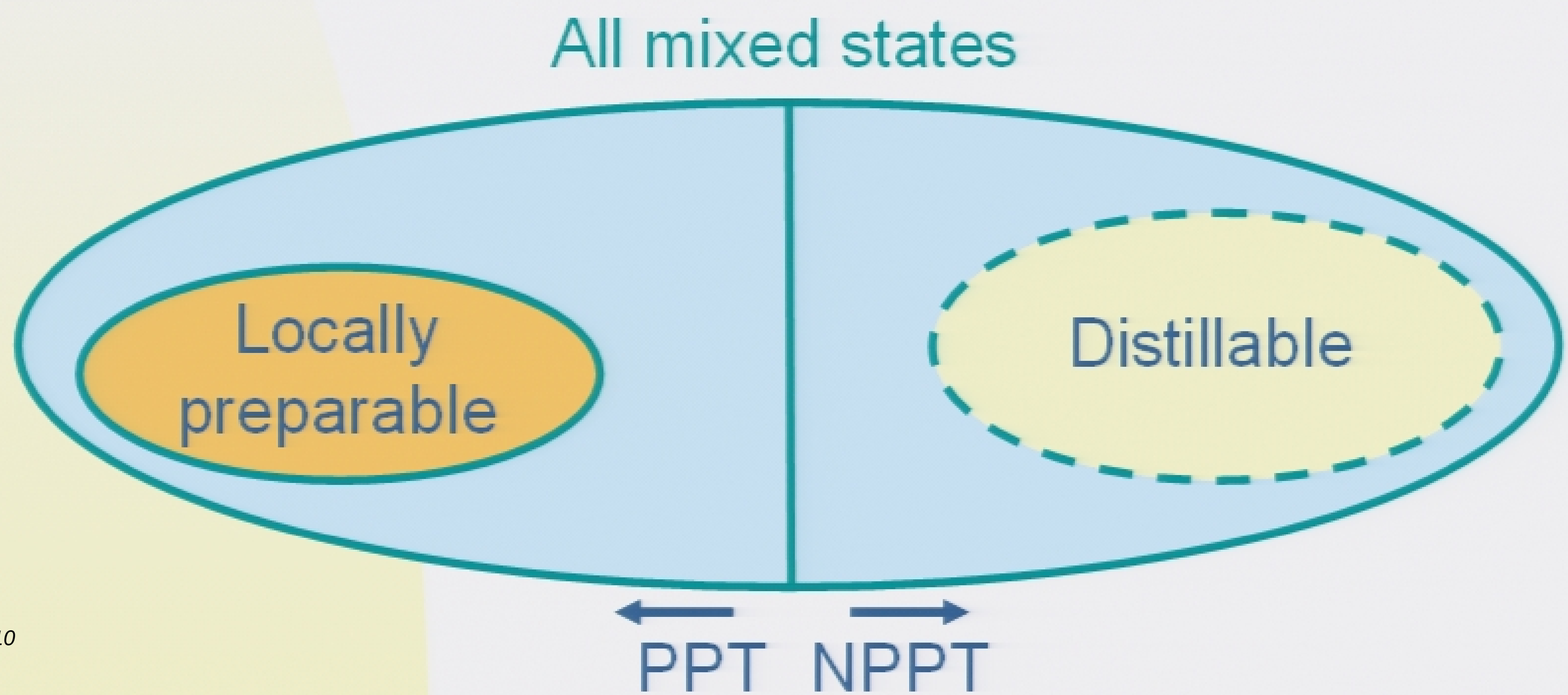

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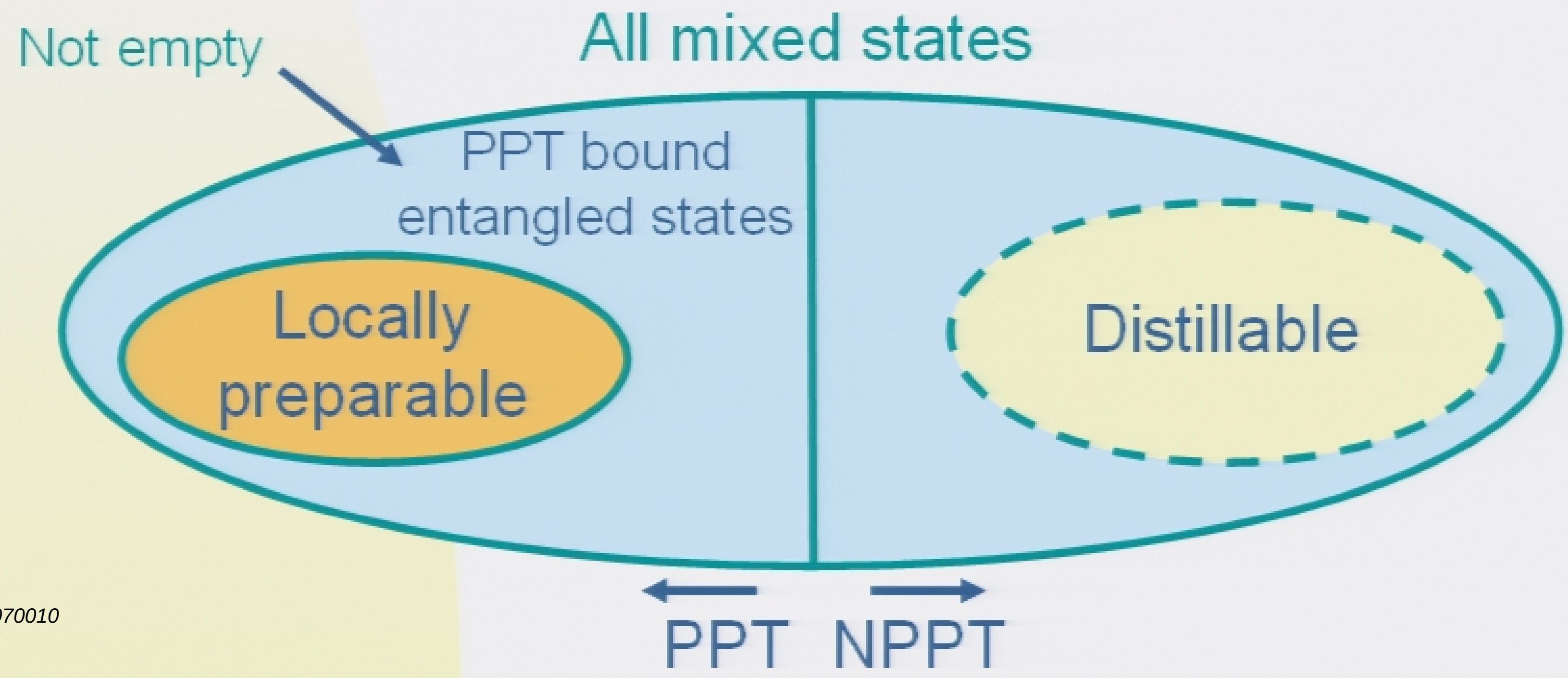

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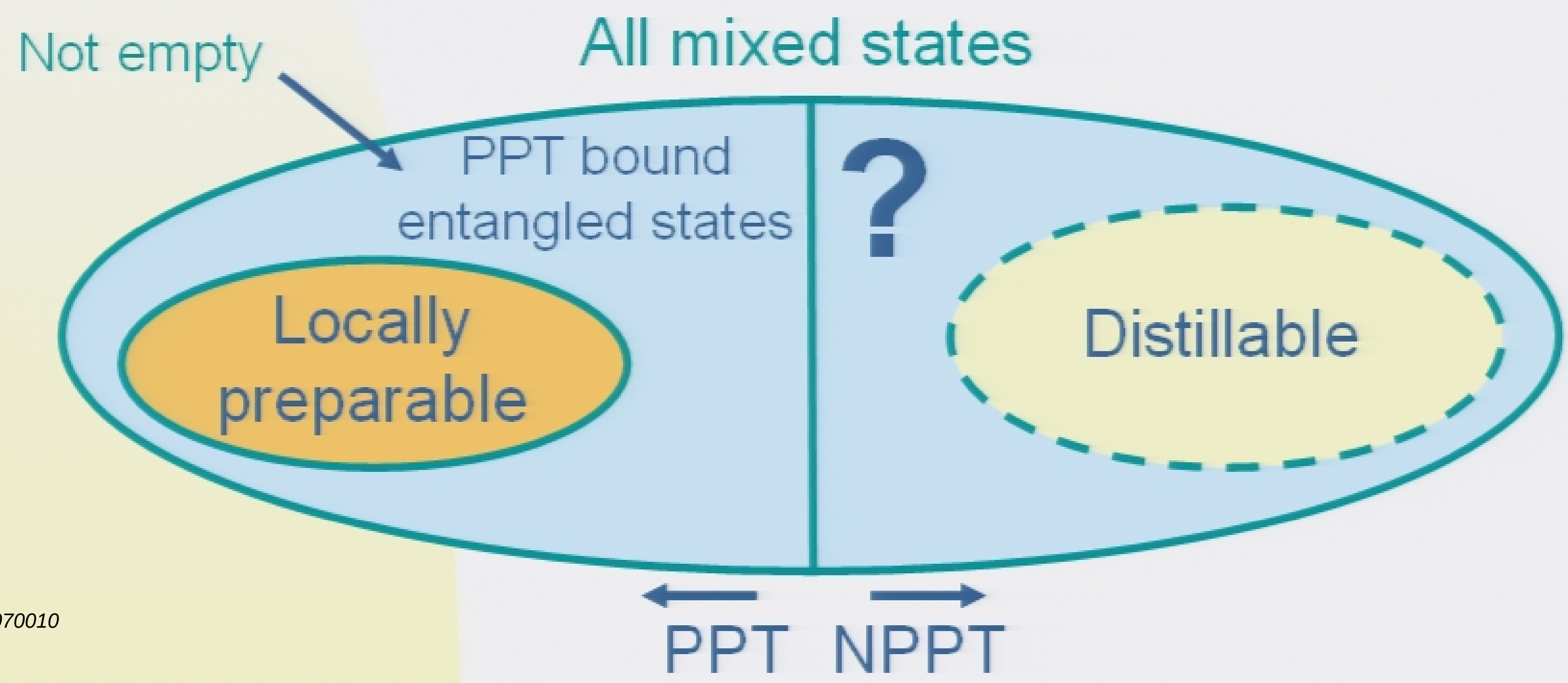
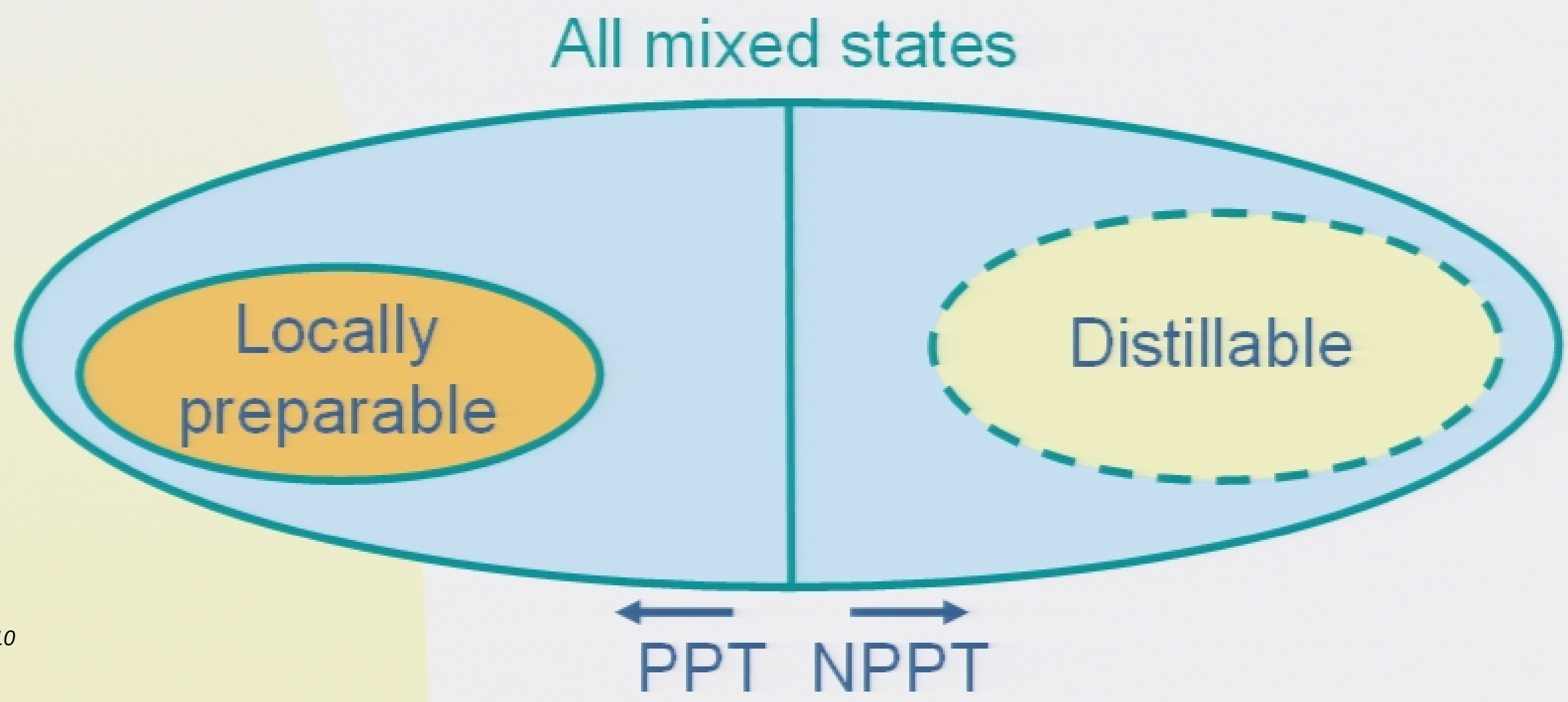


1-distillable

- "Distillable" is too hard to characterise
- Define *1-distillable*: one copy can, with some finite probability, be projected to an entangled 2-qubit state

$$\exists|\Psi\rangle \text{ s.t. } \langle\Psi|\rho^{T_A}|\Psi\rangle < 0$$

- Easy to classify



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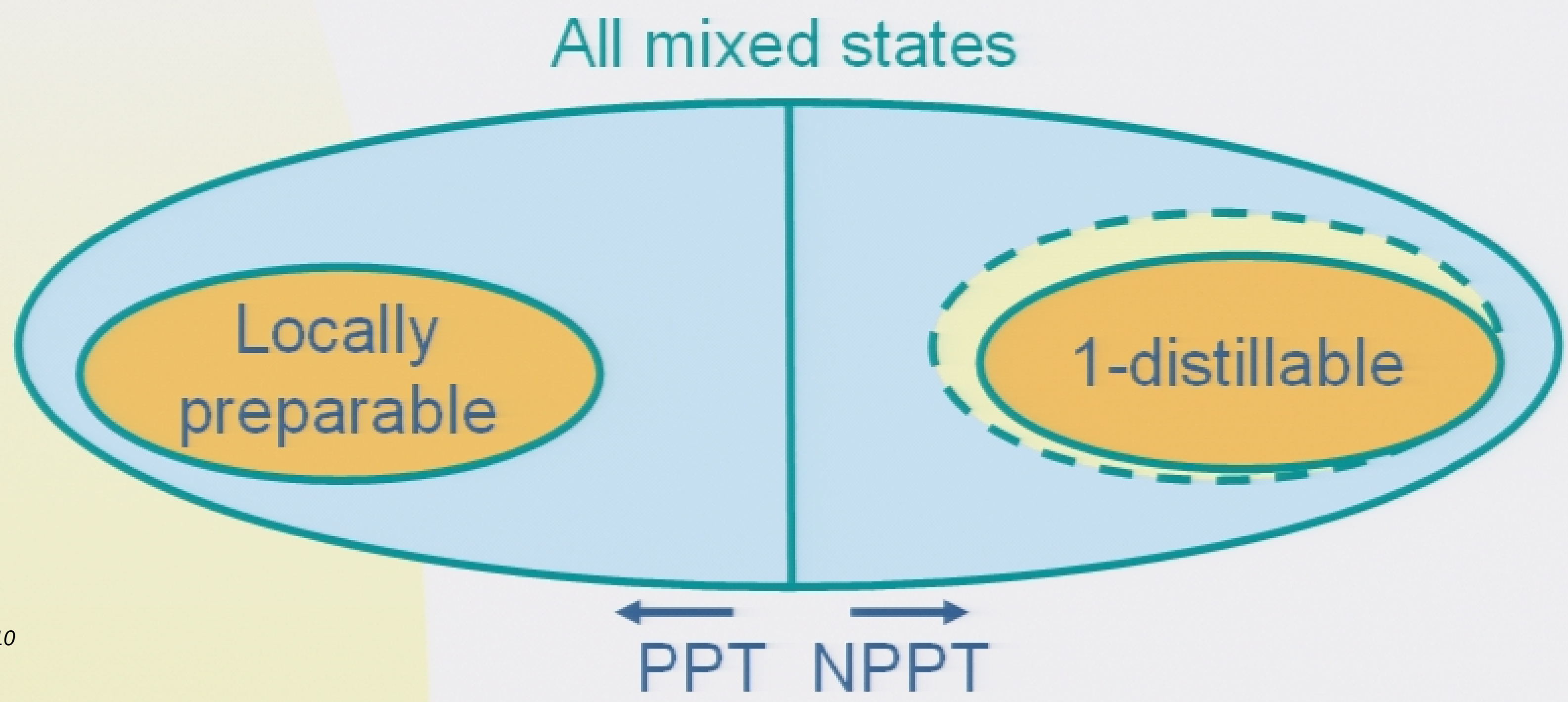

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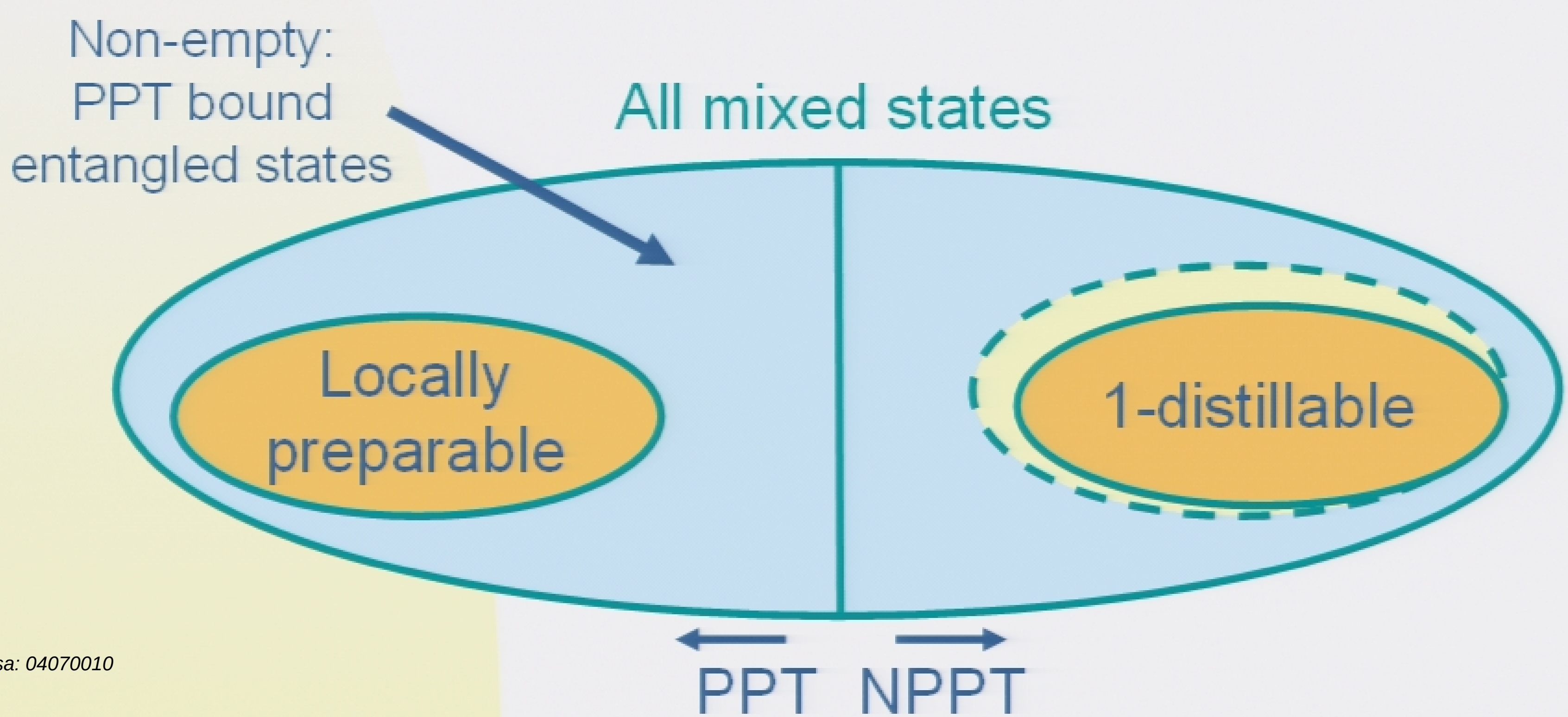

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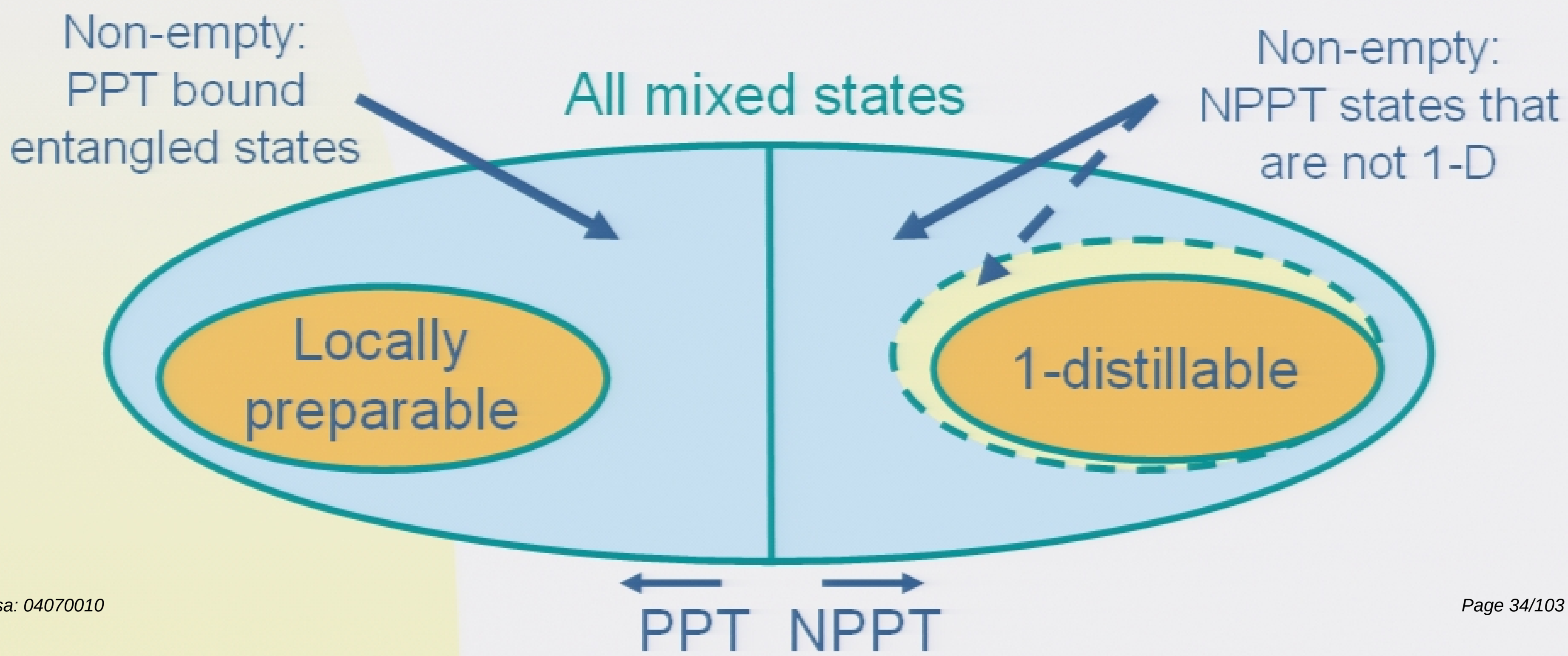
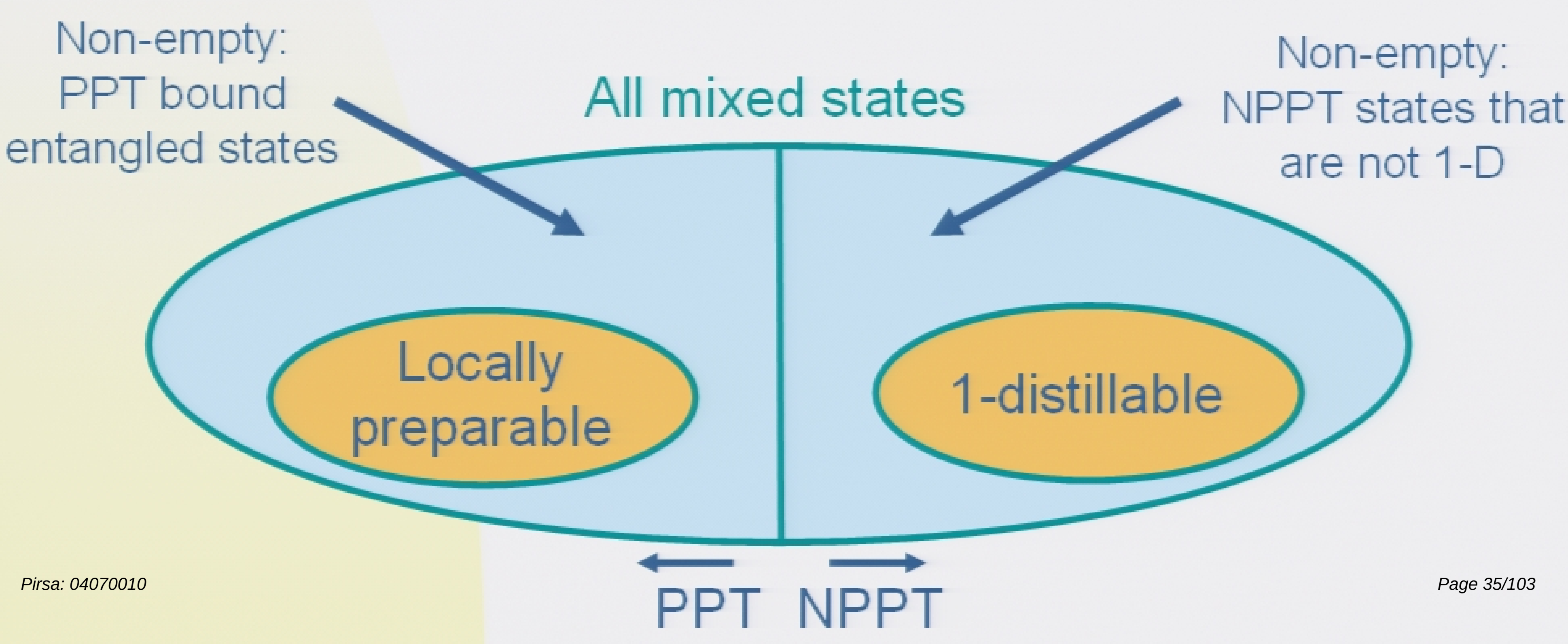


A helpful channel

- Channel C: distributes PPT states (PPT-preserving channel)
- A resource in addition to LOCC – can do more



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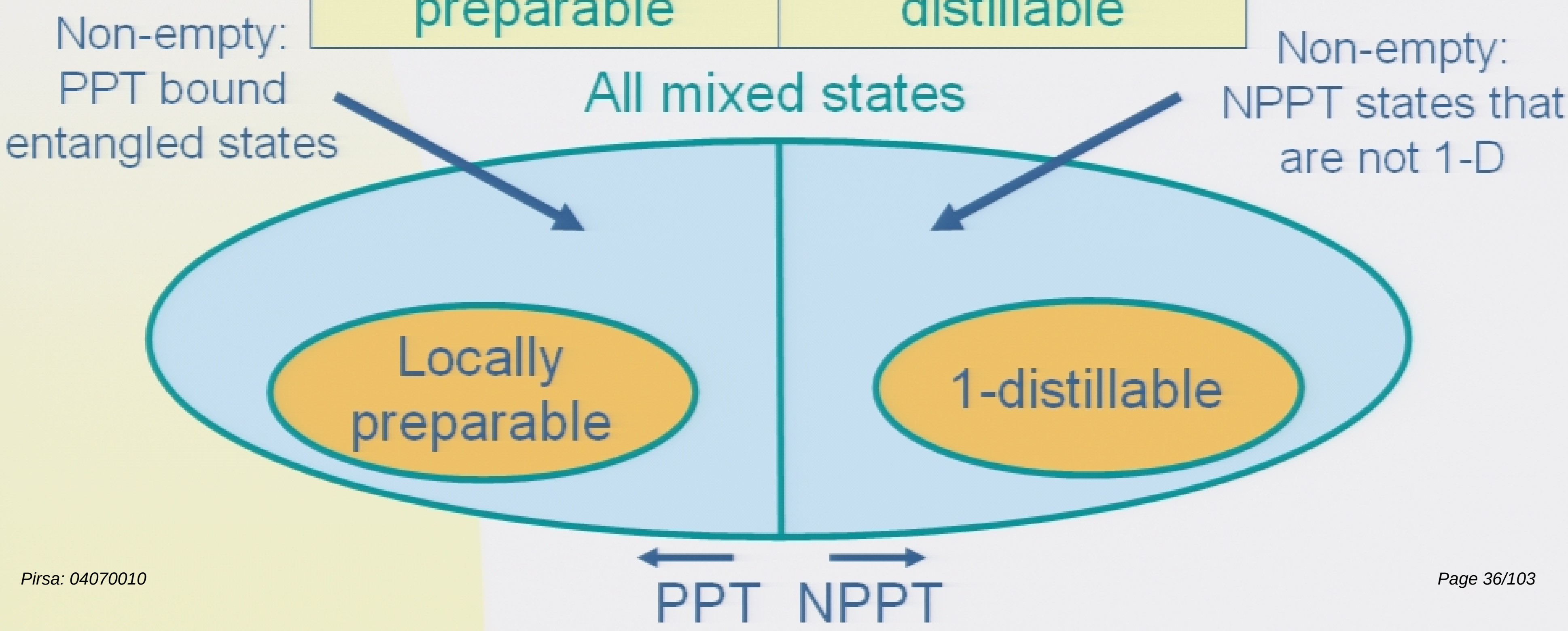

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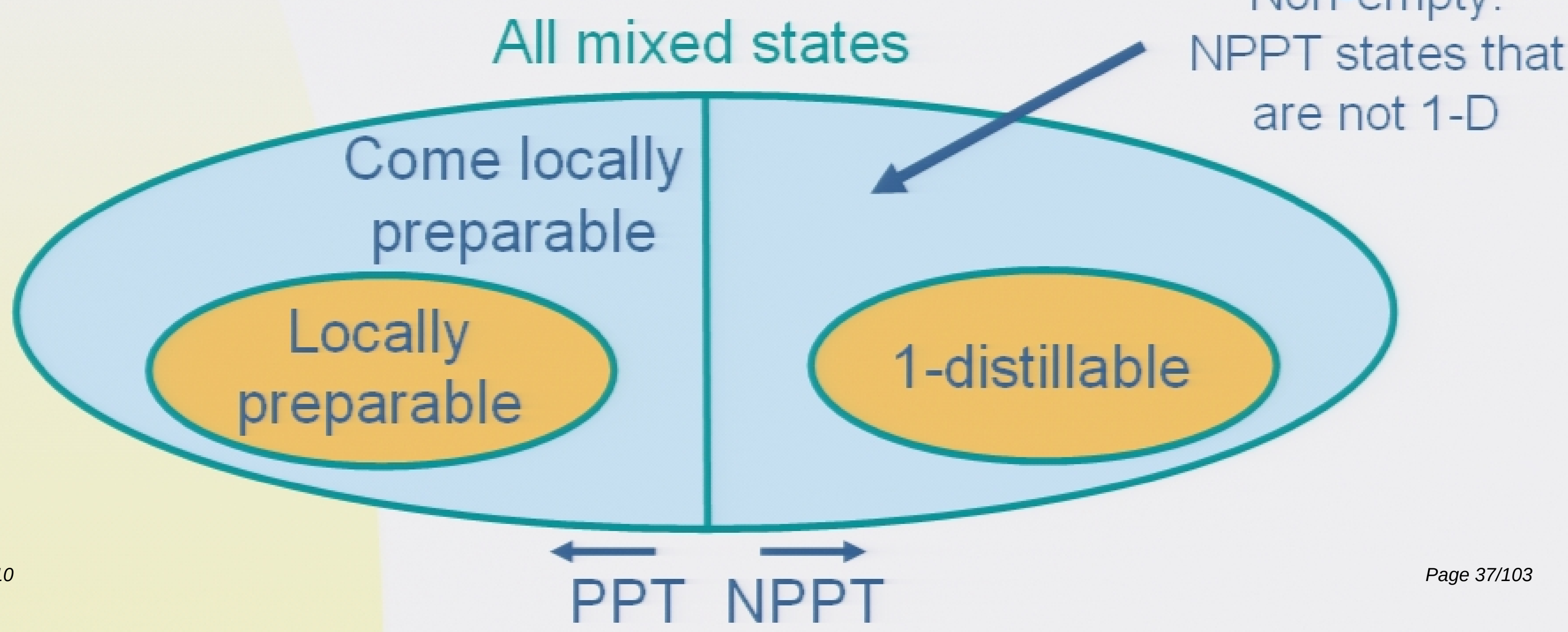

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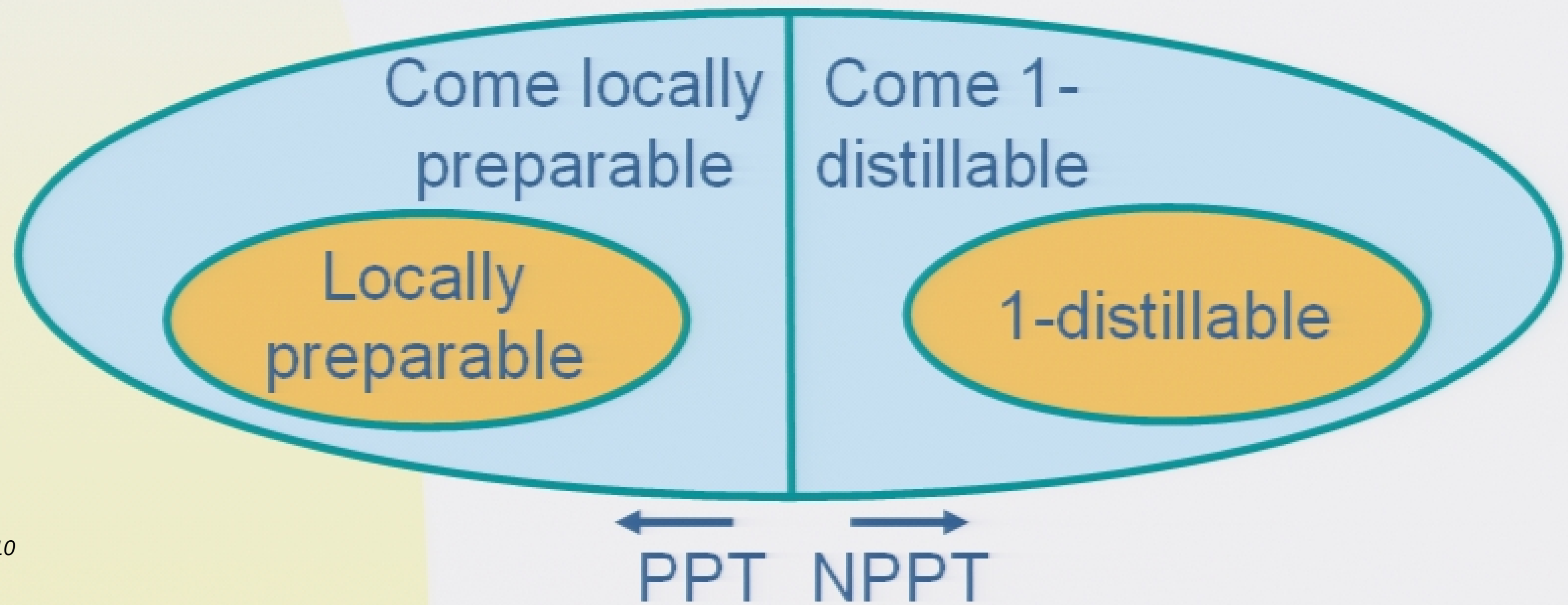

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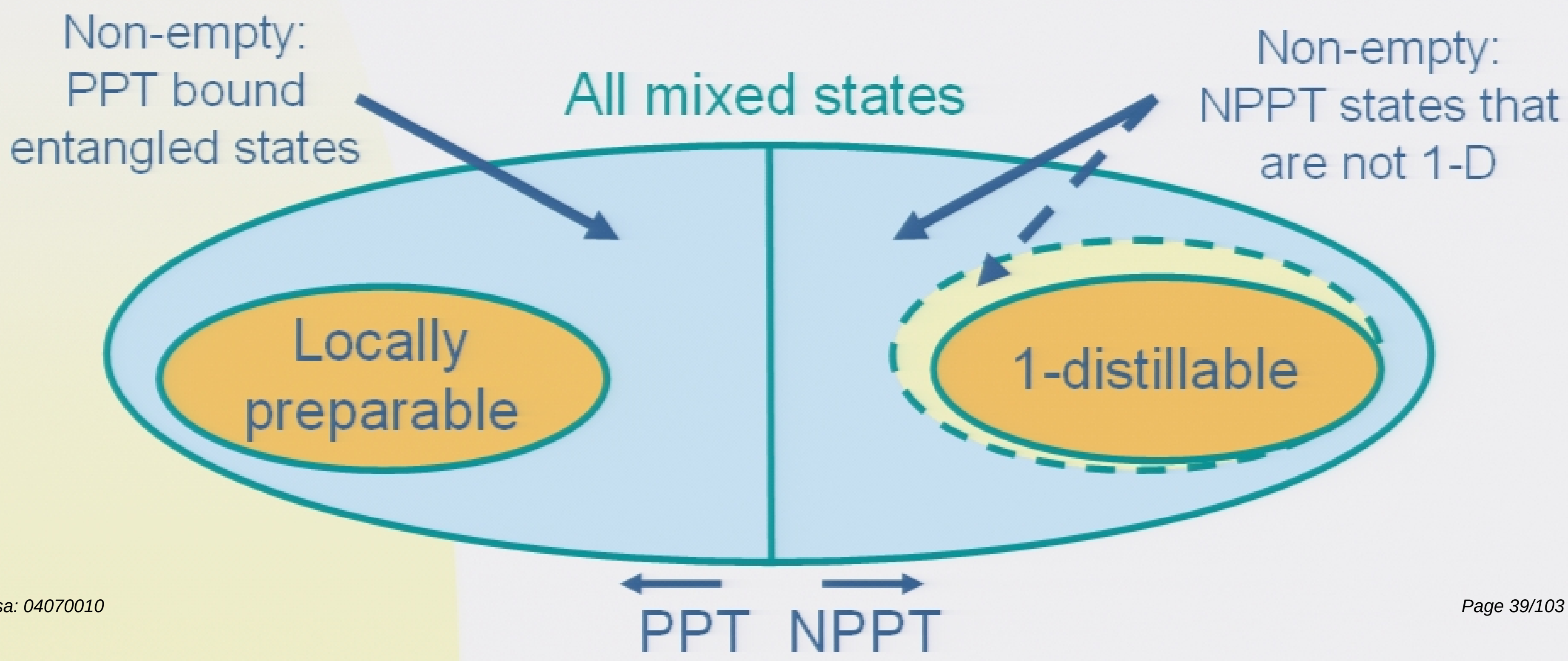

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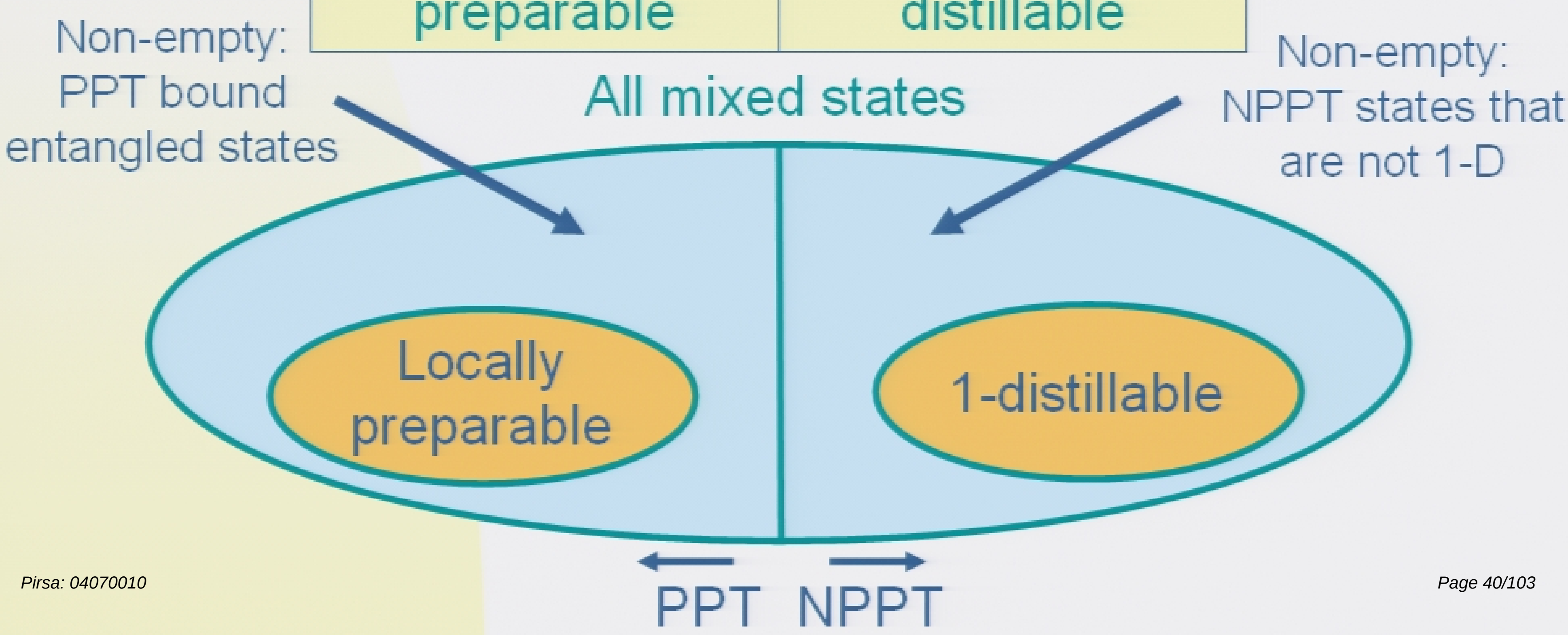

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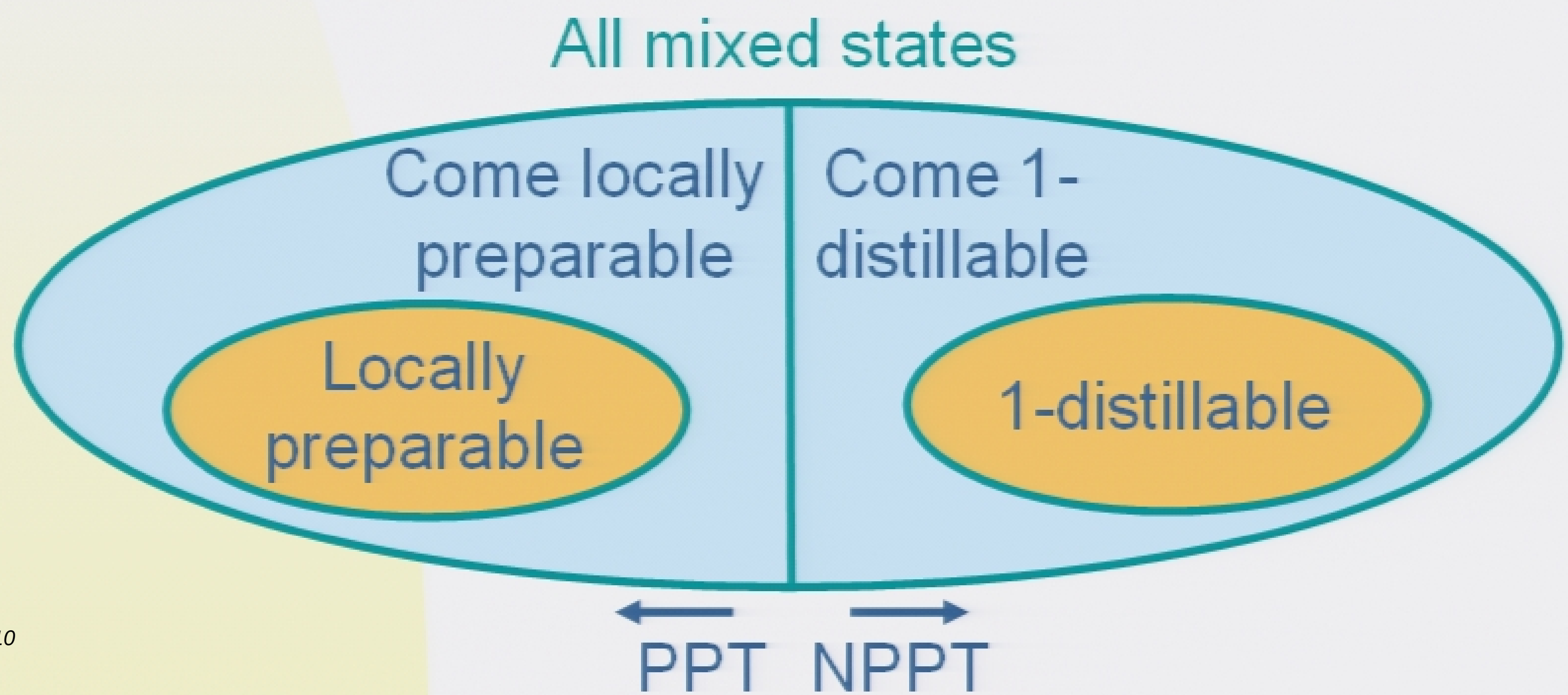
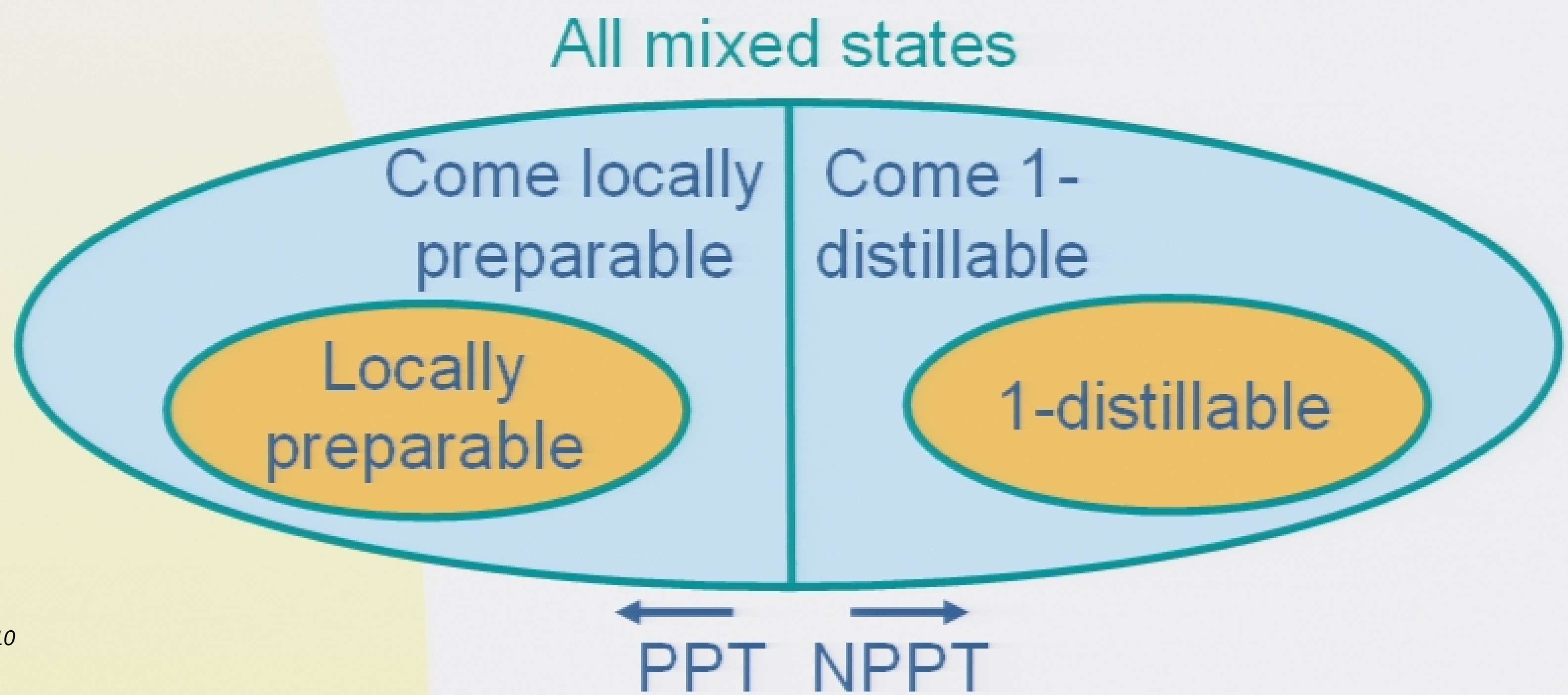


Activation

- Channel *C*: implemented probabilistically using PPT states (Jamiolkowski isomorphism)
- For every "come 1-distillable" state, there exists a "come locally preparable" state that *activates* the 1-distillability



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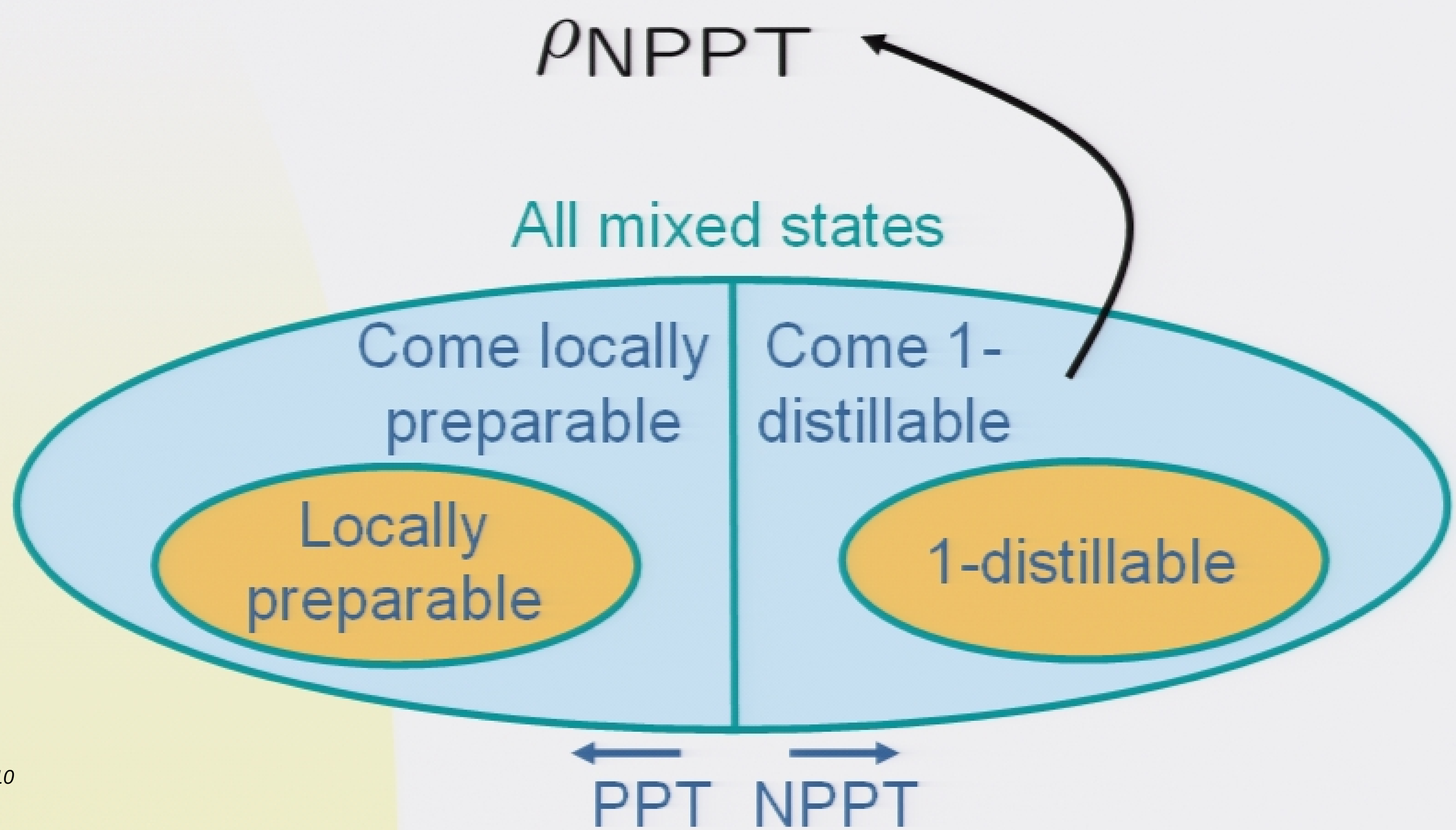

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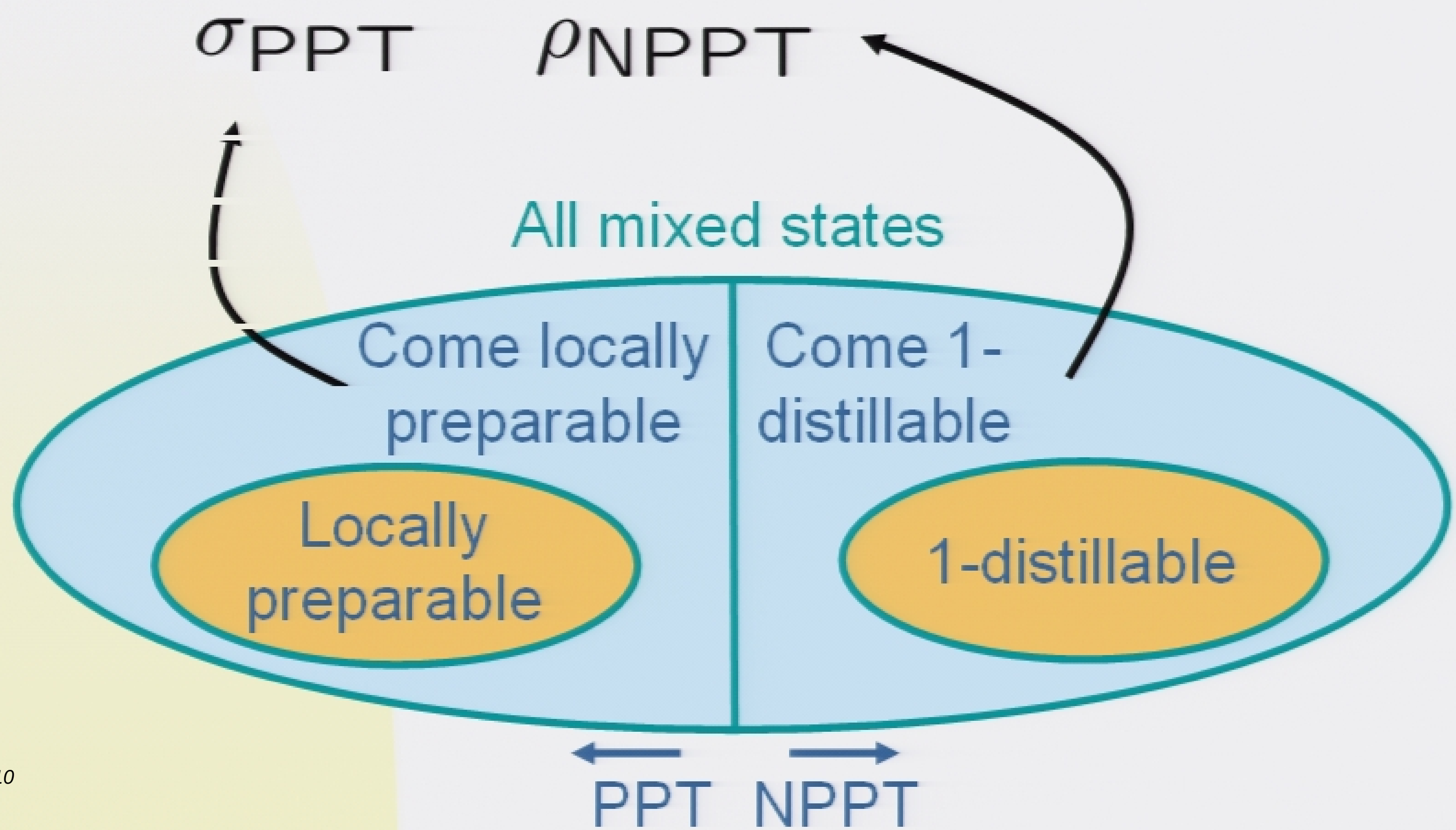

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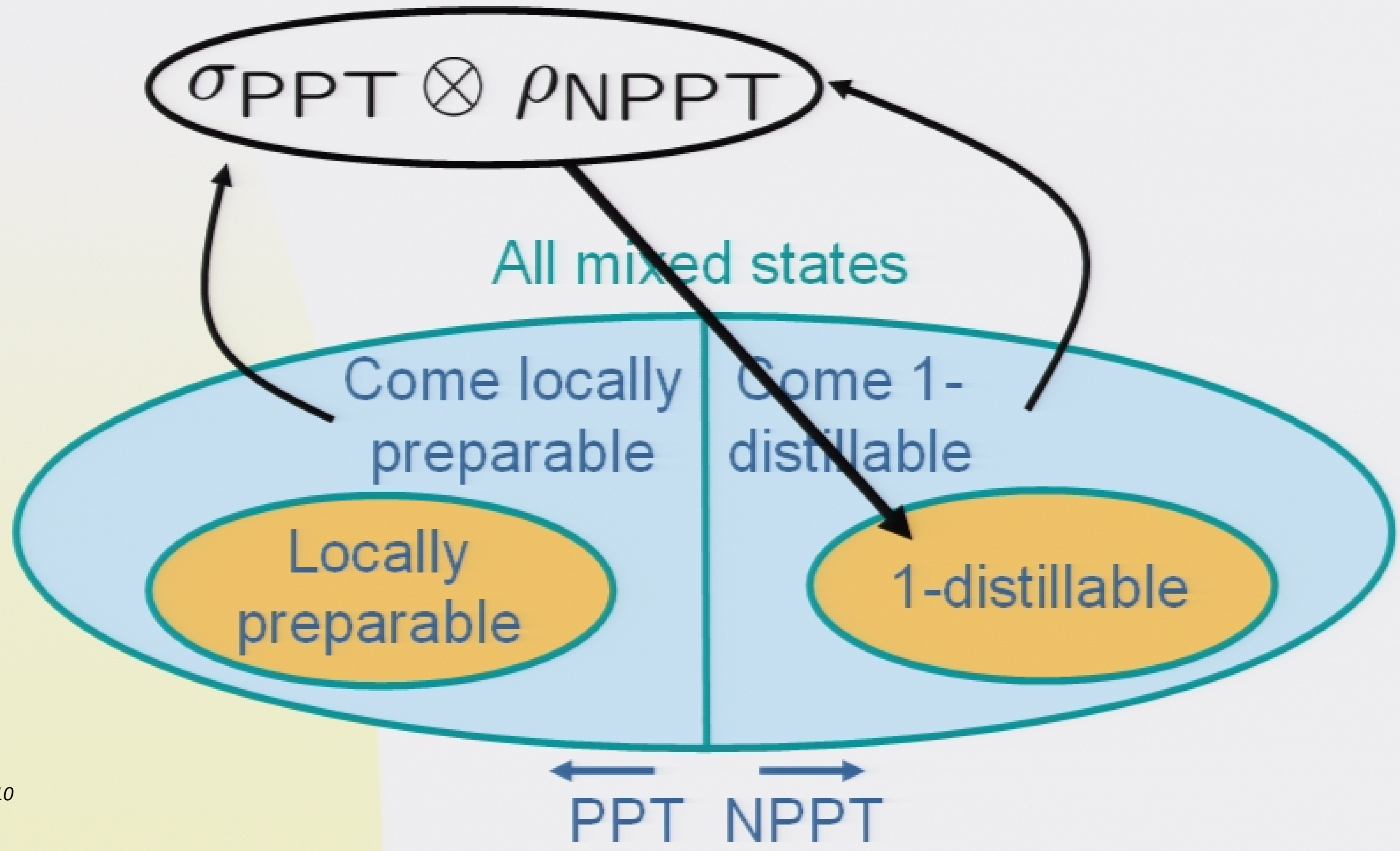
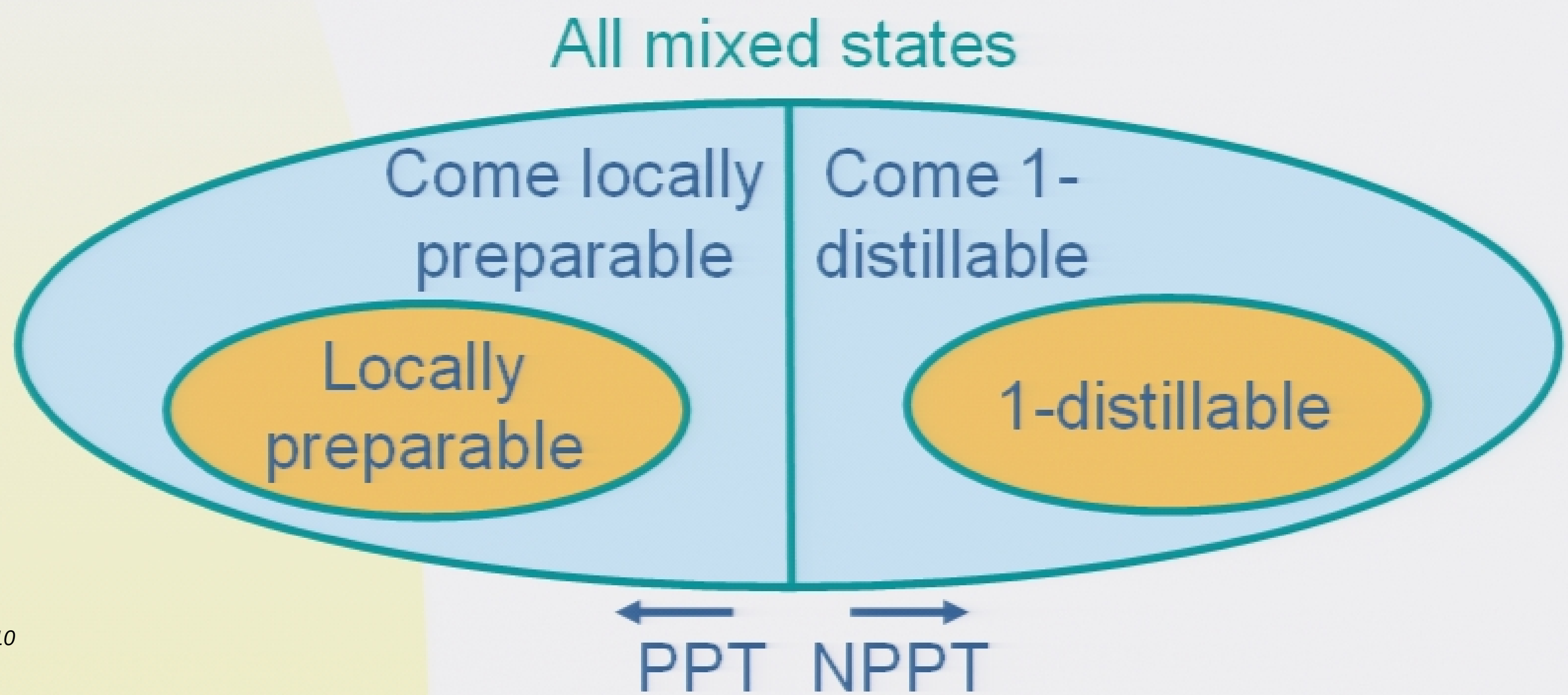


2-distillable and *N*-distillable

- Some "come 1-distillable" states, although not 1-distillable, become 1-distillable when given 2 (or *N*) copies



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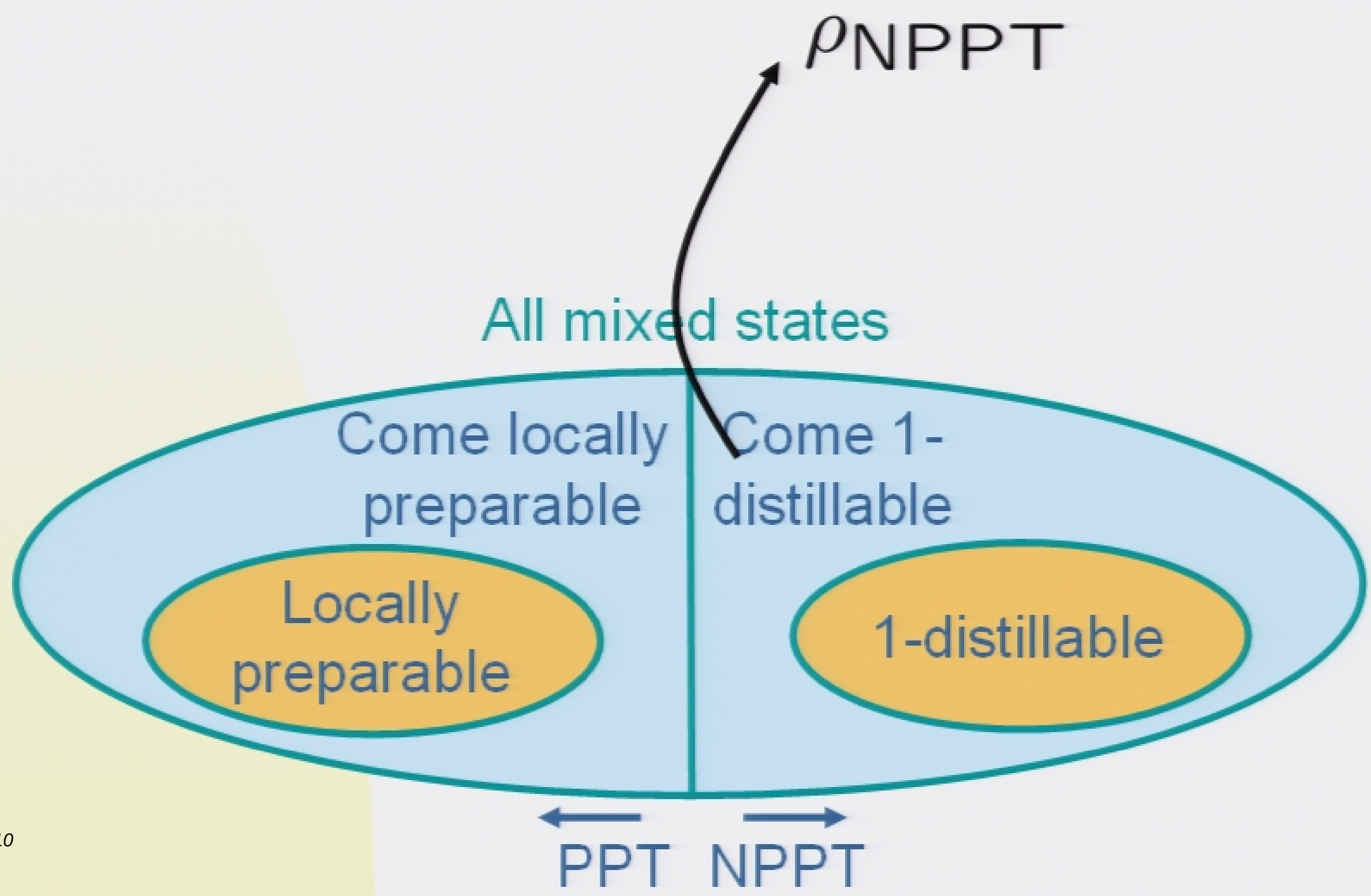

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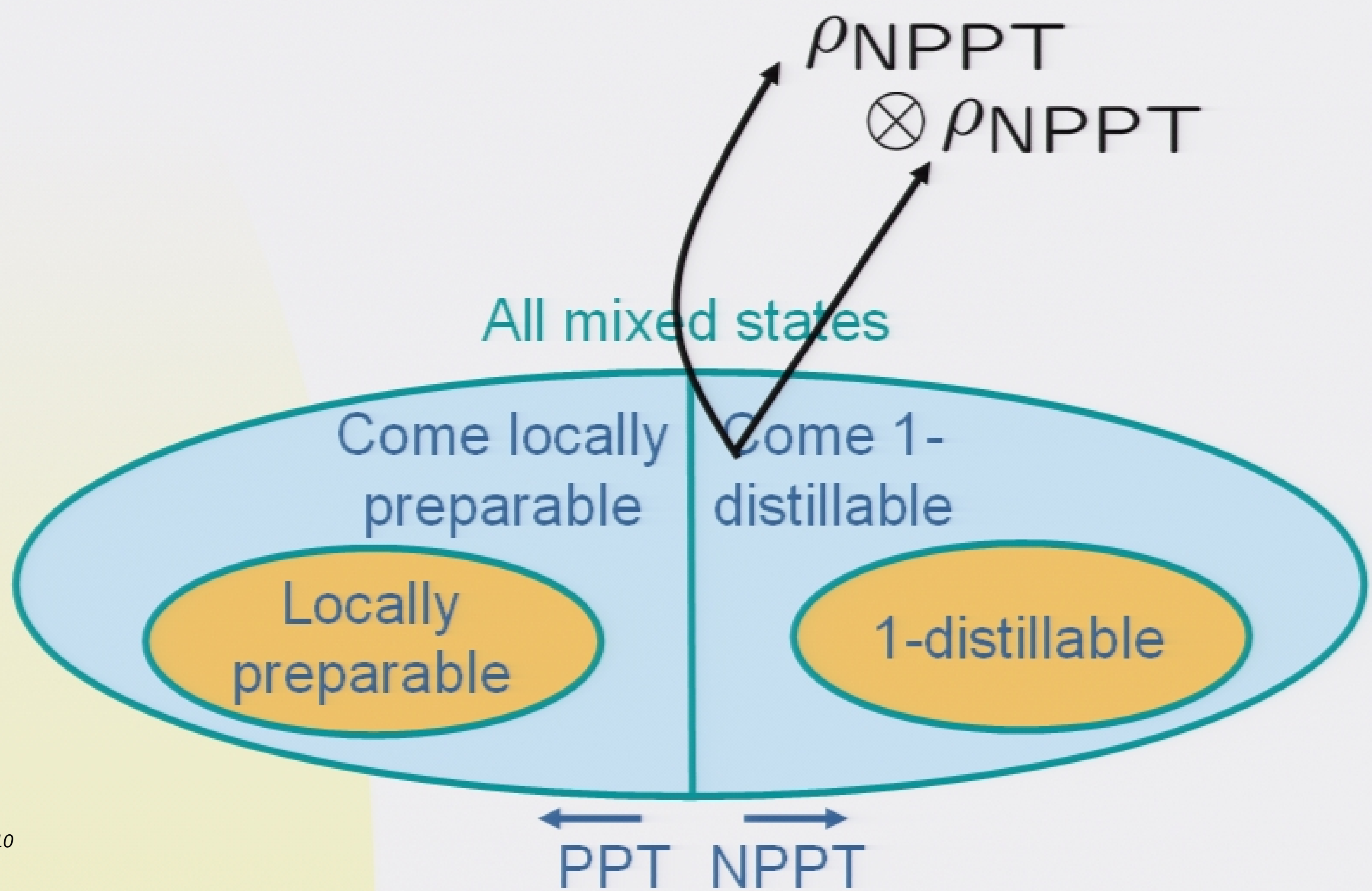

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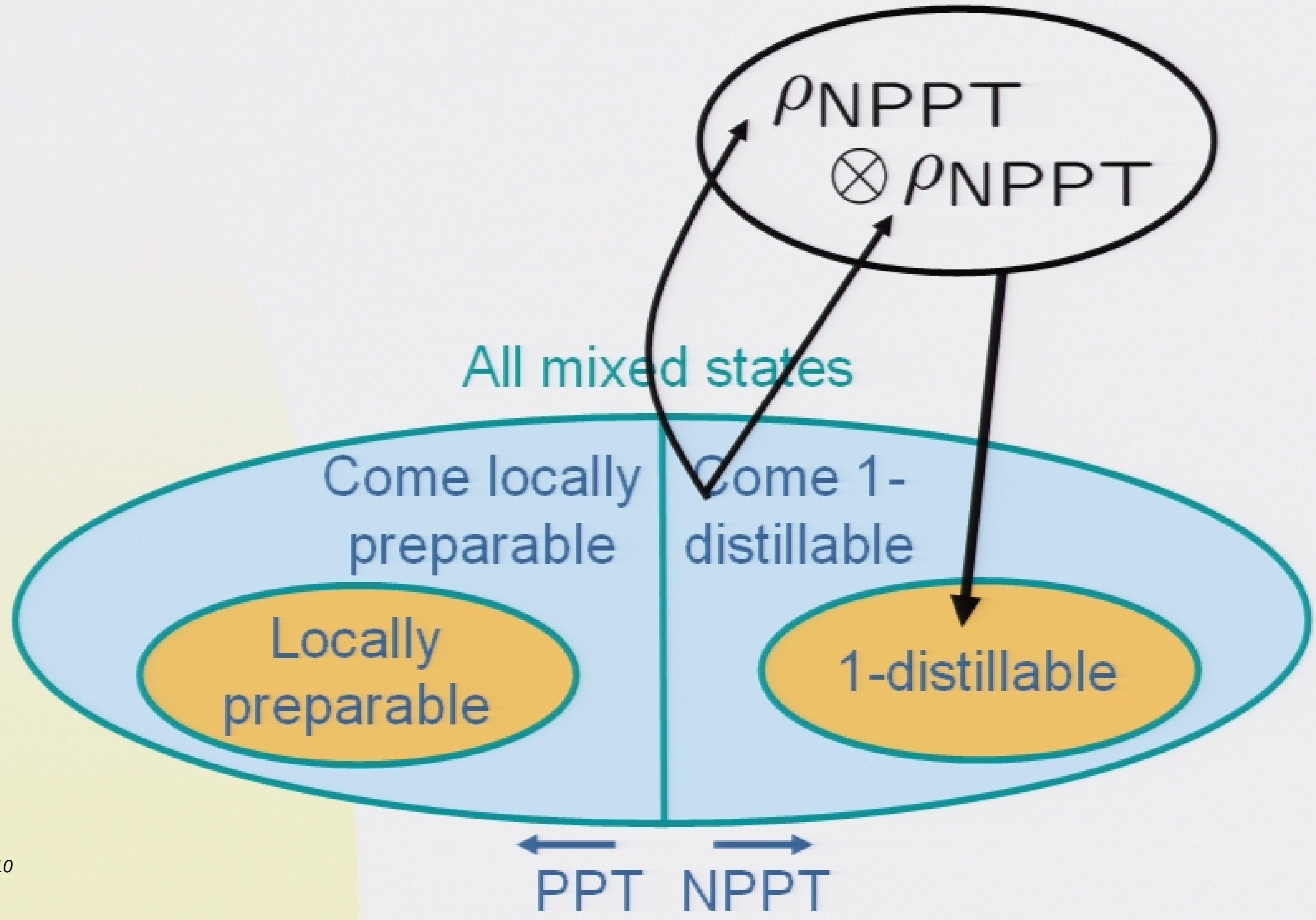
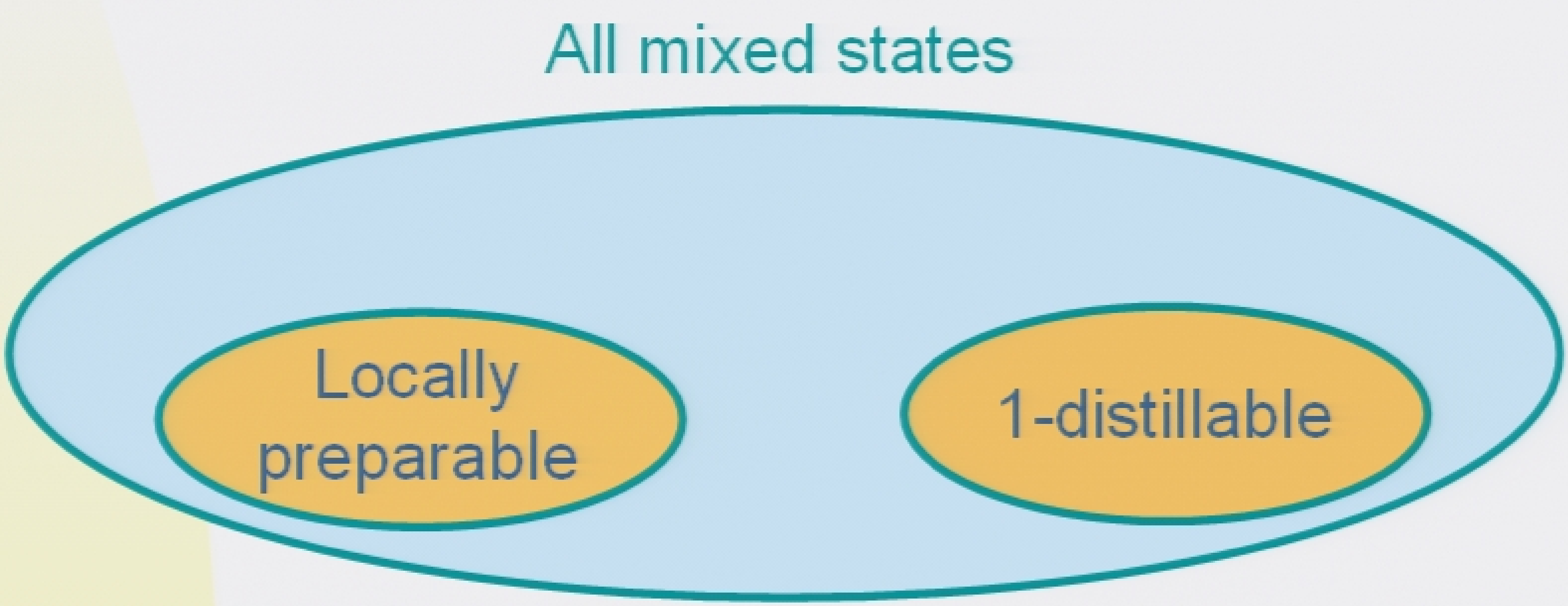


Summary

- Locally-preparable and 1-distillable states



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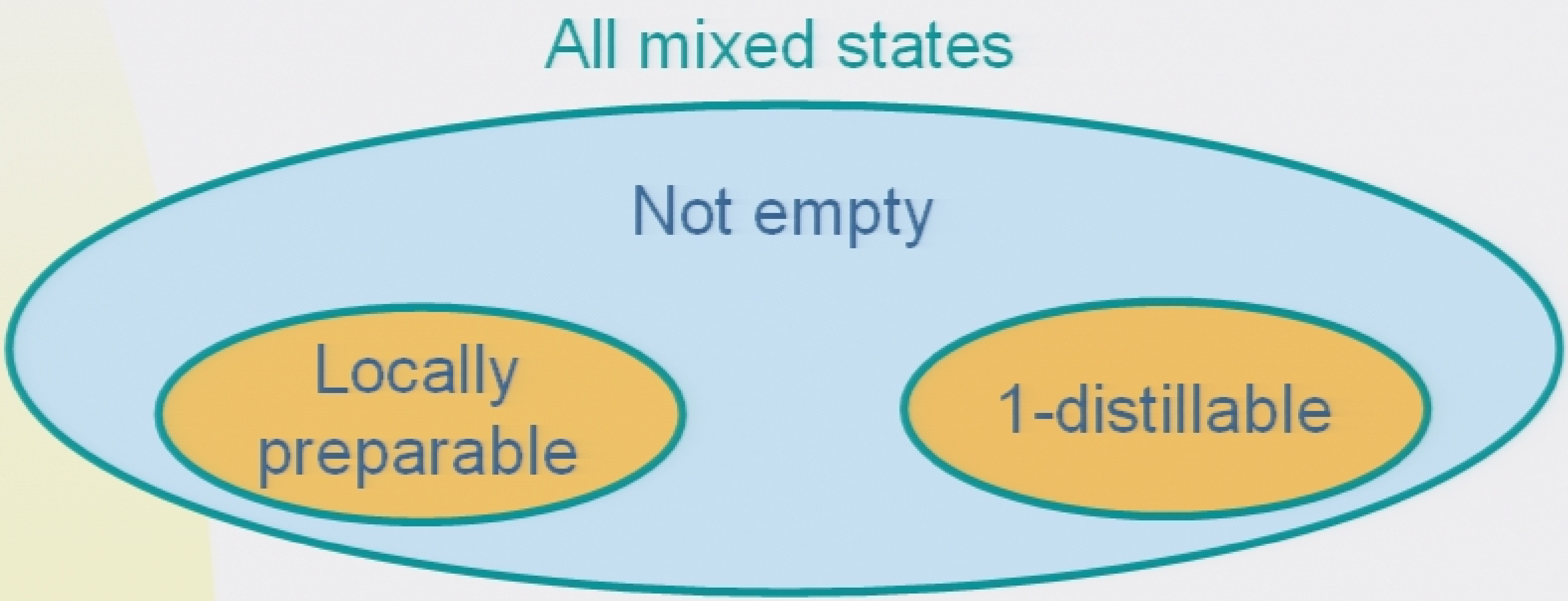

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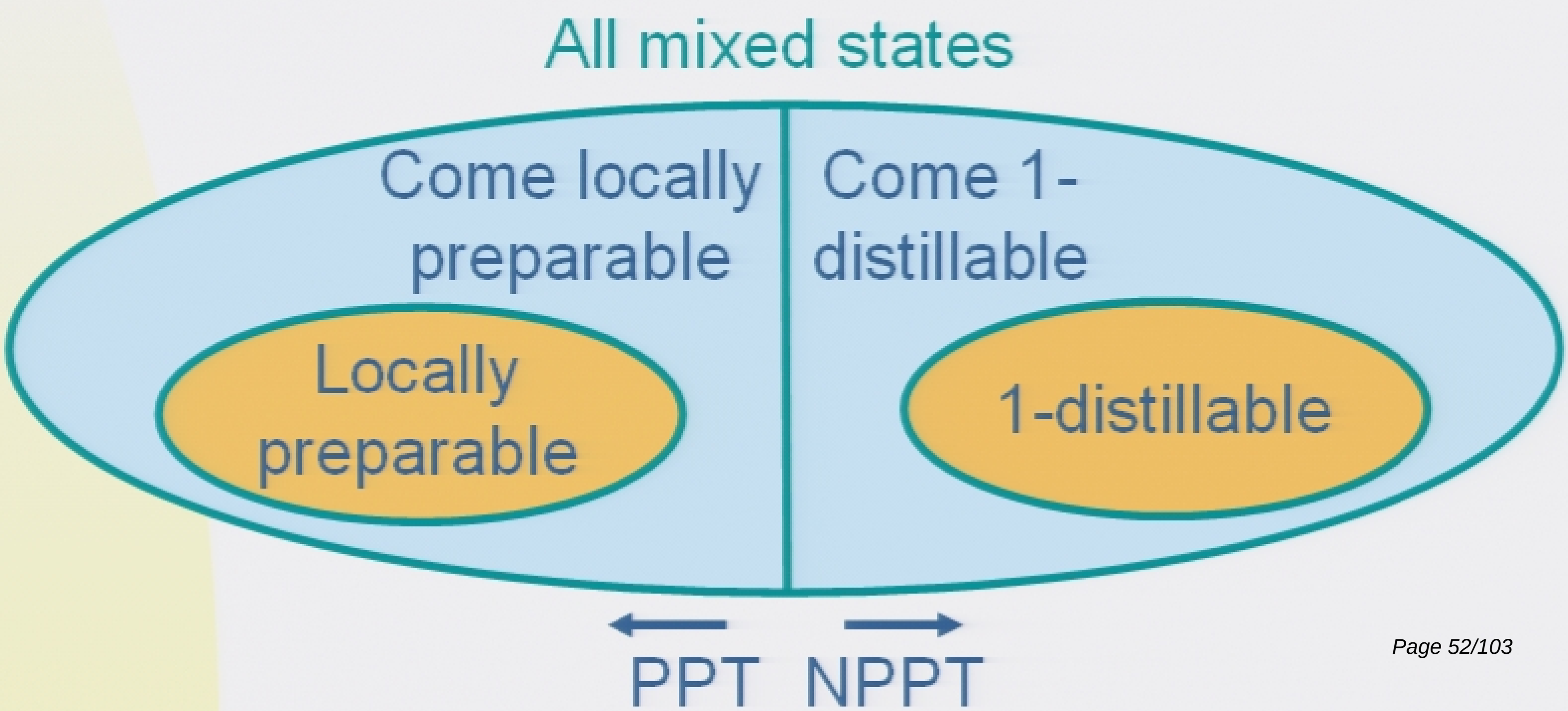

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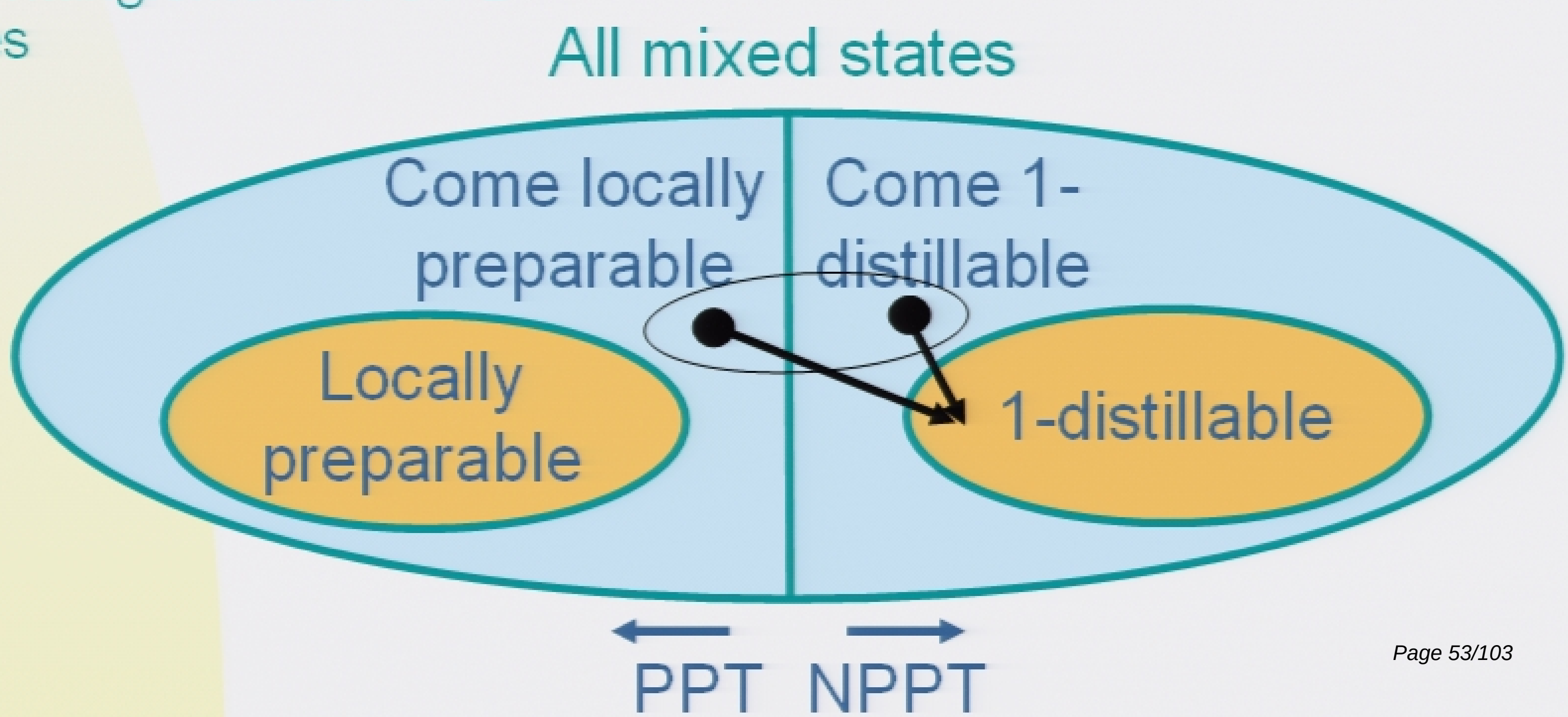

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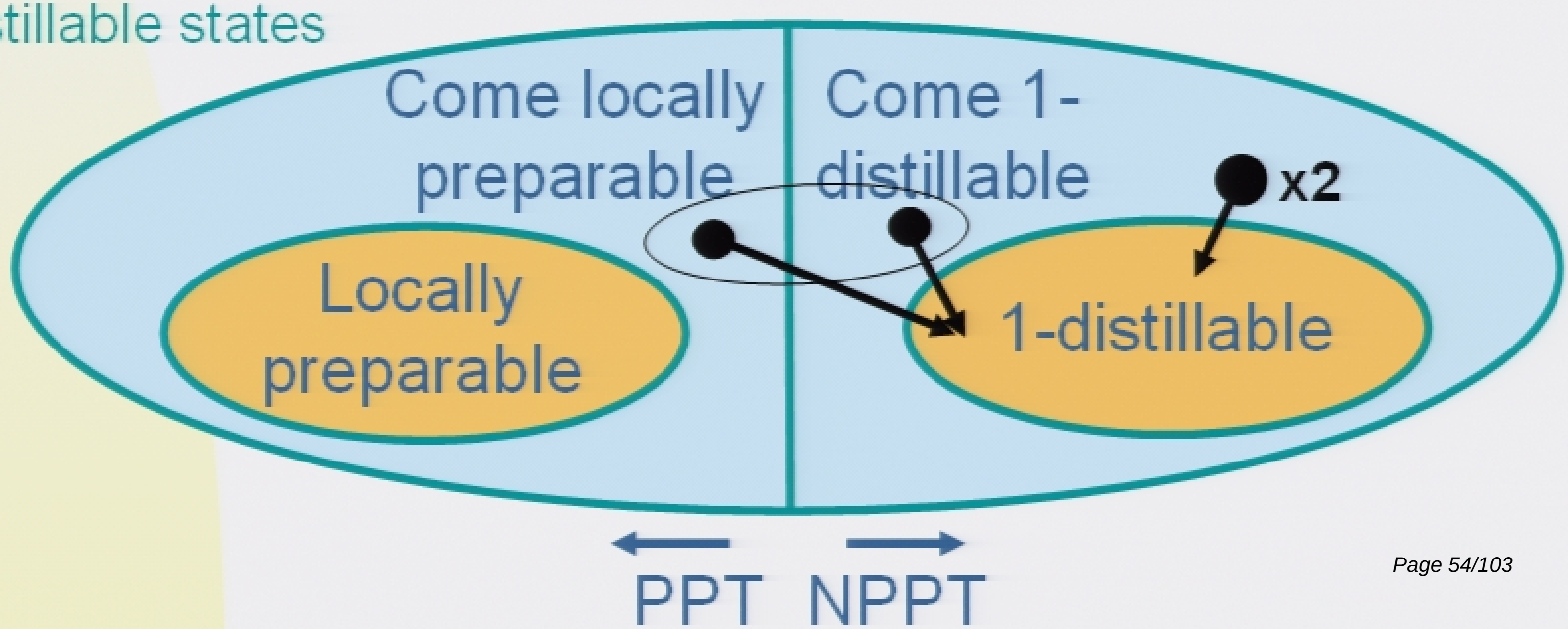
All mixed states



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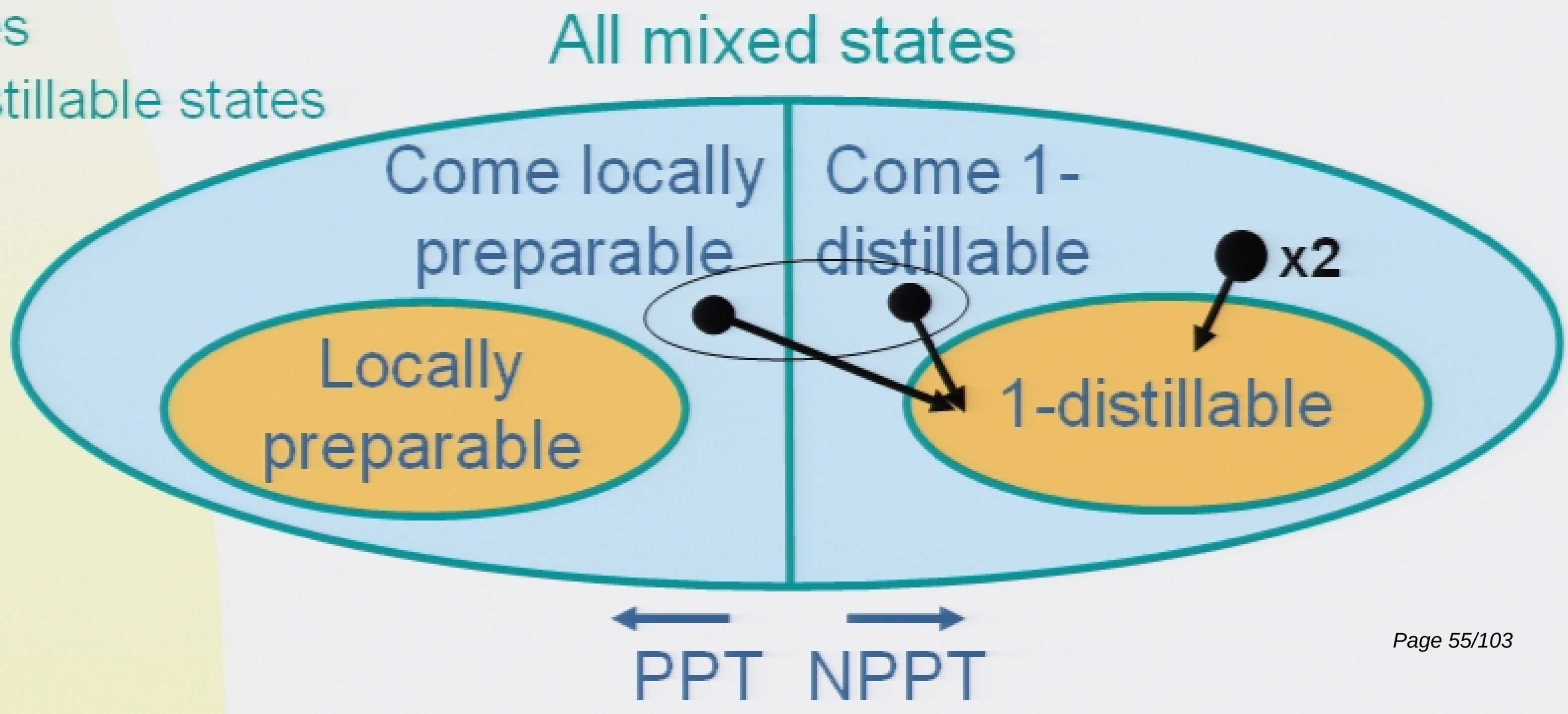
Big open question:

Are all C1-D states distillable?



Entanglement constrained by superselection rules

Operational SSRs

A superselection rule is an additional *restriction*

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G-SSR: Restriction on allowed operations; CP maps $\mathcal{O}$ are *G-covariant*

$$\mathcal{O}[T(g)\rho T^\dagger(g)] = T(g)\mathcal{O}[\rho]T^\dagger(g) \quad \forall\ g \in G\,,\ \rho$$

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- SU(2)-SSR
  - Generated by angular momentum operators

    $$\{\hat{L}_x, \hat{L}_y, \hat{L}_z\}$$
  - All operations are rotationally invariant
  - No direction axis to prepare, measure spins

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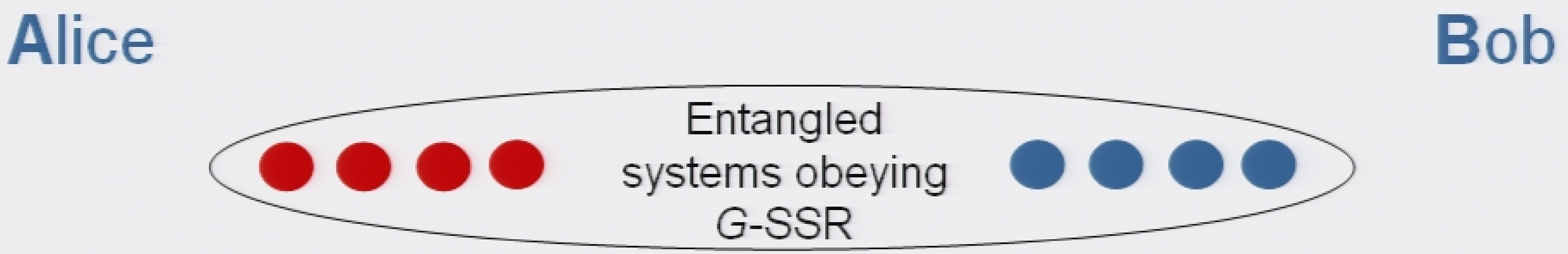

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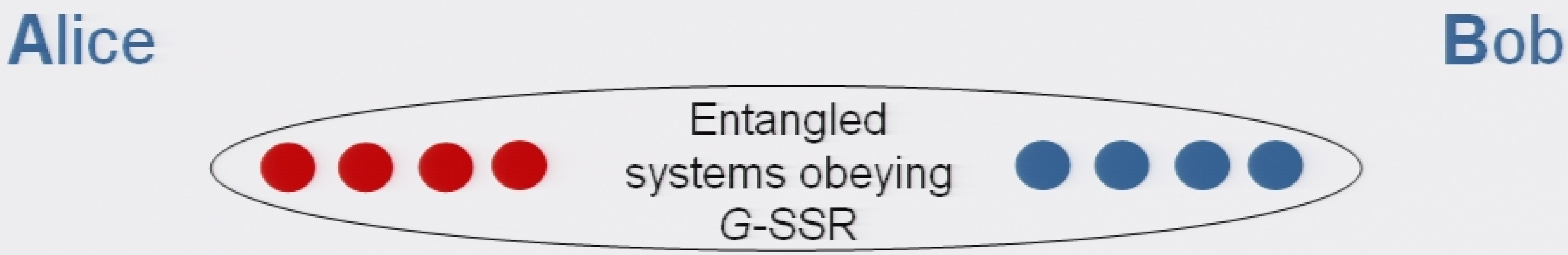

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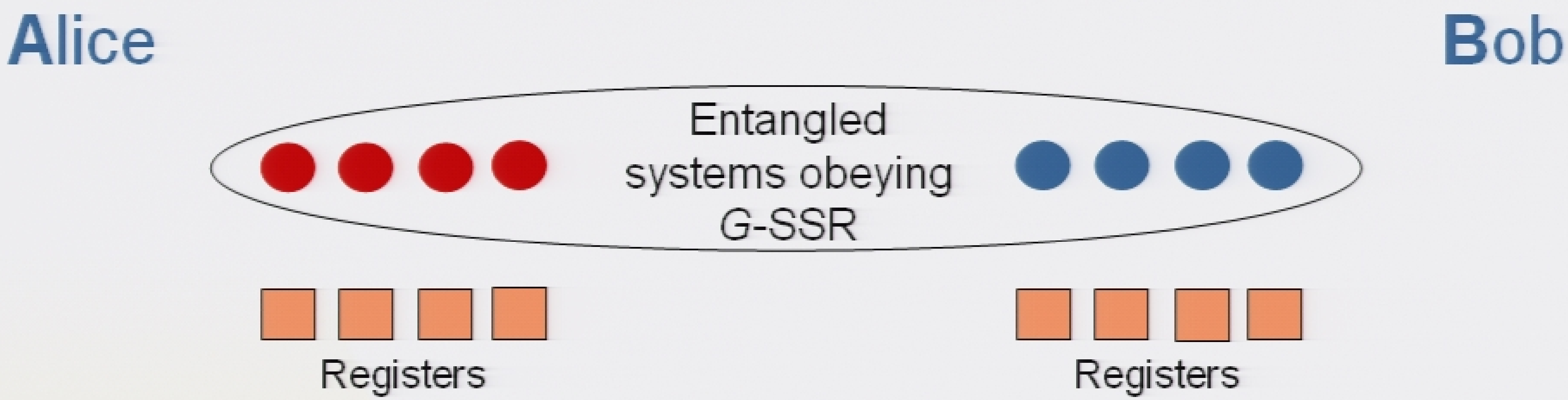

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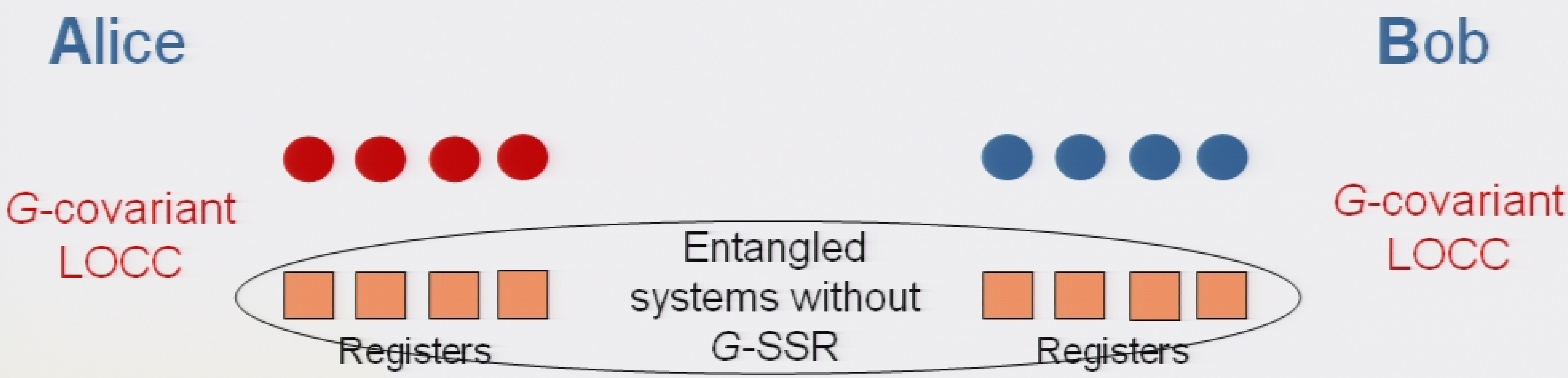

- Quantify entanglement transferable to "standard quantum registers" using LOCC obeying the SSR

Amount of entanglement present in the state ρ in the presence of *G*-SSR	=	Standard entanglement in $(\mathcal{G}_A \otimes \mathcal{G}_B)[\rho]$

Wiseman and Vaccaro, *Phys. Rev. Lett.* **91**, 097902 (2003)

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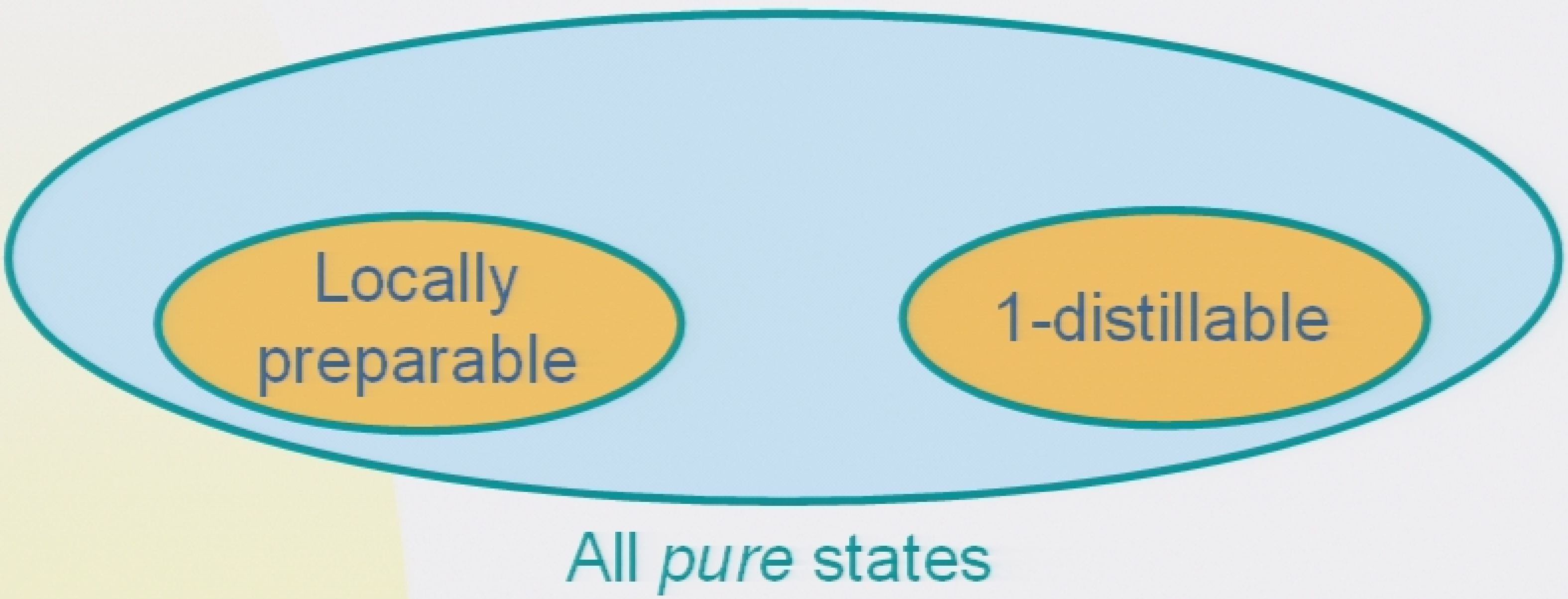
Classification of entanglement with SSRs

- Classification mimics the structure of mixed state entanglement



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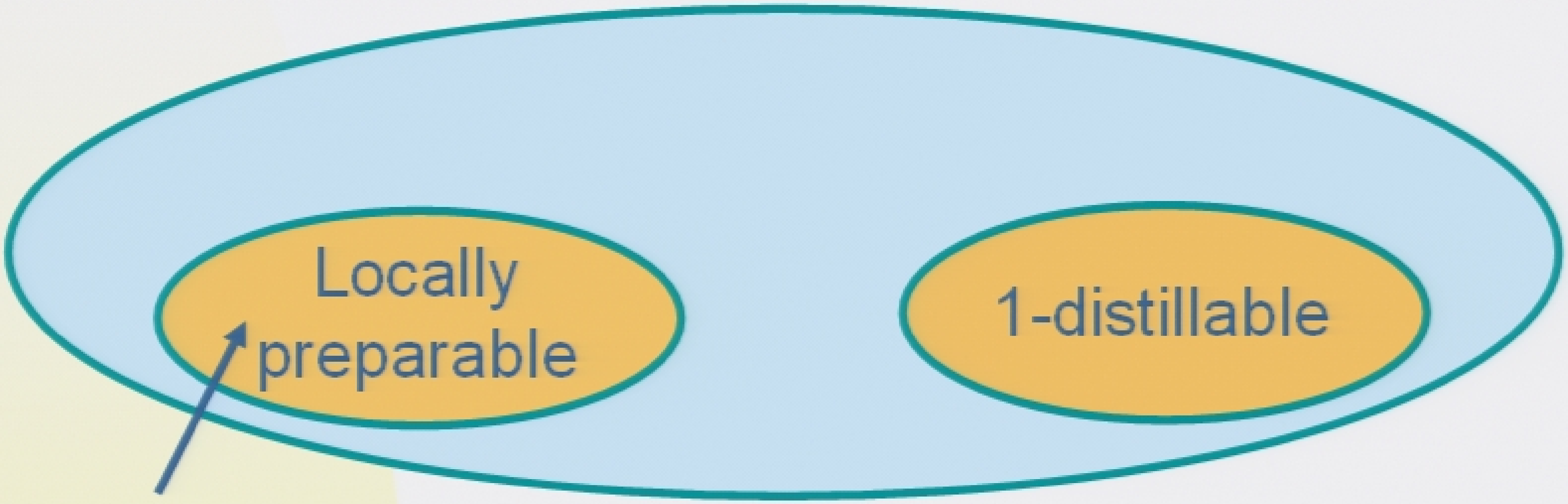

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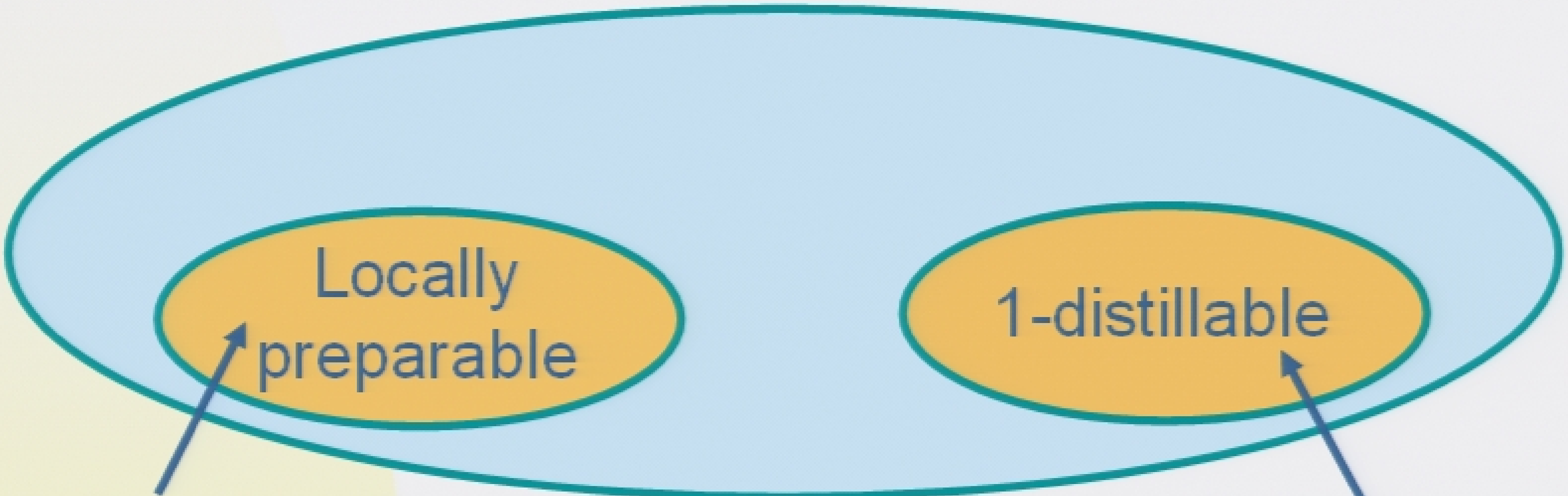

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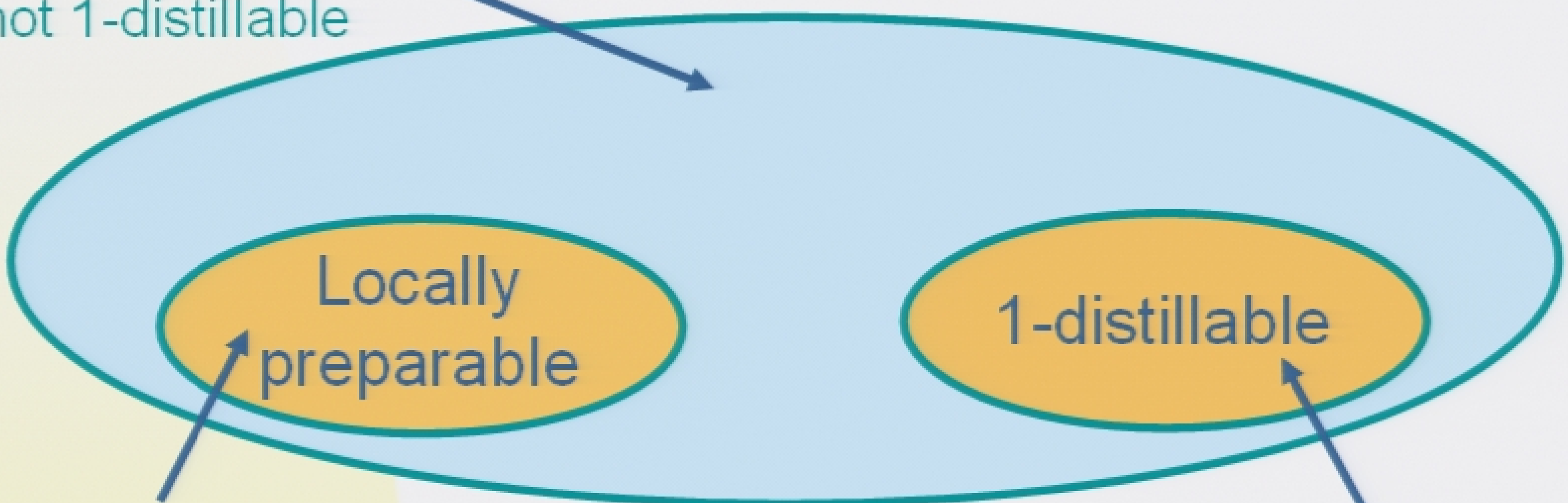

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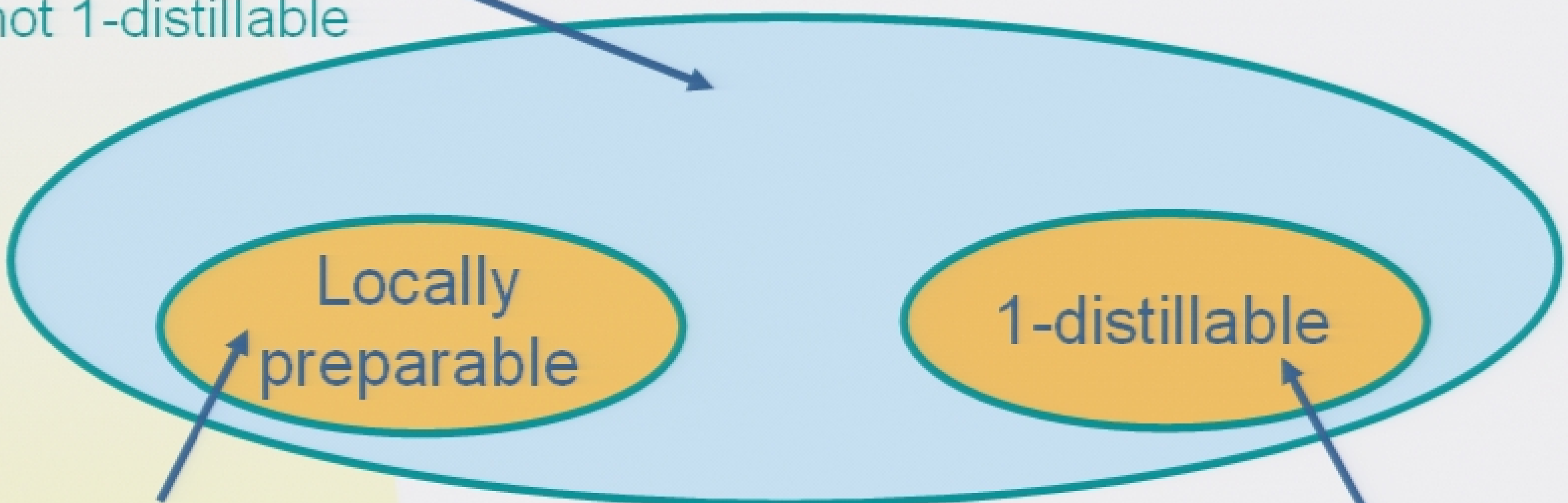

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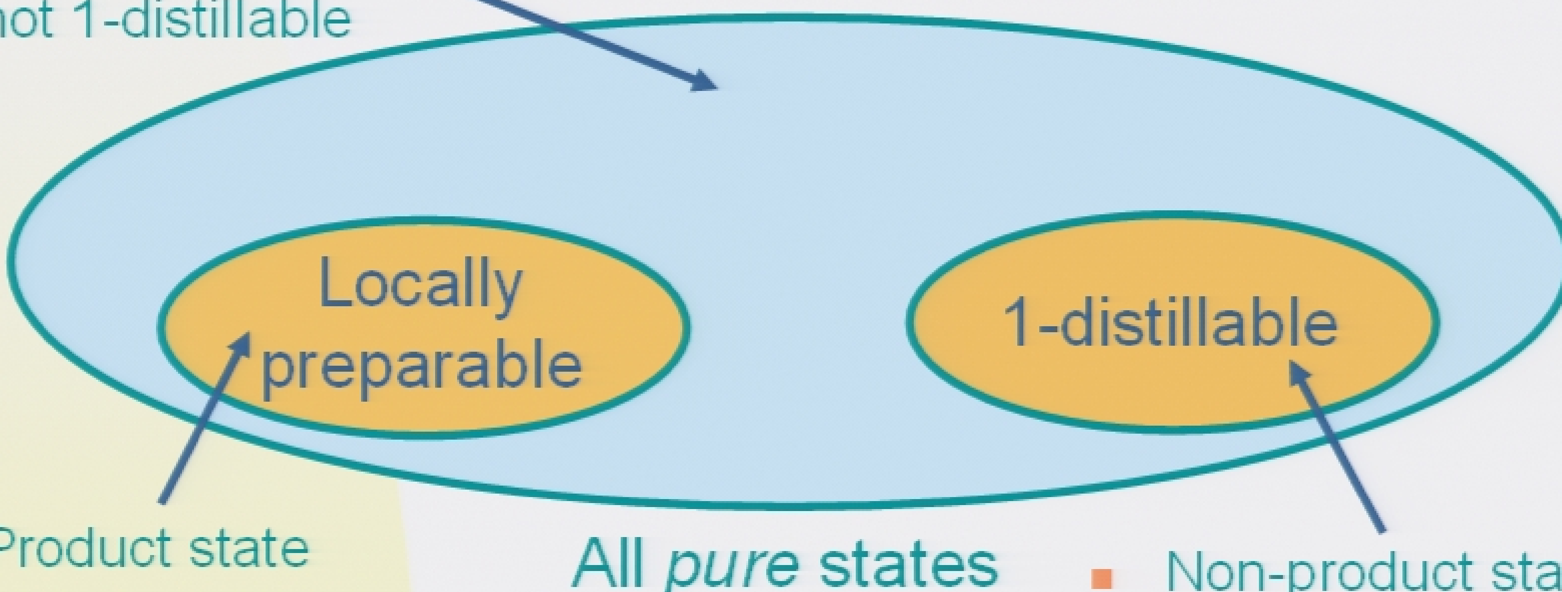

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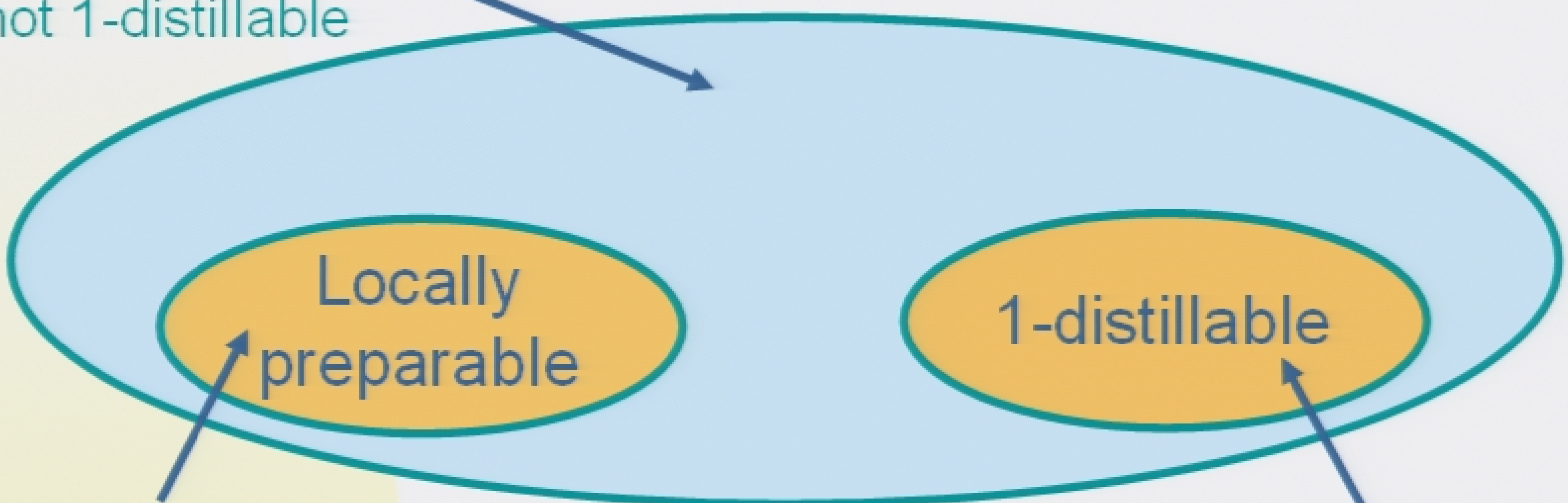

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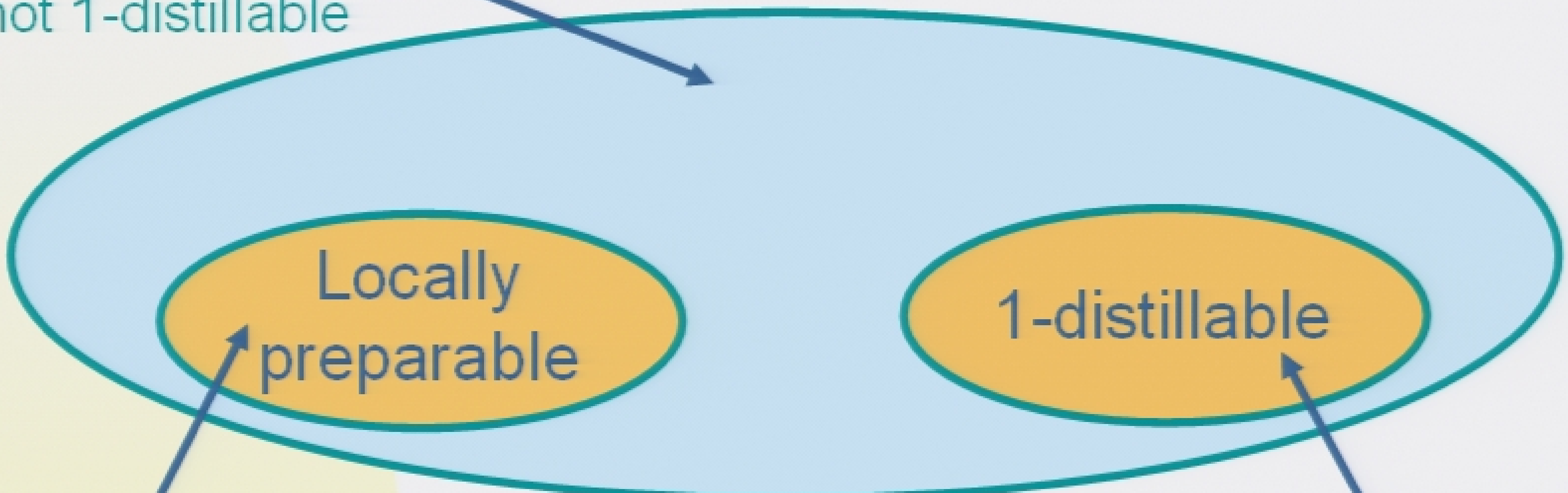

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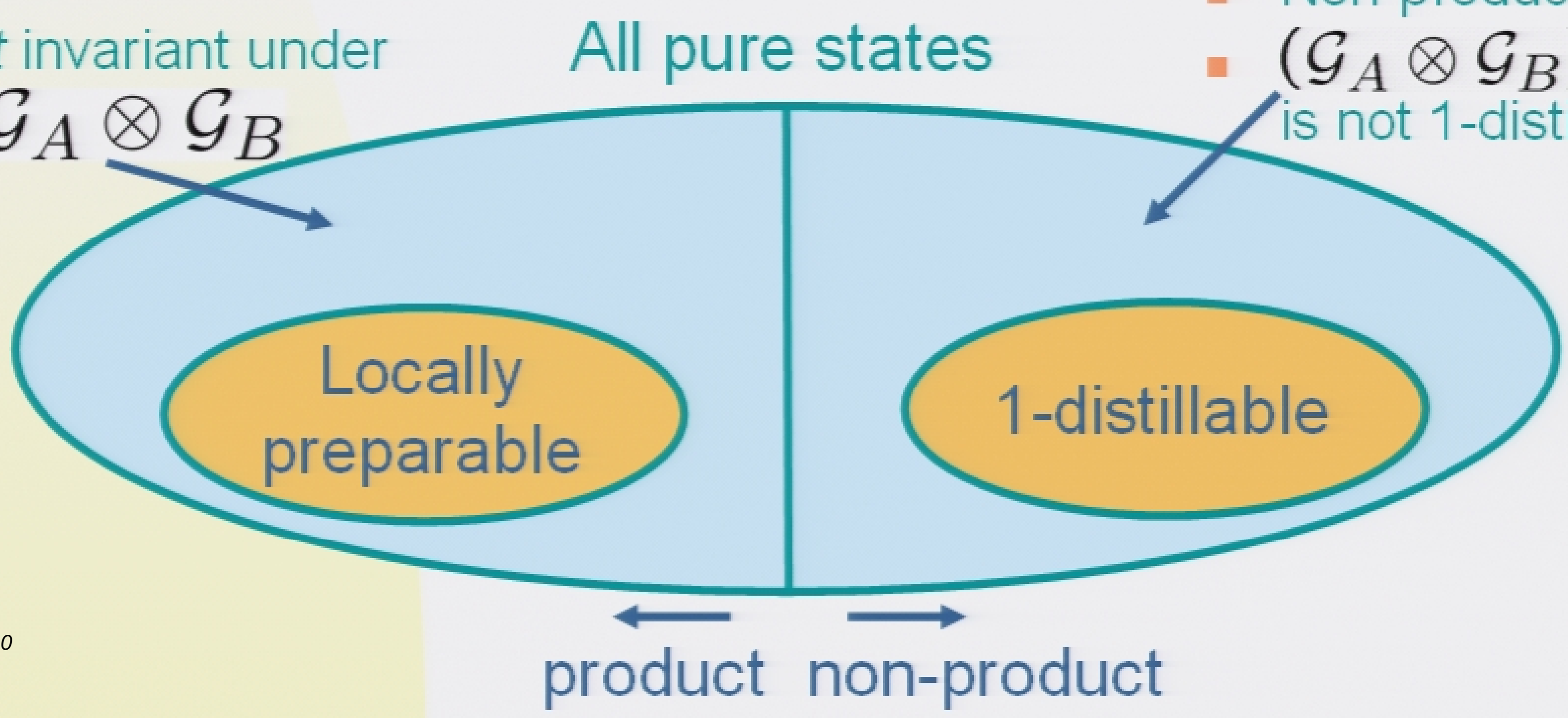

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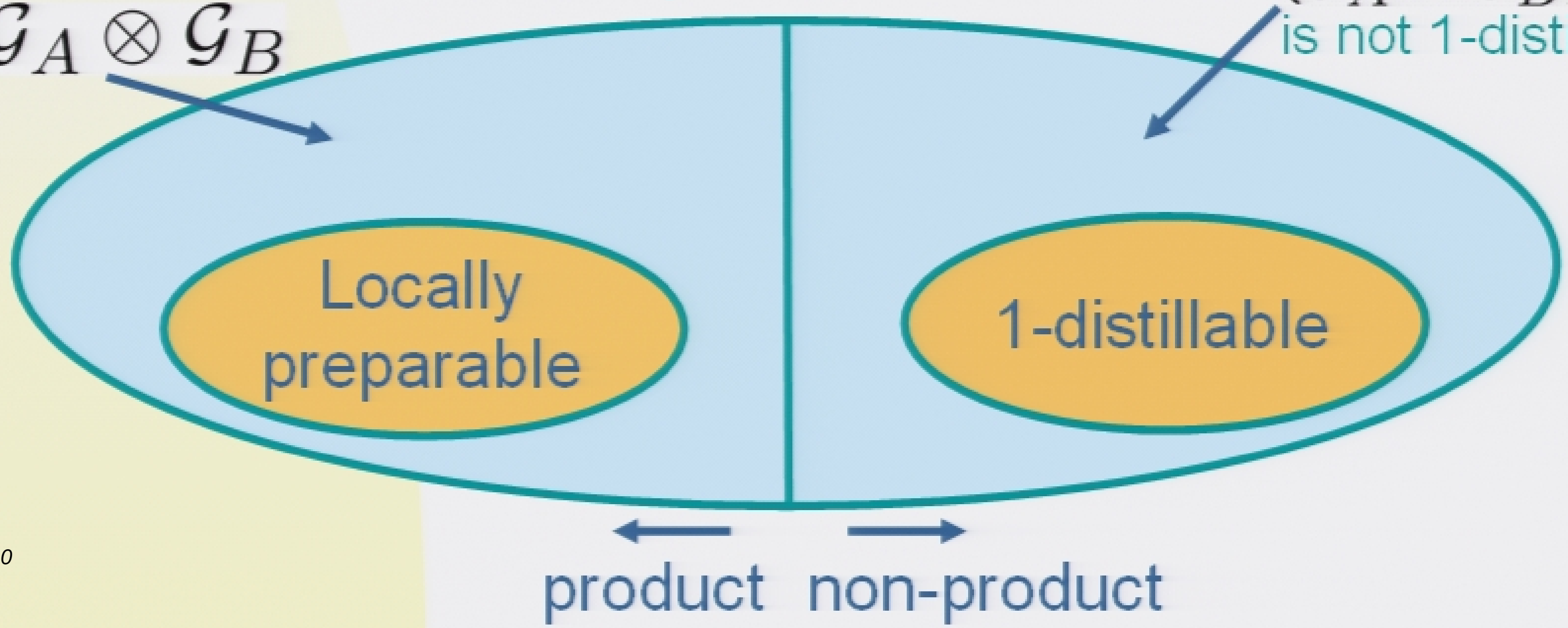

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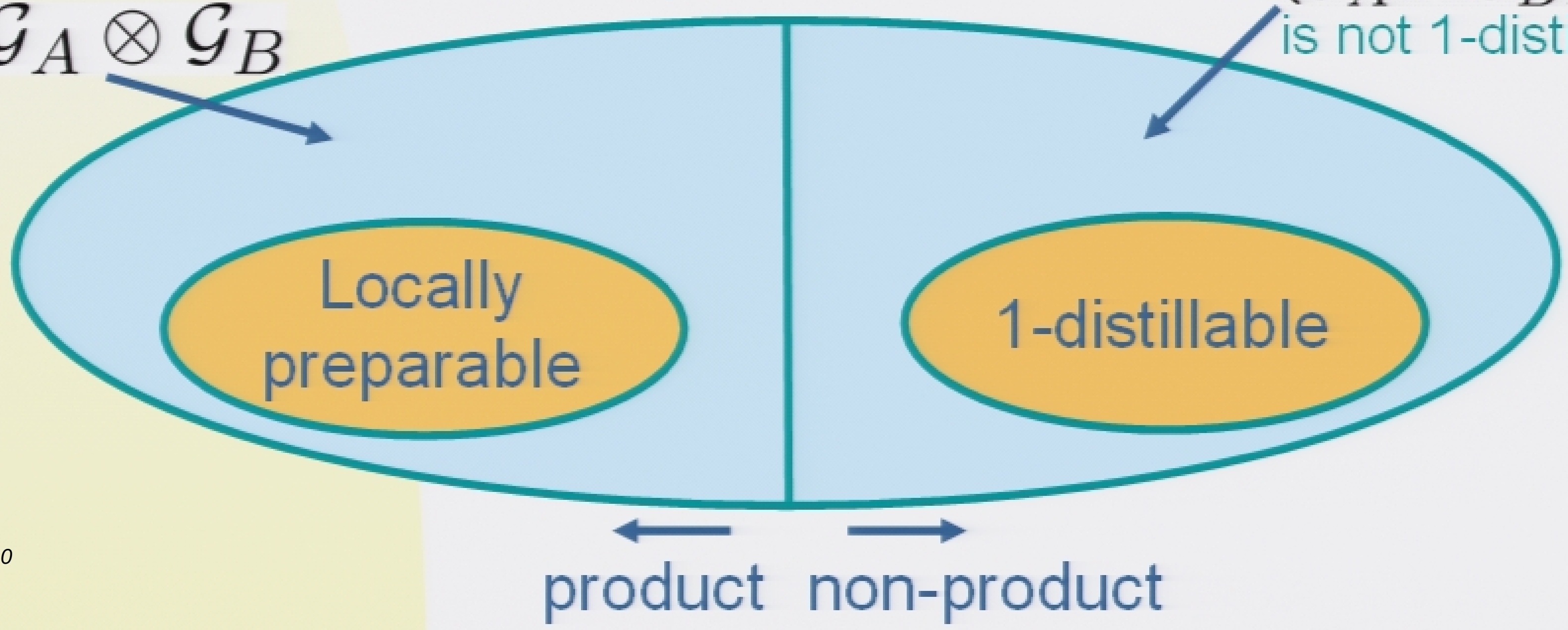

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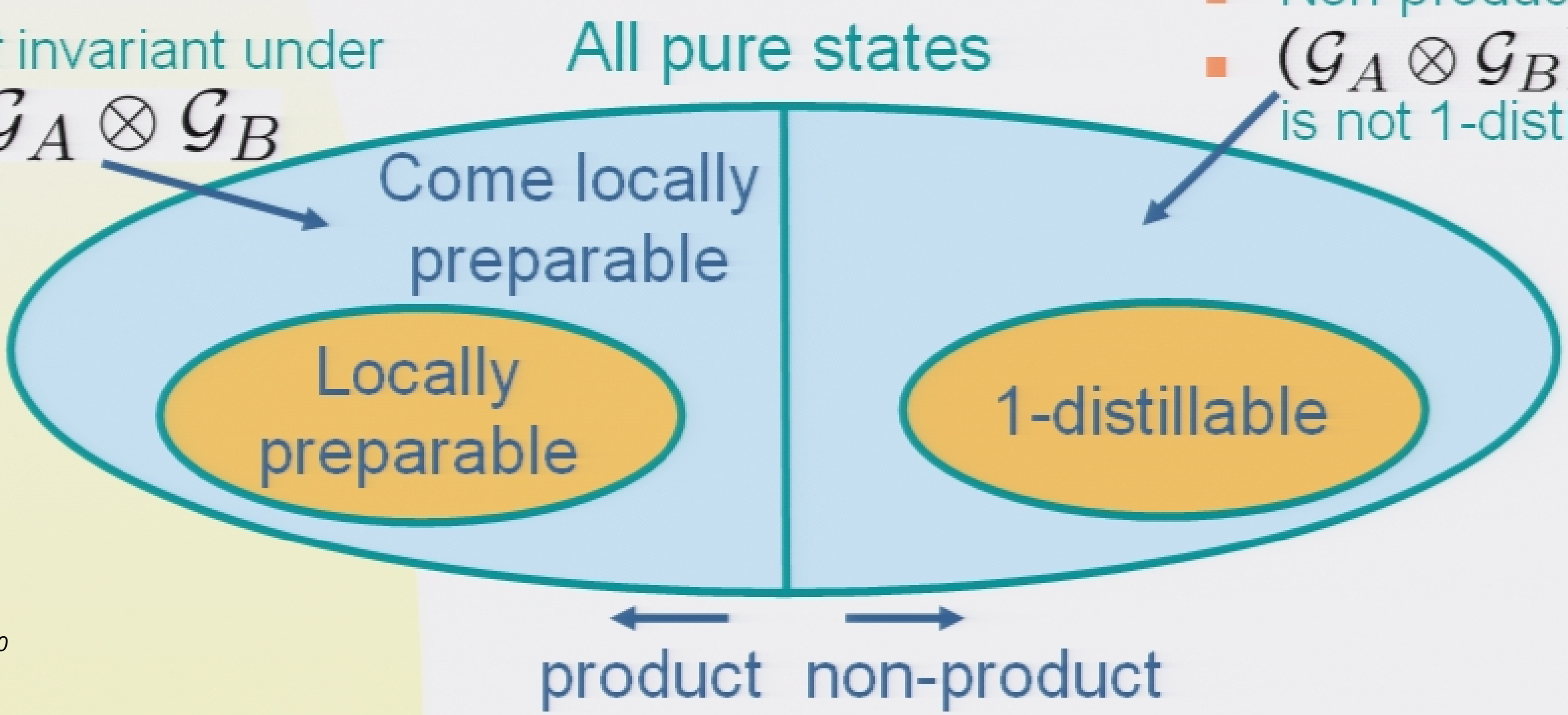

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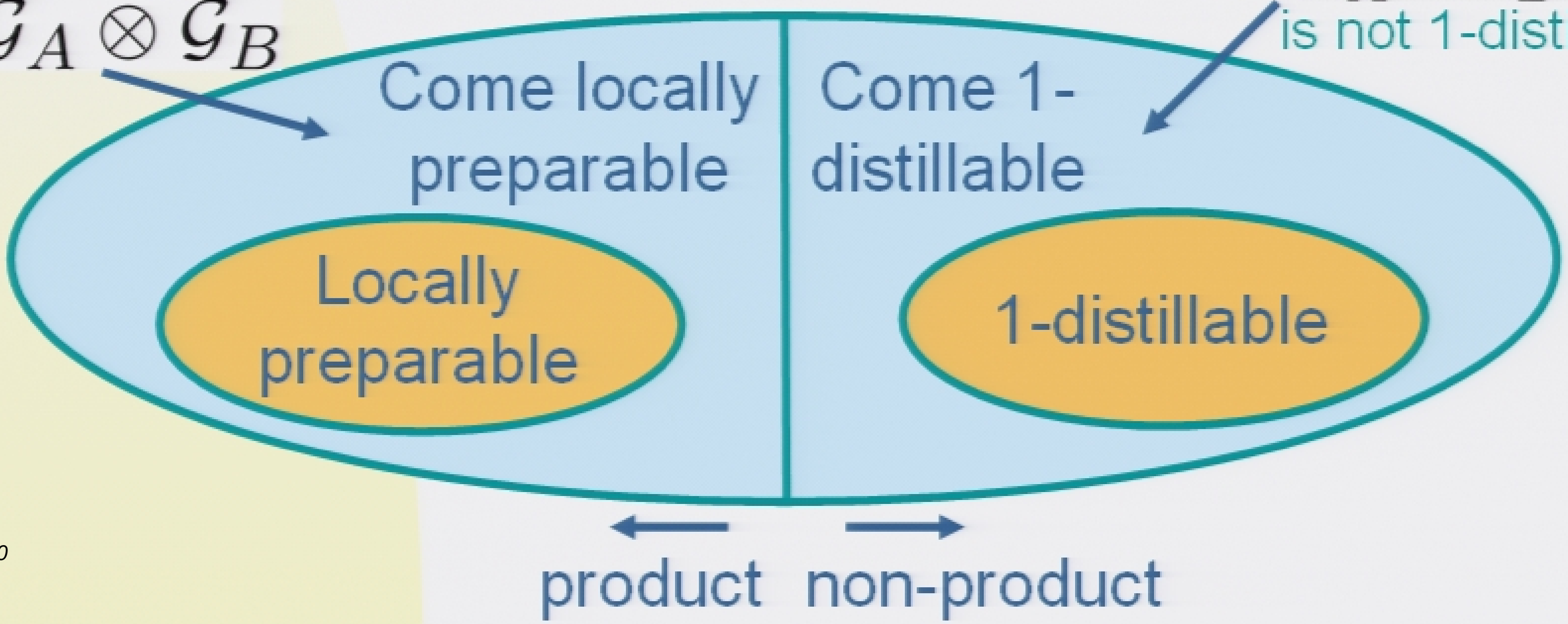

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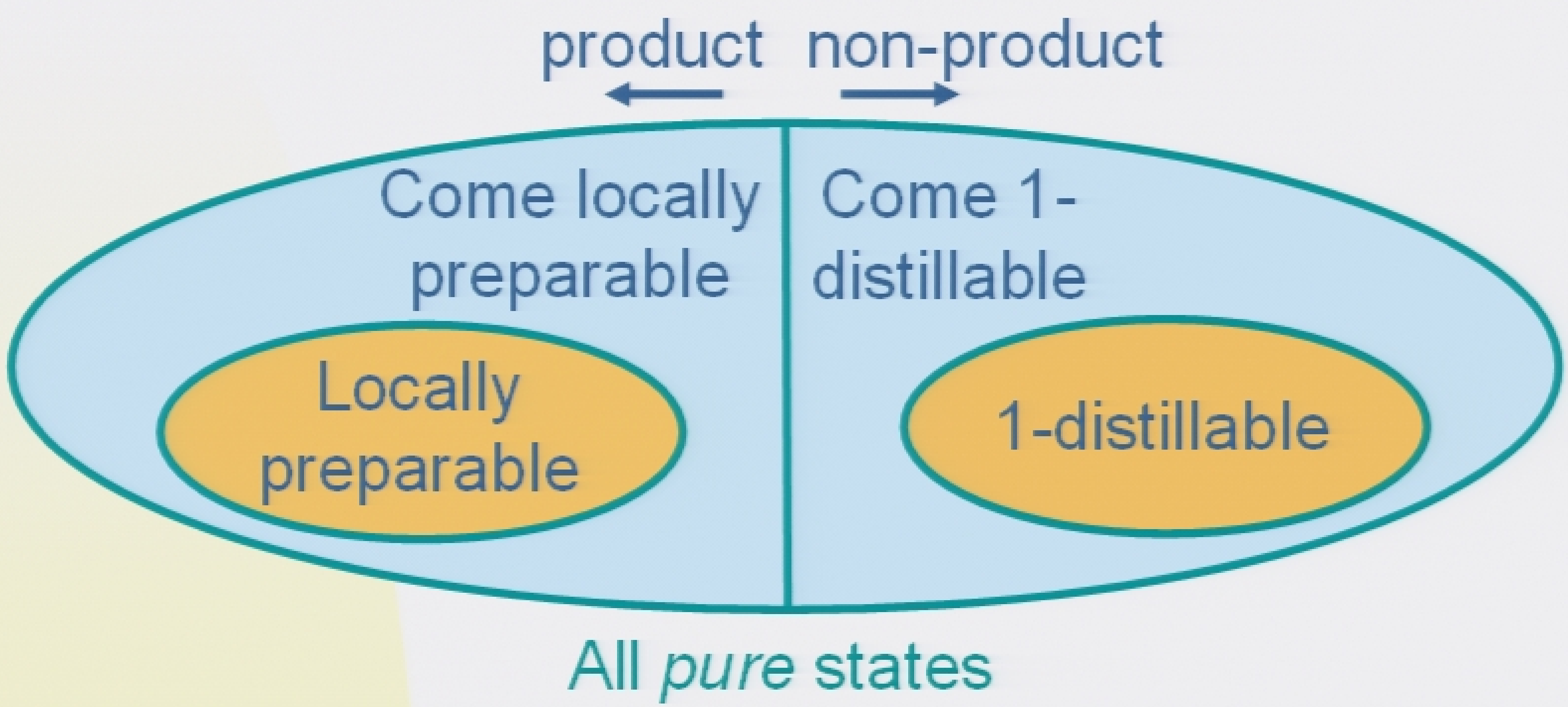

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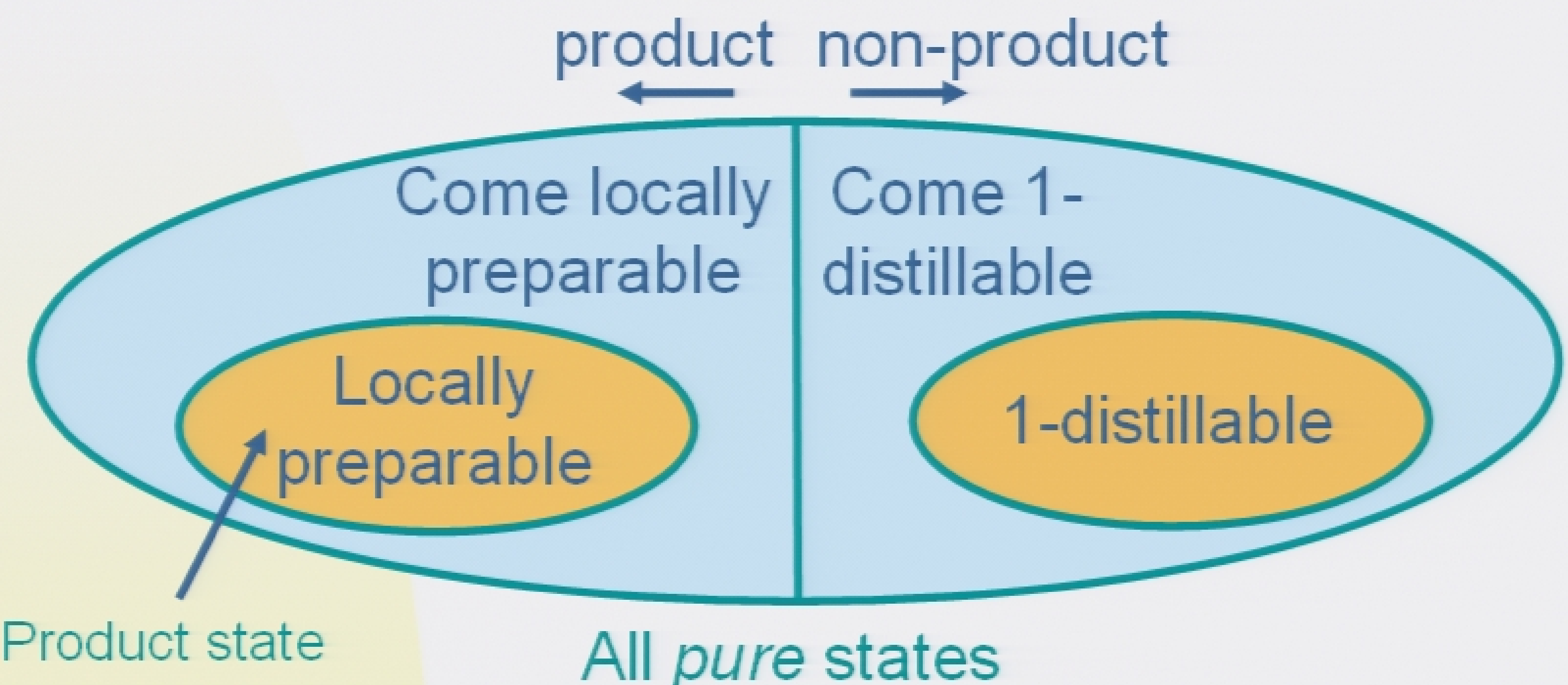

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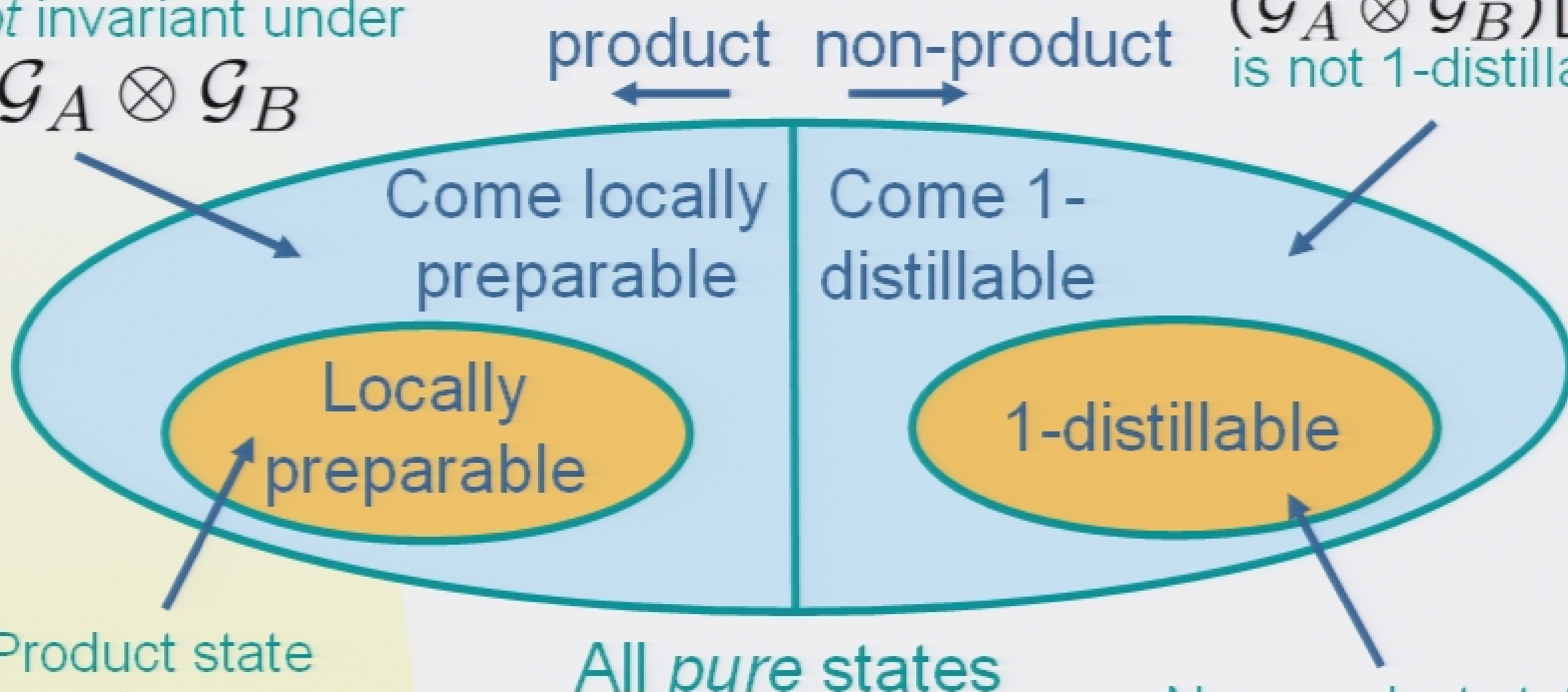

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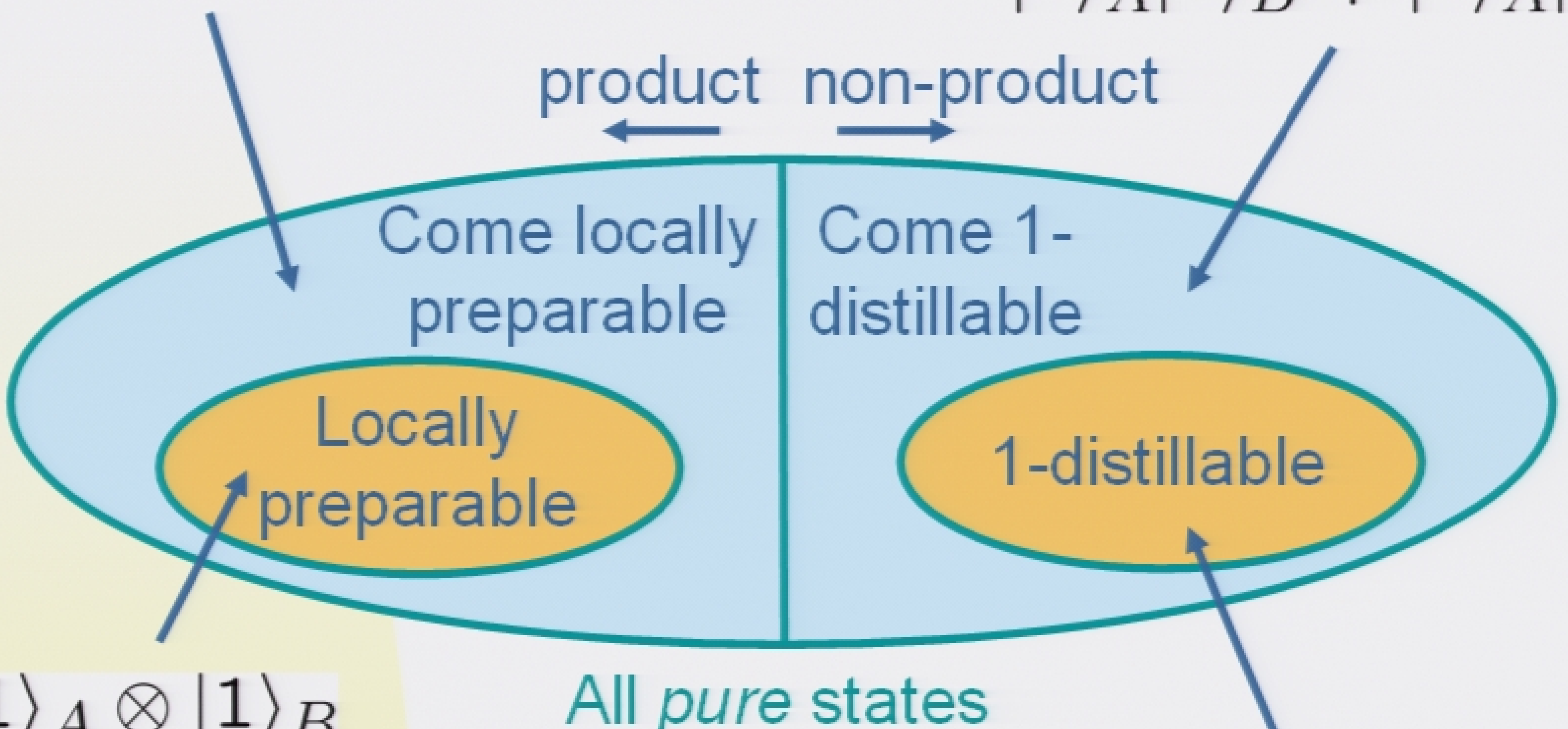

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- Channel *C*: implemented probabilistically using non-invariant product states (Jamiolkowski isomorphism)
- For every "come 1-distillable" state, there exists a "come locally preparable" state that serves as a finite shared reference frame and *activates* the 1-distillability

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Resulting state: $$|0,1\rangle_A|0,1\rangle_B + |1,0\rangle_A|1,0\rangle_B$$

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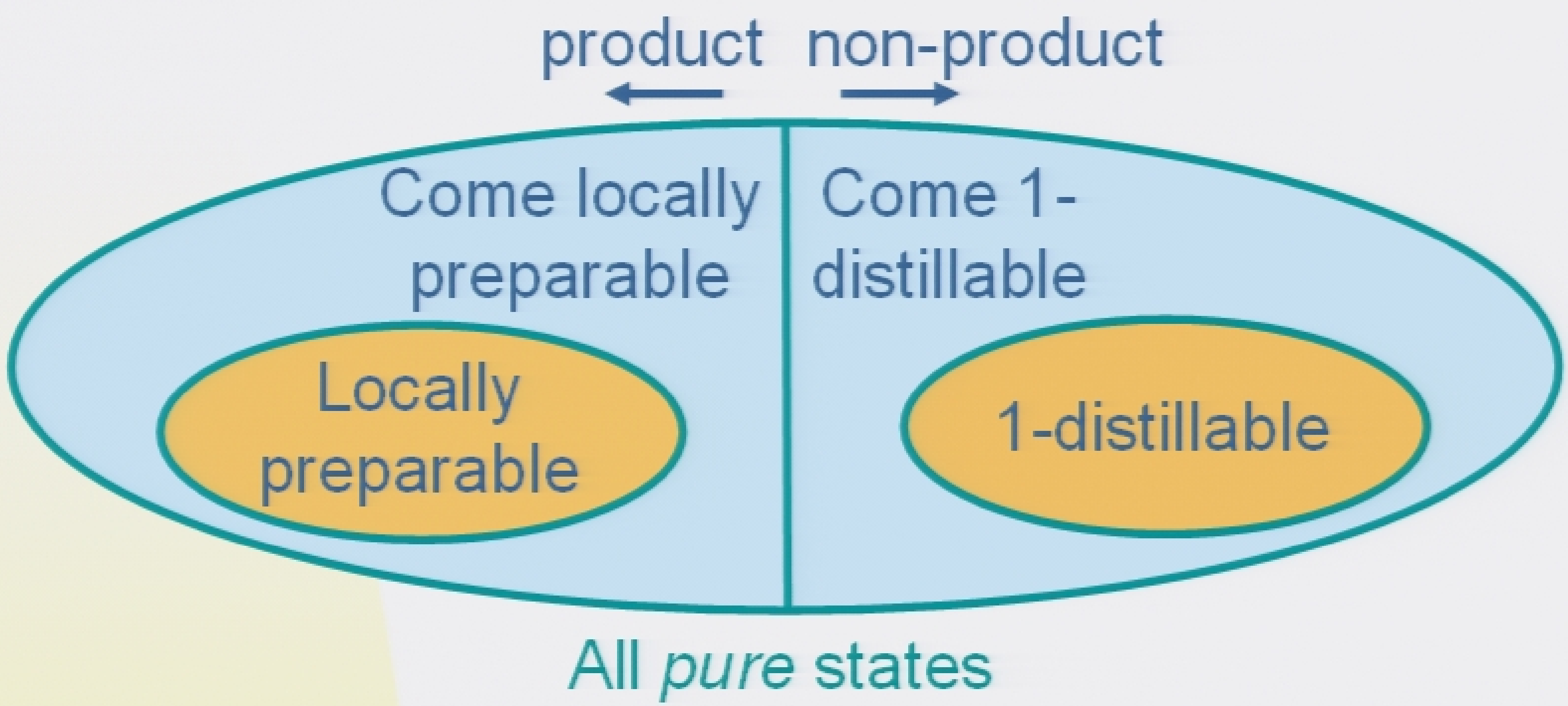

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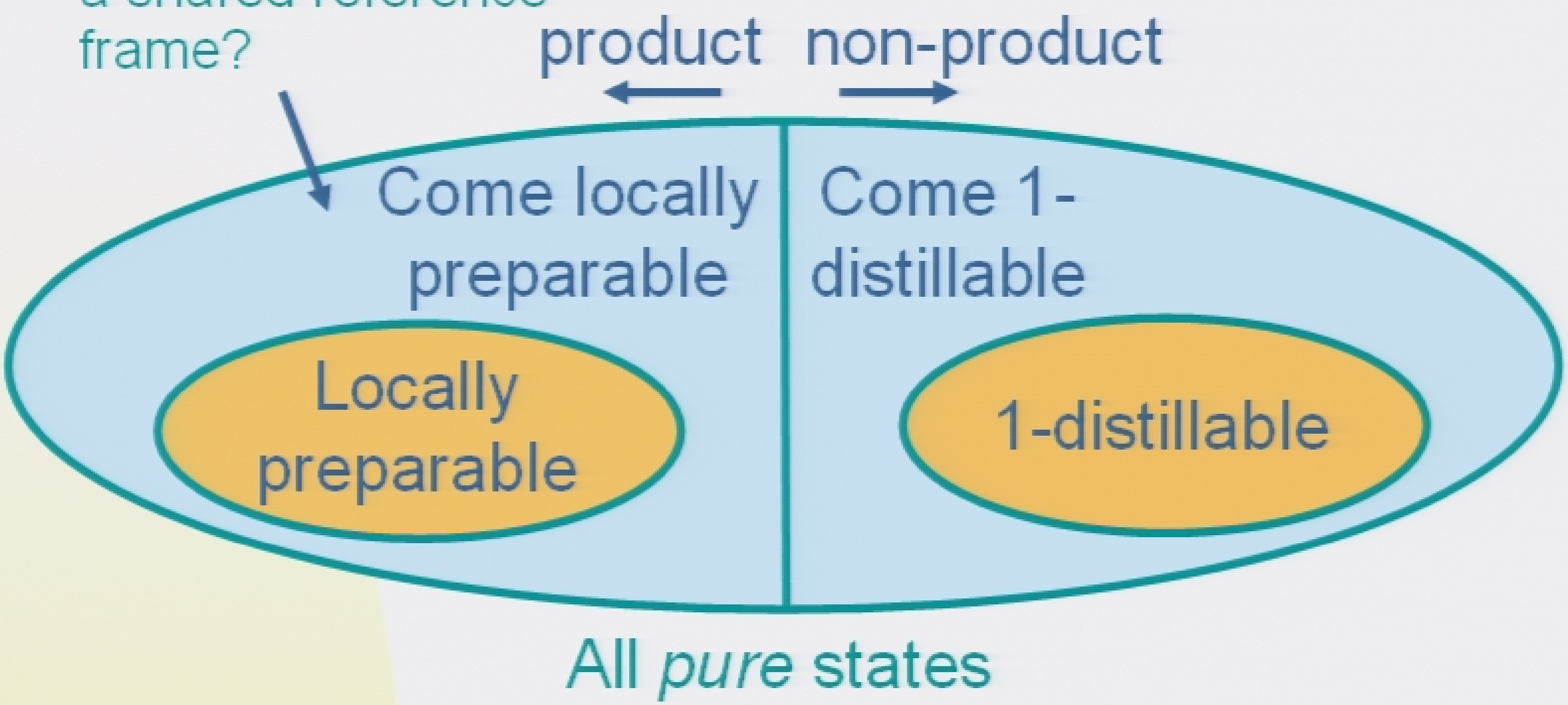

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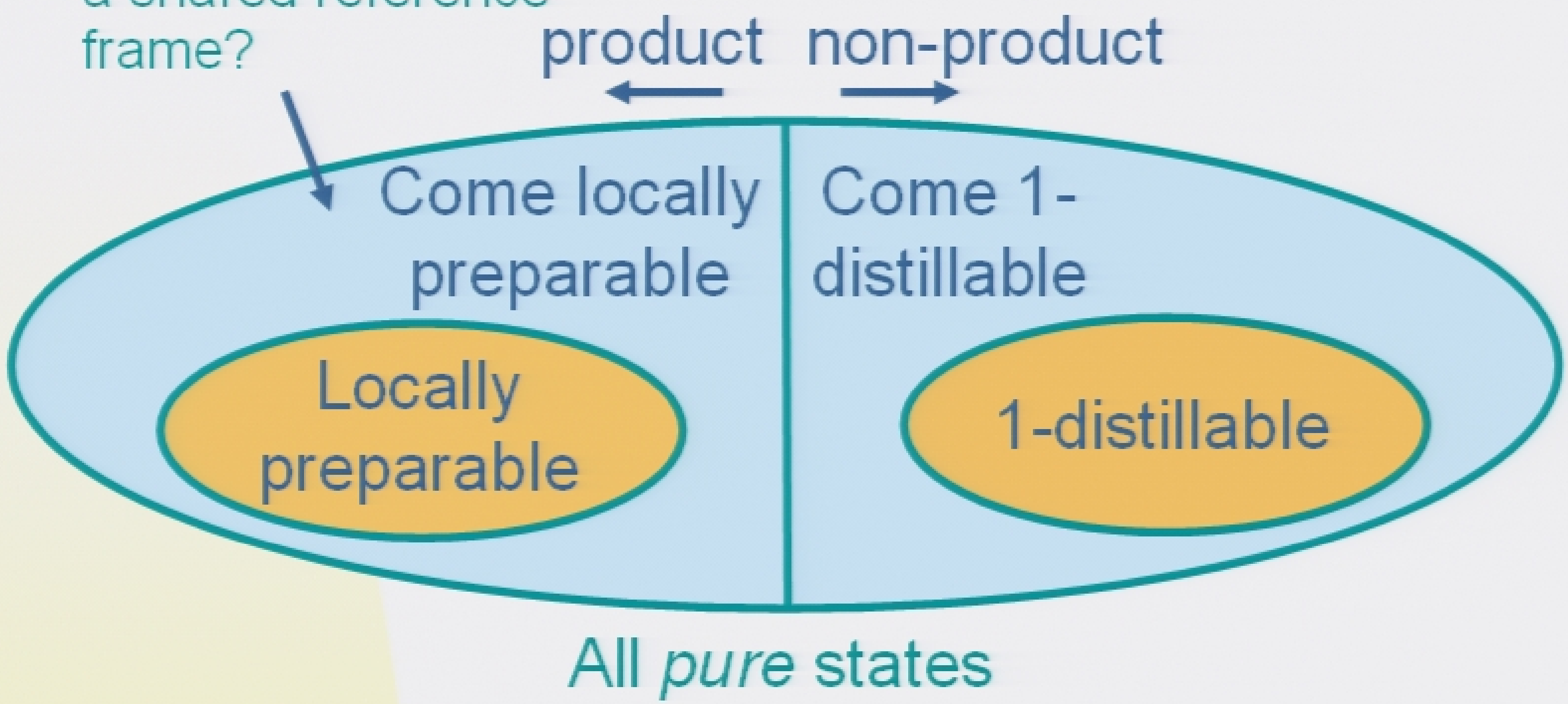

No! Some C1-D states can activate entanglement in other states, and themselves…

Activation / 2-distillability

- C1-D states can possess some "reference frameness"
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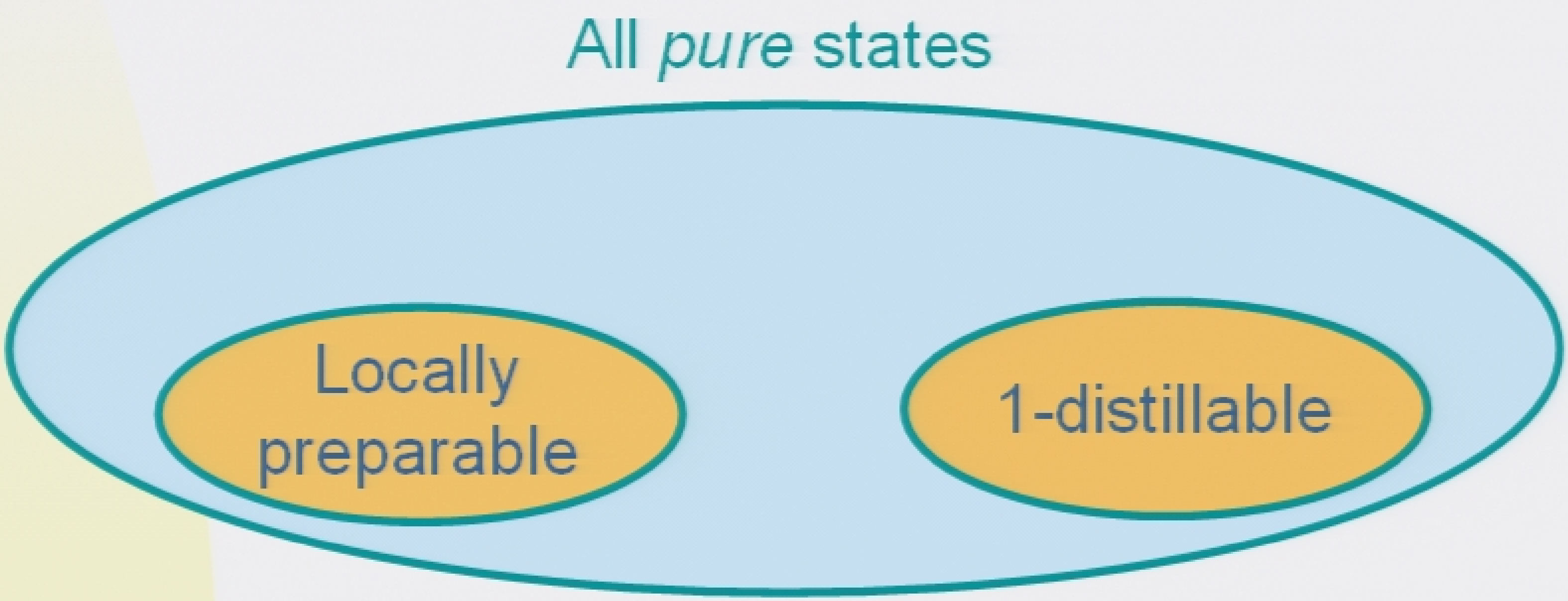
Summary

- Locally-preparable and 1-distillable states



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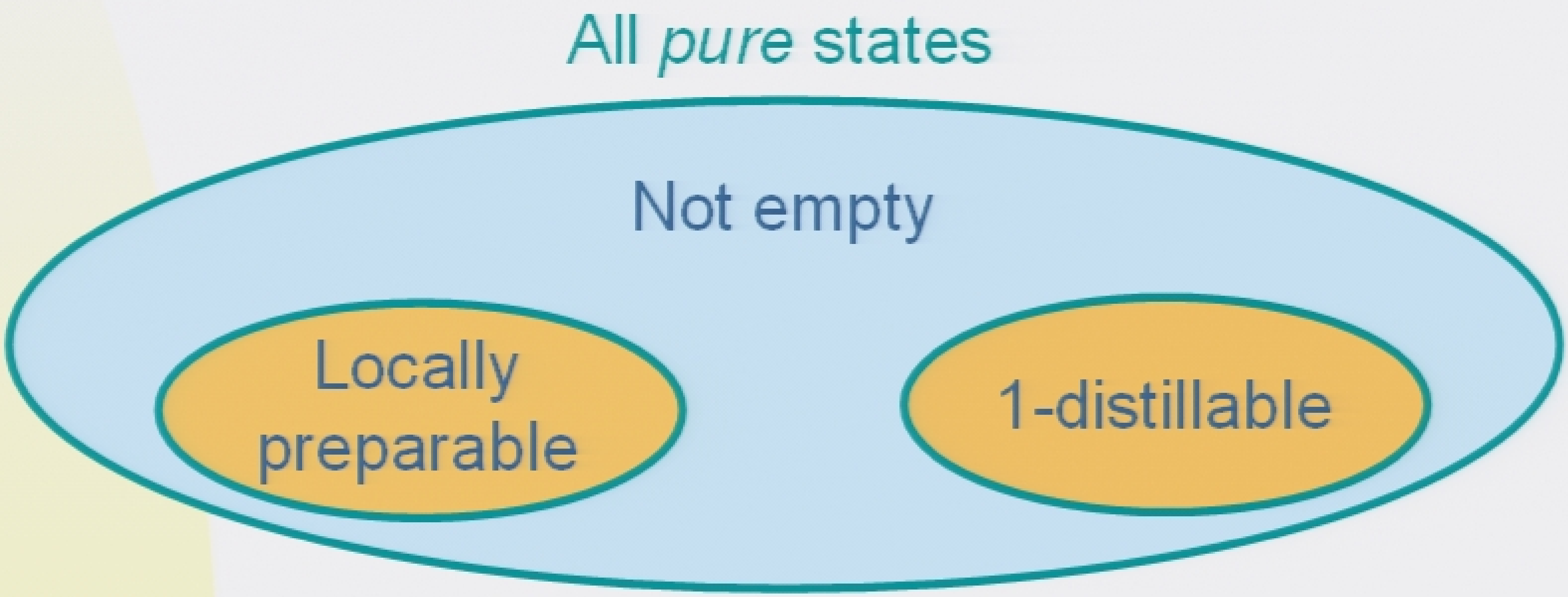

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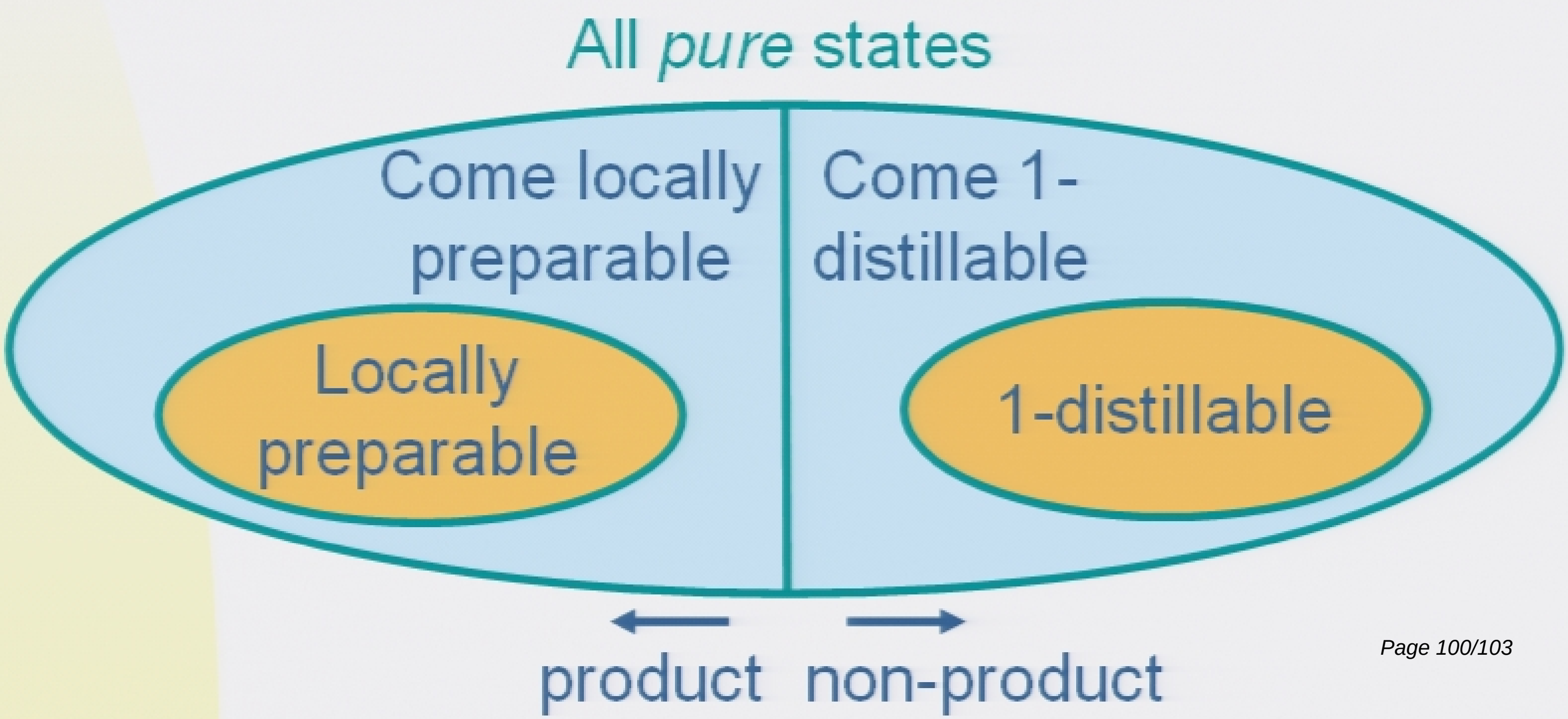

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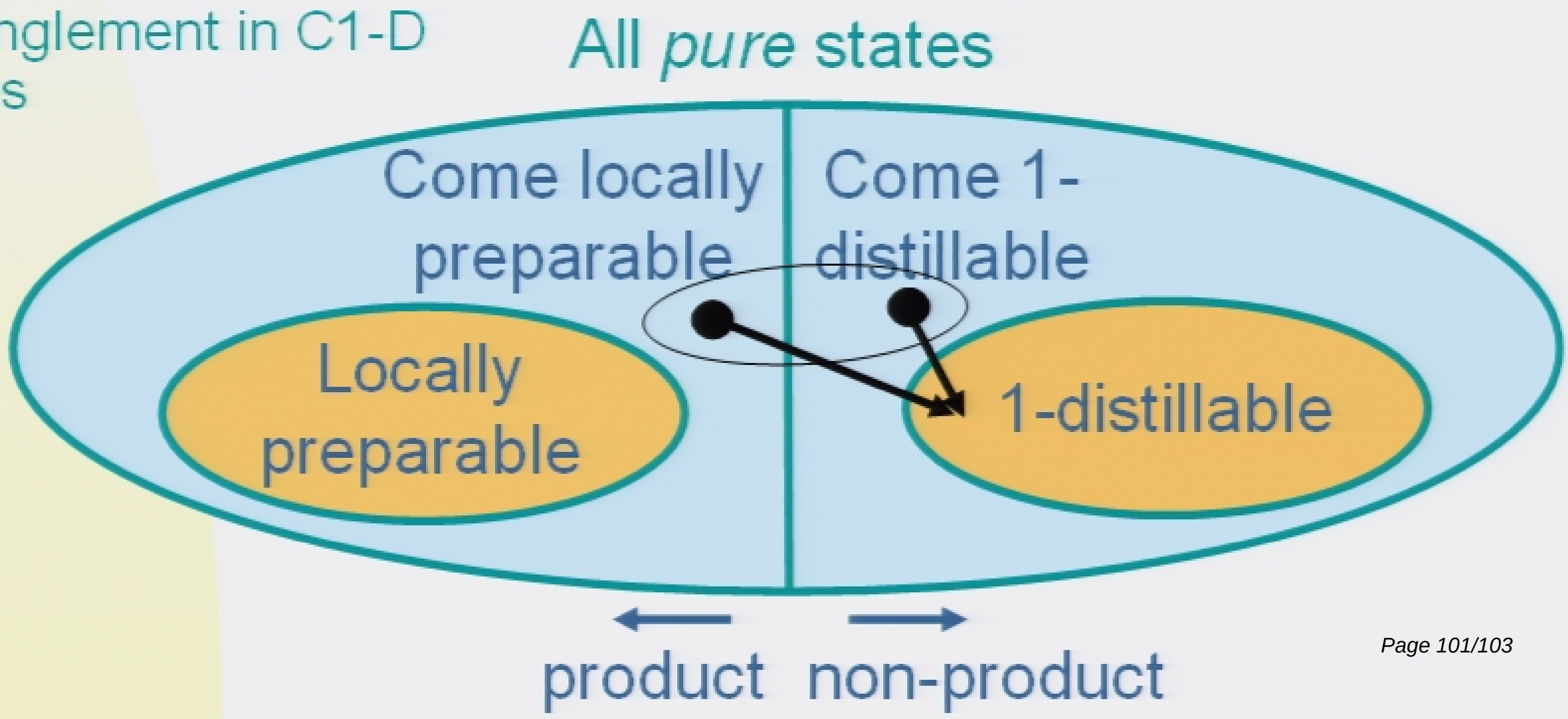

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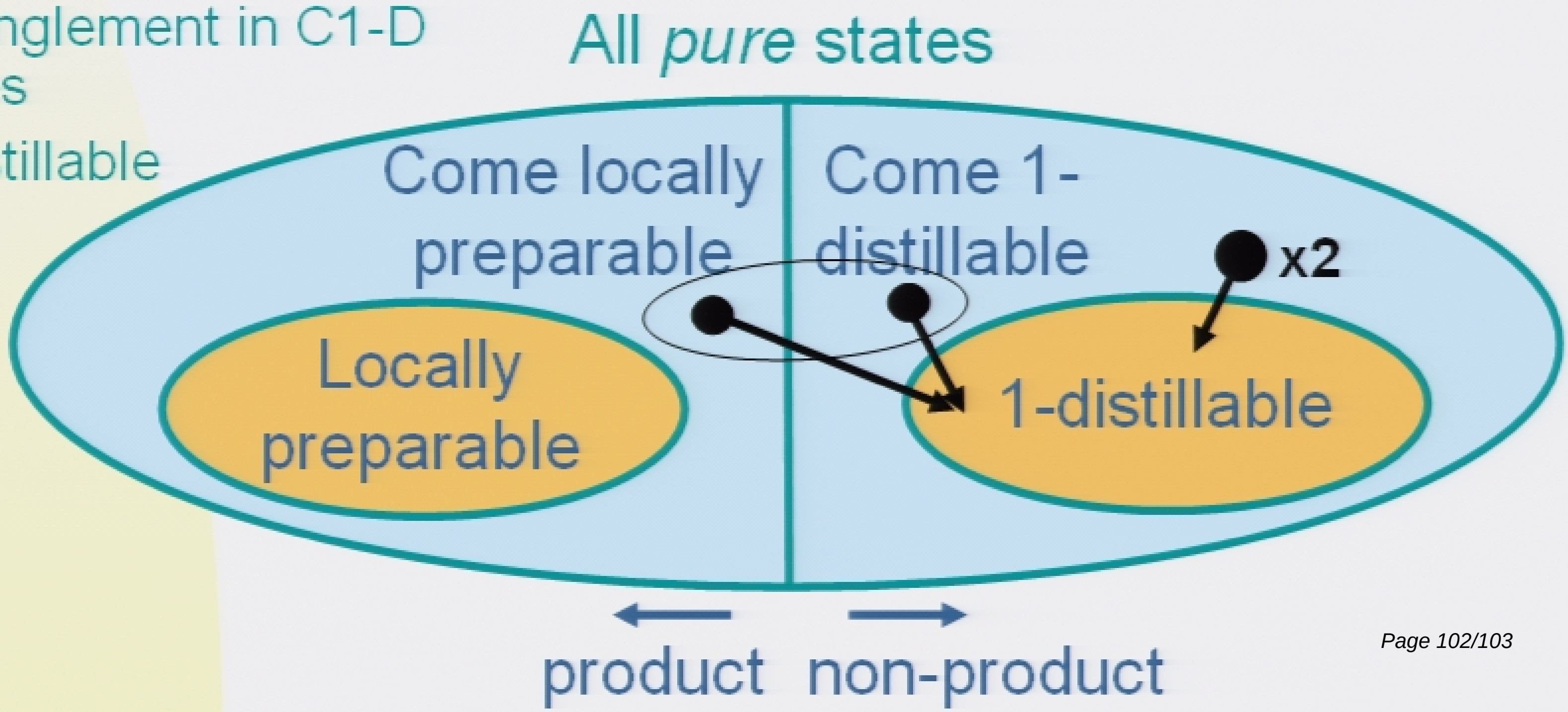

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Useful to think of bound entanglement in terms of restrictions and reference systems