Title: Efficient Alignment of Reference Frames and without Shared Entanglement

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Abstract: Quantum Information Theory

Efficient alignment of reference frames with and without entanglement

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In collaboration with: M. Baig & R. Muñoz-Tapia

Ariadne, Theseus, the Minotaur

[Theseus] got from Ariadne, who had fallen in love with him, the famous thread, and that having been instructed by her how to make his way through the intricacies of the Labyrinth, he slew the Minotaur and sailed off with Ariadne and the youths. " –*Plutarch, Theseus 19.1*

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GIQ Grup d'Informació Quàntica

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  - A related problem: estimation of one-qubit gates

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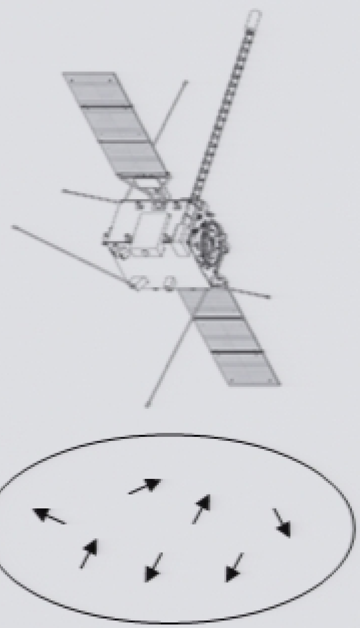
- Saving resources
- Summary and conclusions

The problem

Point of view 1

A communicates a spatial frame to B by sending N spin ½ particles. B is allowed to perform generalized measurements.
•No classical communication allowed.

A



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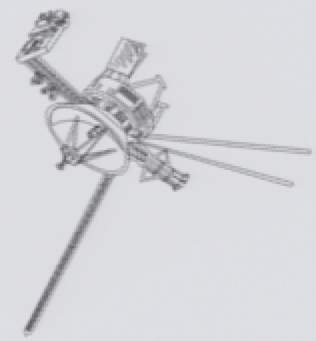

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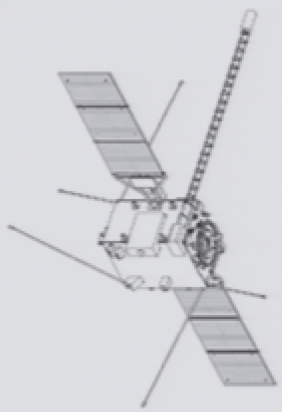

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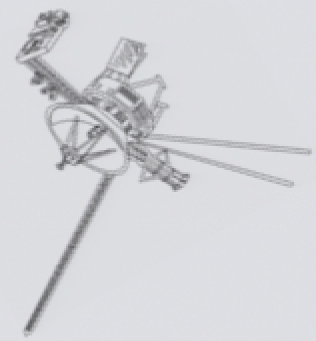

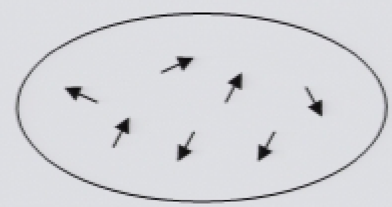

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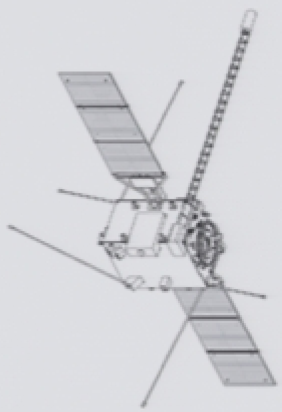

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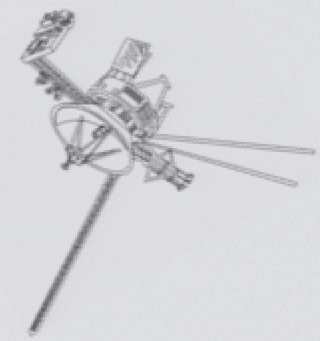
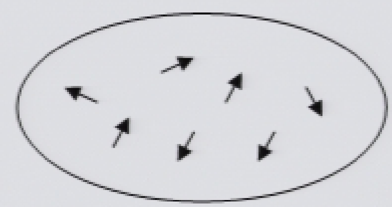
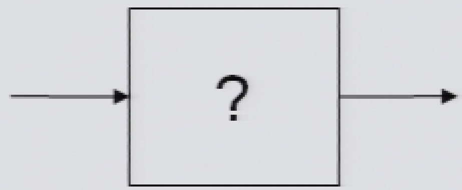




The problem

Point of view 2 (***Reverse engineering***)**:** estimation of unknown 1-qubit gates



We are allowed to make N calls to the unknown gate (but input state made out of as many qubits as we need). We may use generalized measurements.

The problem

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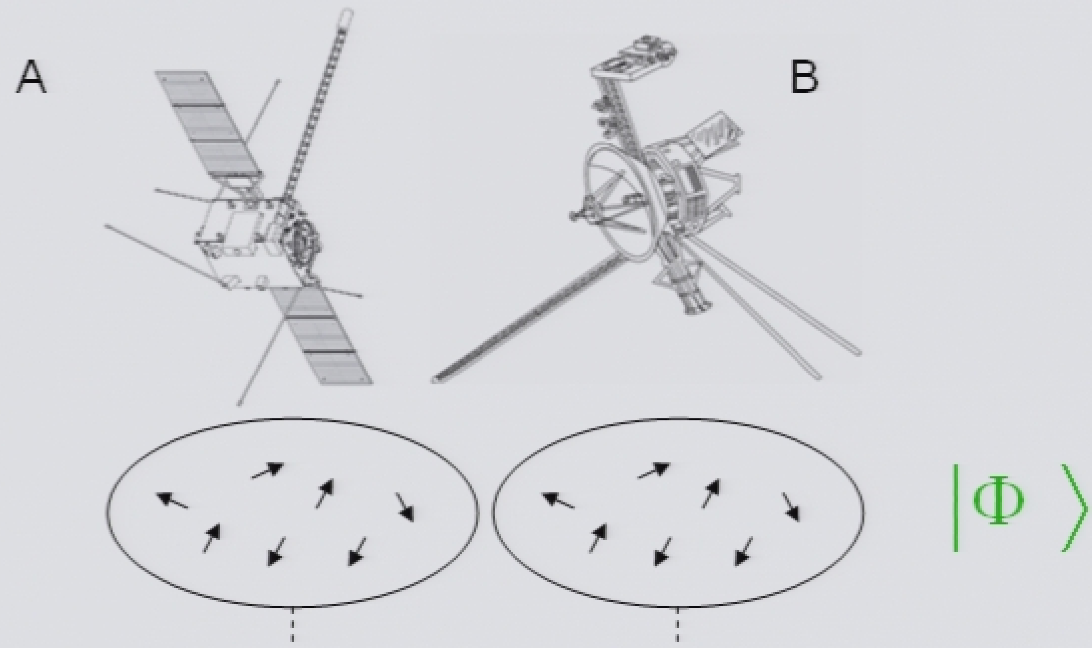
The optimal scheme

(dense covariant coding)

Point of view 1: Preparation

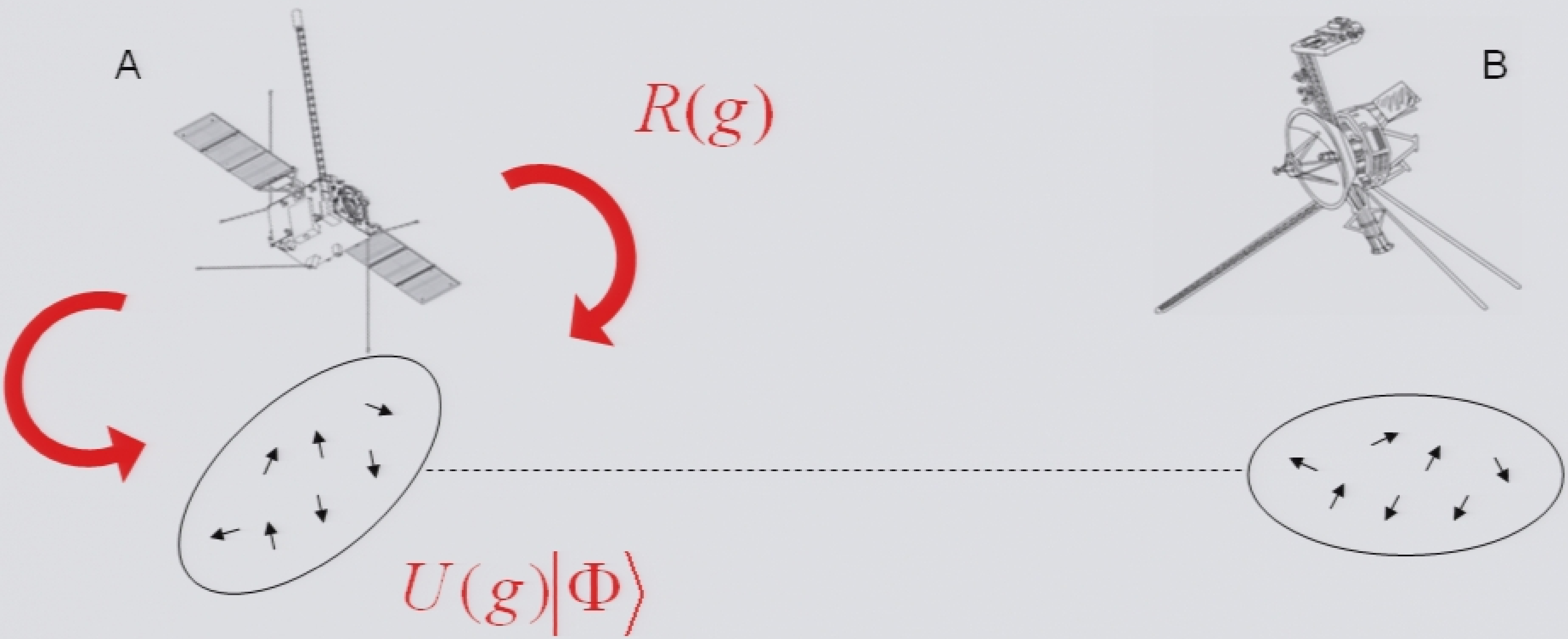


Additional resources: *entanglement* + *N extra spins*

The optimal scheme

(dense covariant coding)

Point of view 1: The journey

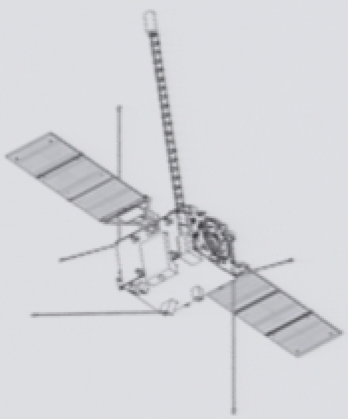


The optimal scheme

(dense covariant coding)

Point of view 1: communication, measurement and alignment

A



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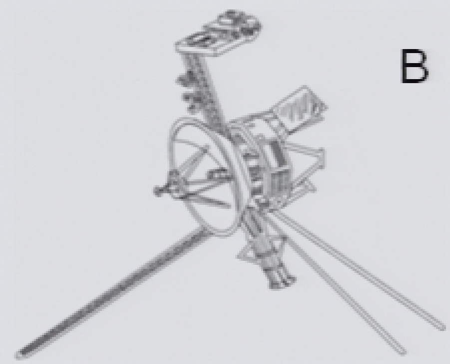

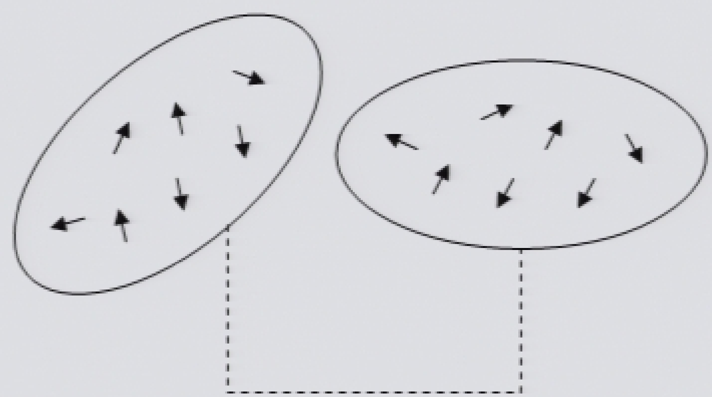

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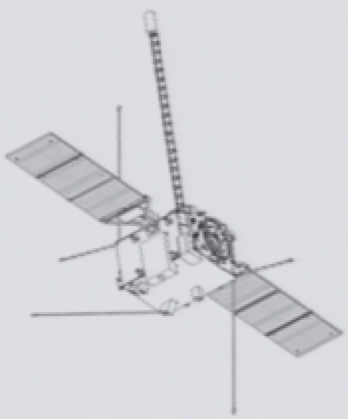

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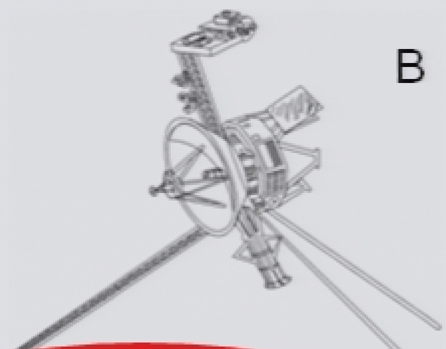
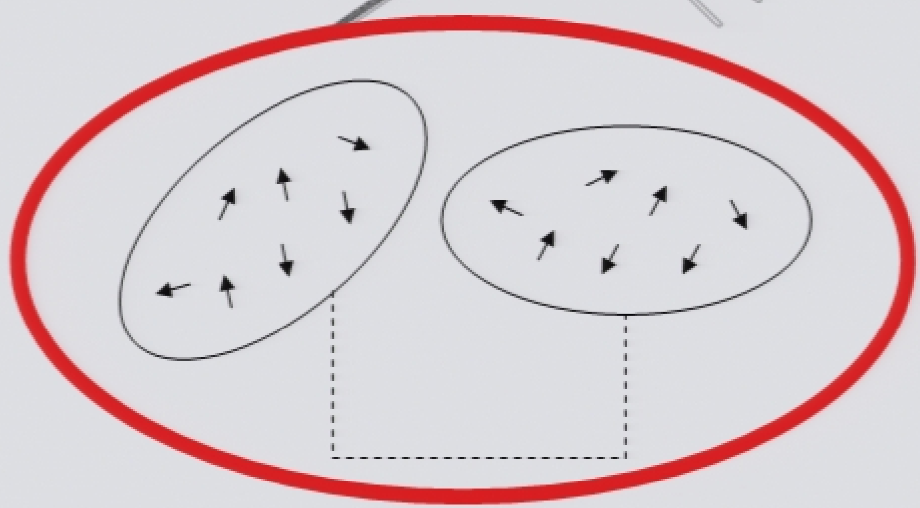


$\{O_x\}$



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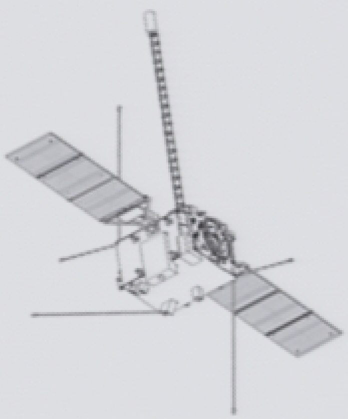

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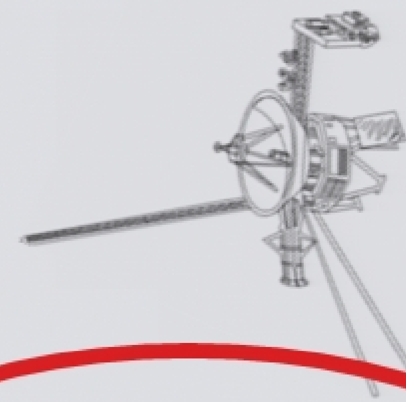

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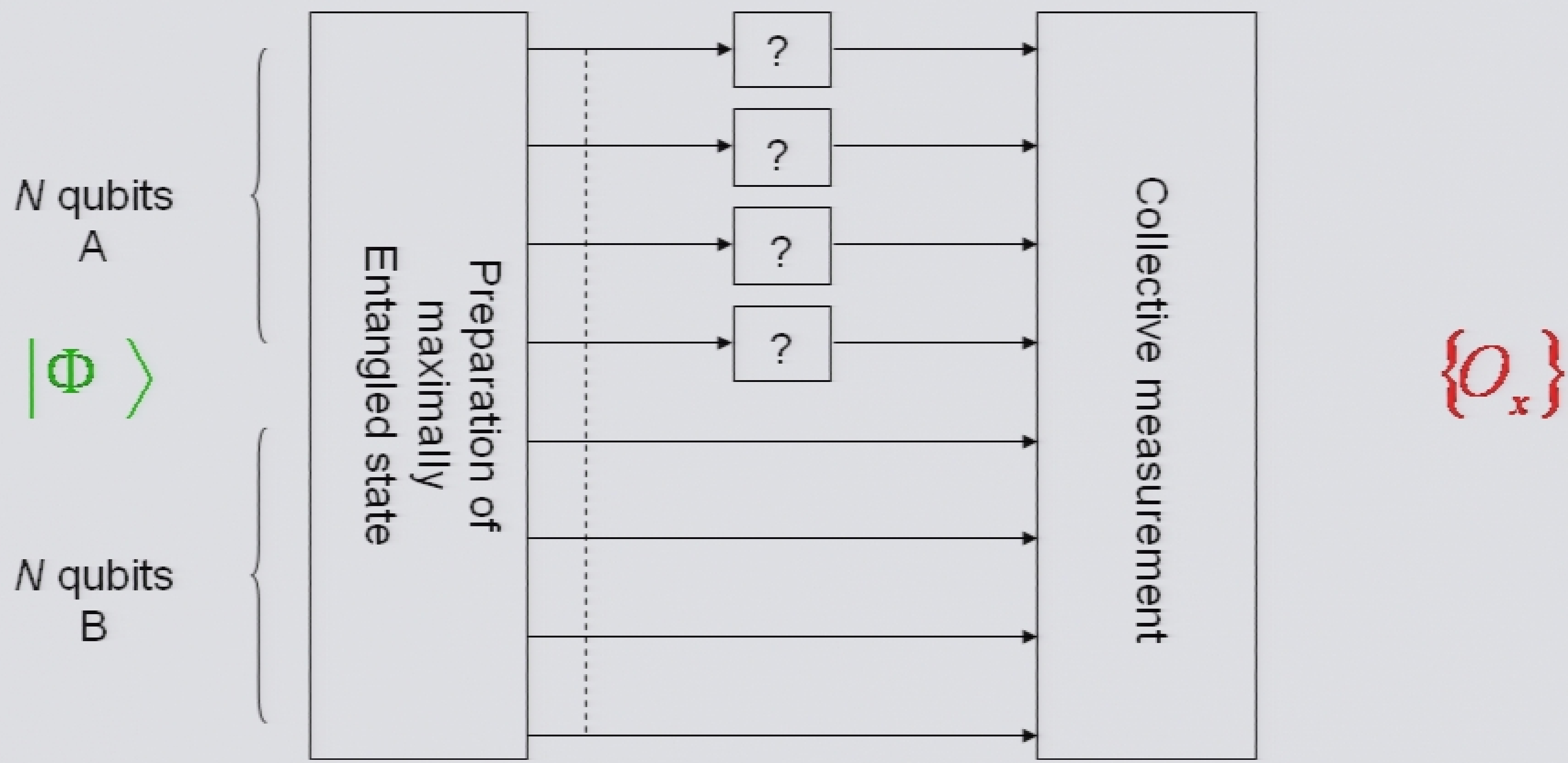
Optimal scheme

Point of view 2:



Optimal scheme

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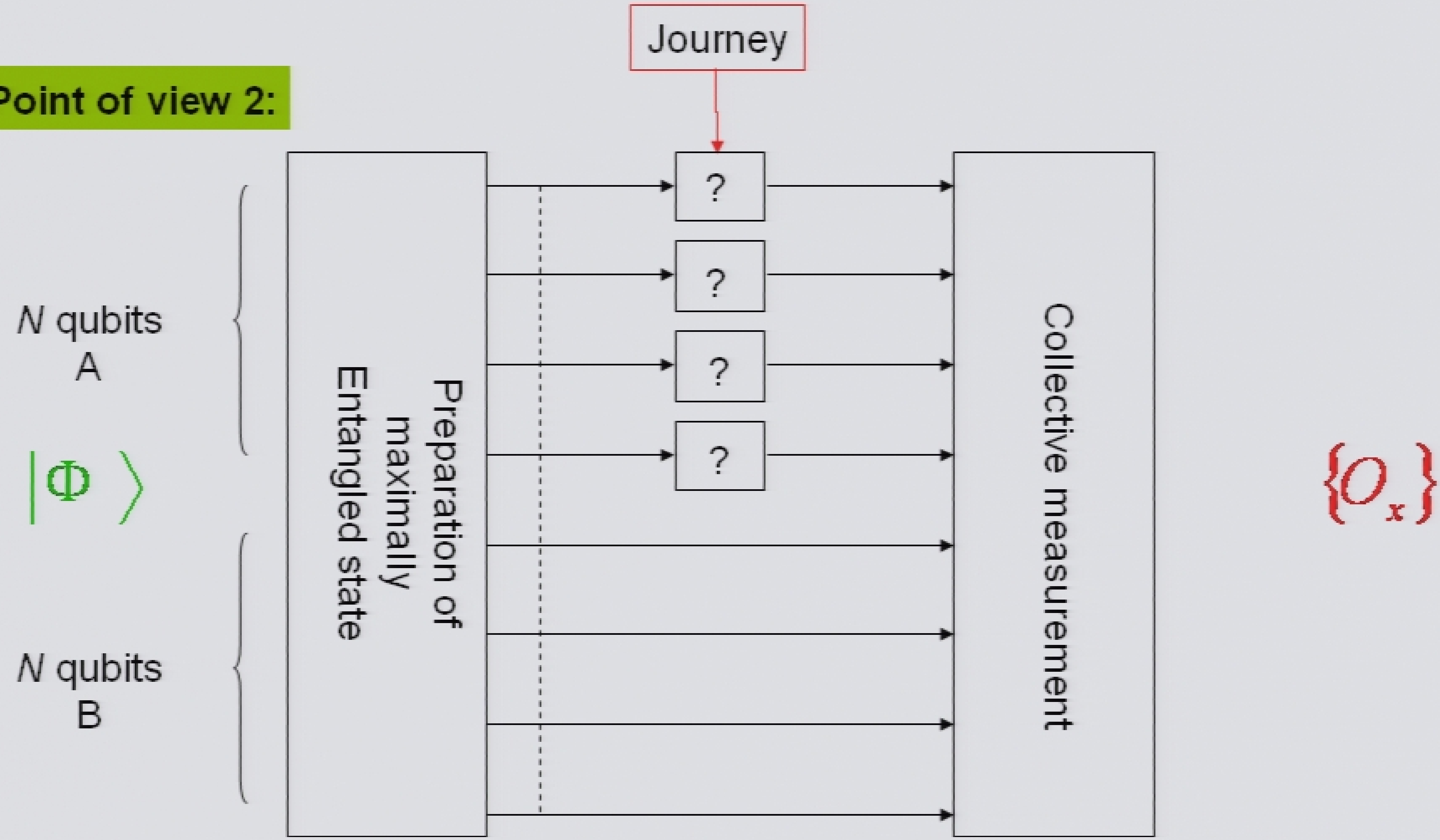

Optimal scheme

Communication, measurement and alignment

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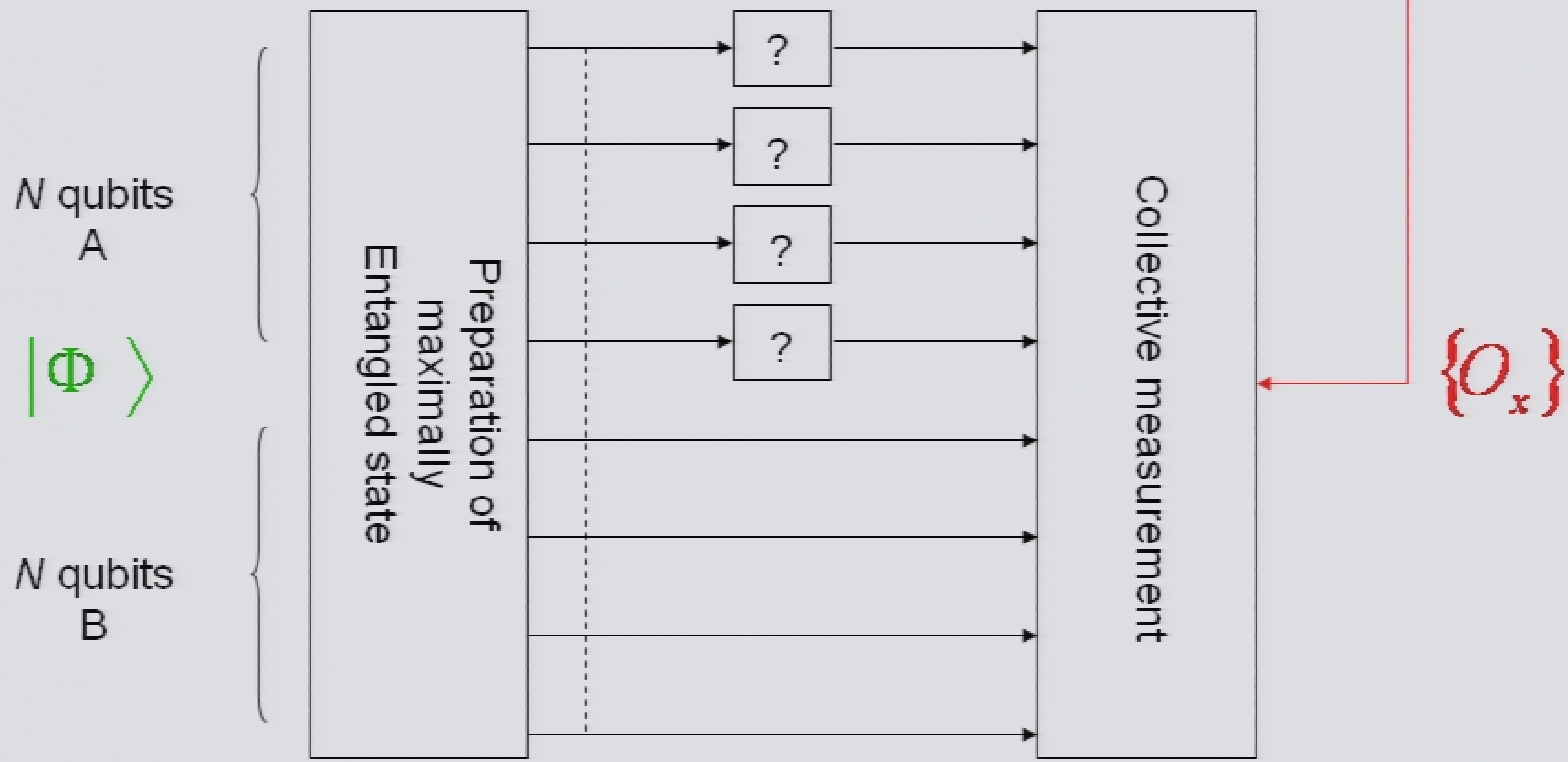

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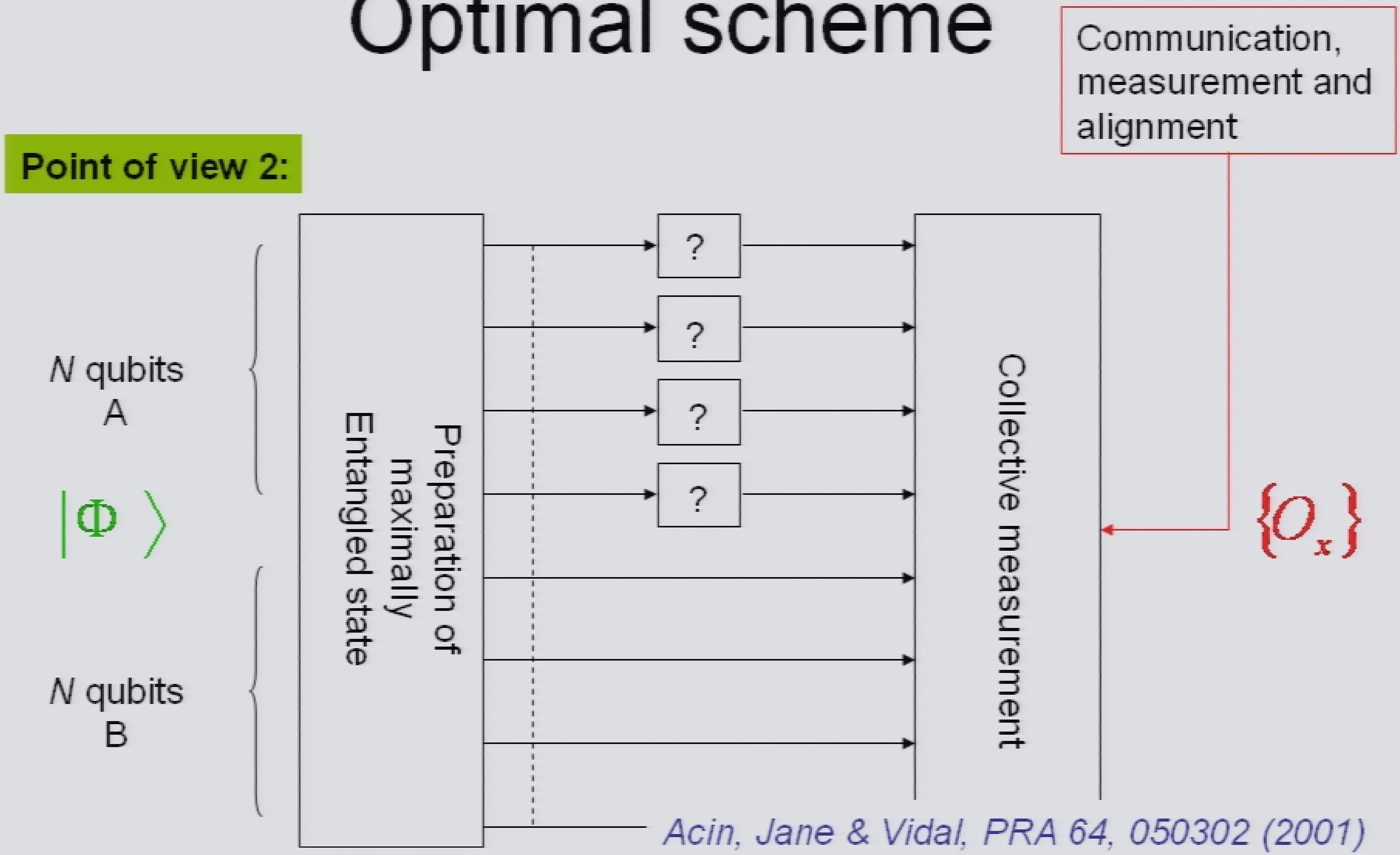


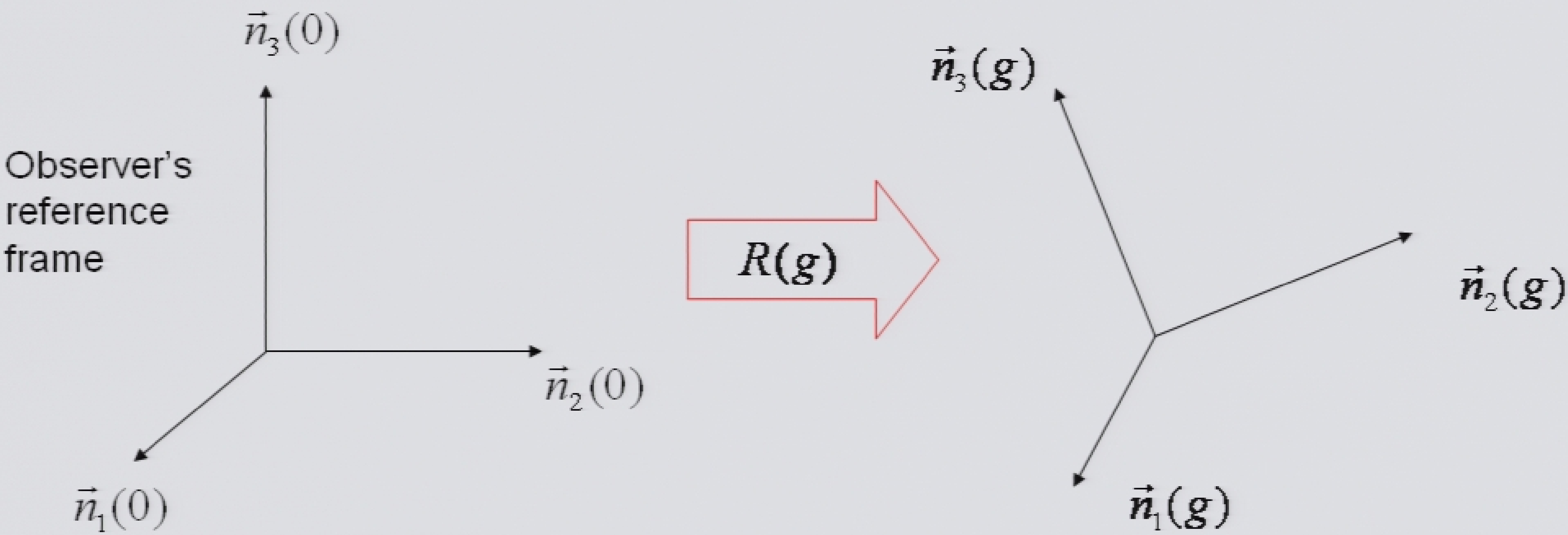
Acin, Jane & Vidal, PRA 64, 050302 (2001)

Figure of merit

Point of view 1: *Holevo error*

$$h(g, g_x) = \sum_{a=1}^{3} \left| \vec{n}_a(g) - \vec{n}_a(g_x) \right|^2$$

x = outcomes of measurement $\{O_x\}$



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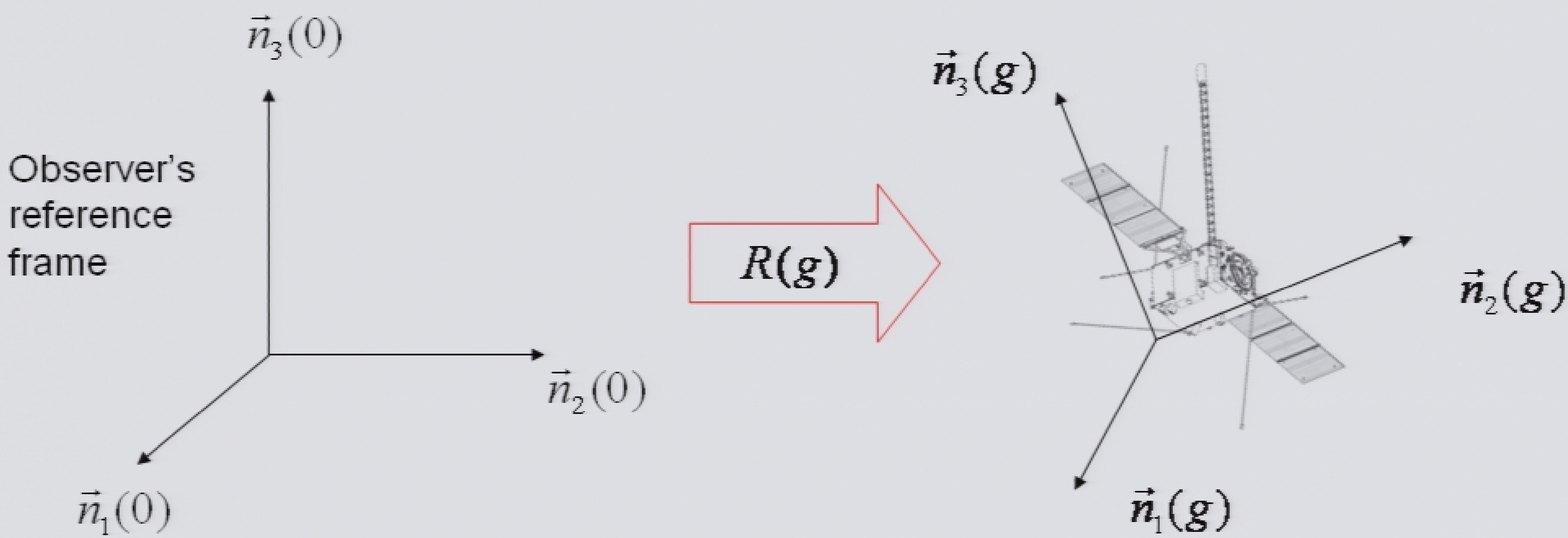

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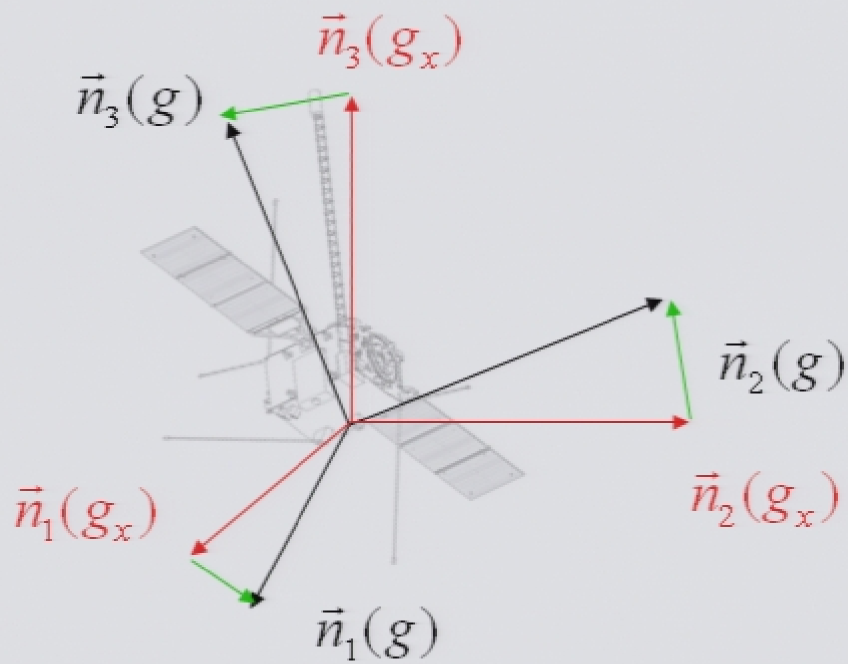

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Averaged Holevo error

$$\langle h\rangle=\sum_x\int dg\,h(g,g_x)\,p(x\,|\,g)$$

x Outcomes of measurement; g_x Guess based on x

dg SU(2) Haar measure $d(gg')=d(g'g)=dg$

$p(x\,|\,g)=\mathrm{tr}\left[O_x\rho(g)\right]$; $\rho(g)=U(g)|\Phi\rangle\langle\Phi|U^+(g)$

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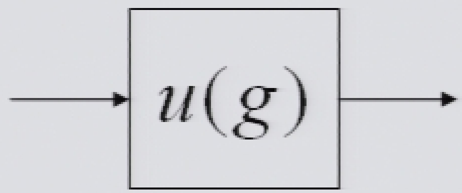
Figure of merit

Point of view 2: *Fidelity*

$$F(g, g_x) = \left|\mathrm{tr}\left[u^+(g_x)u(g)\right]\right|^2 / 4$$



x Outcomes of measurement; g_x Guess based on x

It is a measure of how well $u(g)|\phi\rangle$ compares to $u(g_x)|\phi\rangle$

in average (over all possible states $|\phi\rangle$)

Figure of merit

Point of view 2: *Fidelity*

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$$\langle F\rangle = \sum_x \int dg\, F(g,g_x)\, p(x\,|\,g)$$

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Figure of merit

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$$h(g, g_x) = 6 - 2\chi_1(g_x^{-1}g)$$

- Point of view 2

$$F(g, g_x) = \frac{1 + \chi_1(g_x^{-1}g)}{4}$$

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$$8 \geq h(g,g_x) \geq 0 \leftarrow -1 \leq \chi_1(g) \leq 3 \rightarrow 0 \leq F(g,g_x) \leq 1$$

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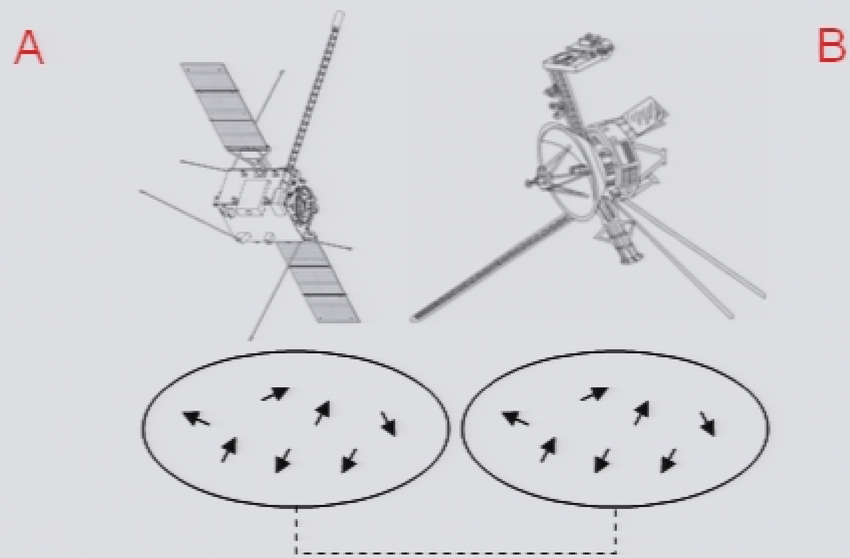
Preparation

$$|\Phi\rangle = \sum_j \frac{a_j}{\sqrt{d_j}} \sum_{m=-j}^{j} |jm\rangle\ |jm\rangle$$



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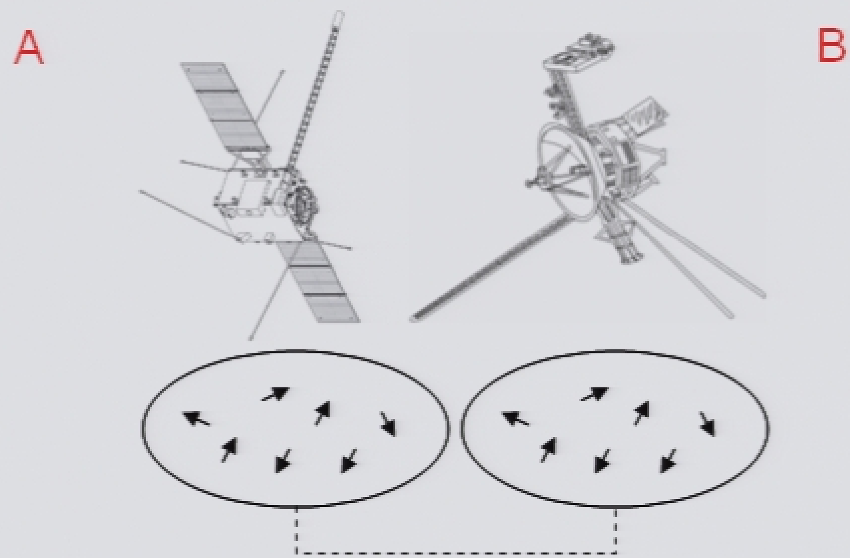
Referred to B reference frame



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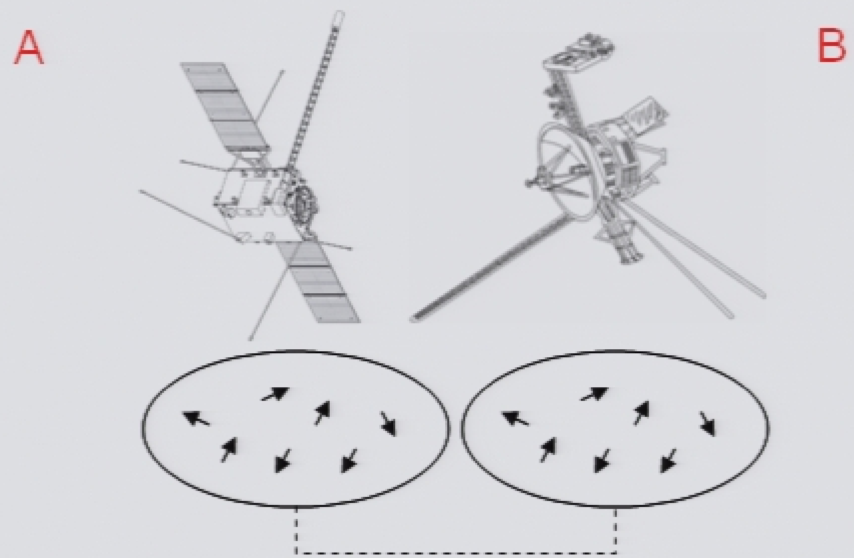
Referred to B reference frame

Maximally entangled state in each irreducible representation of SU(2)



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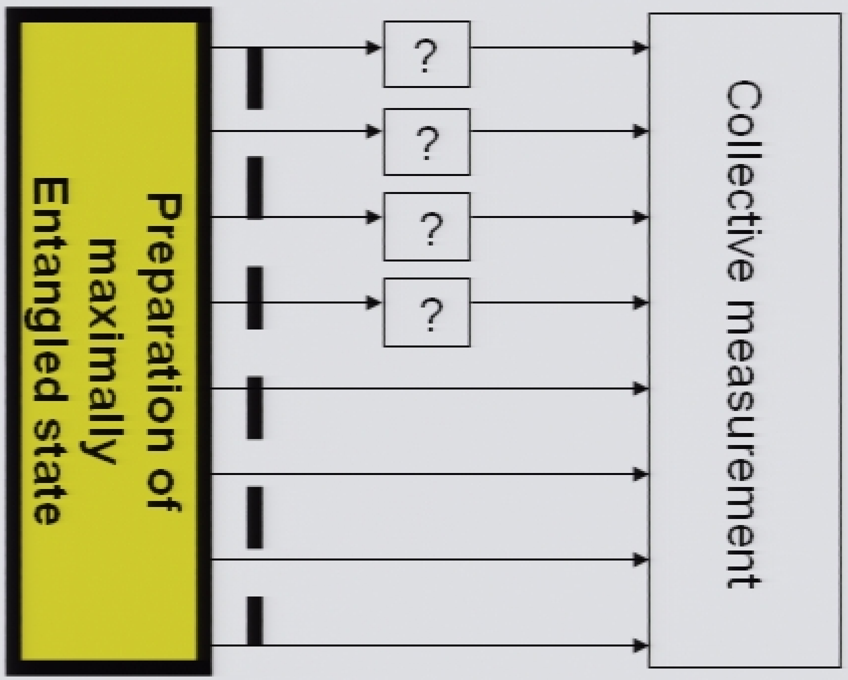

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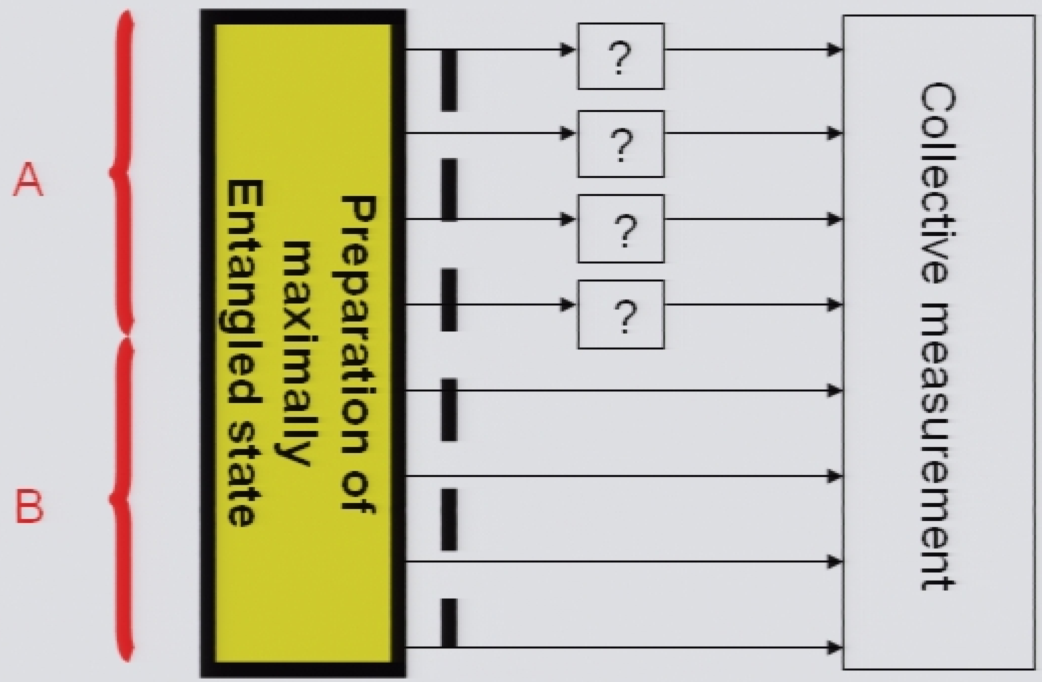
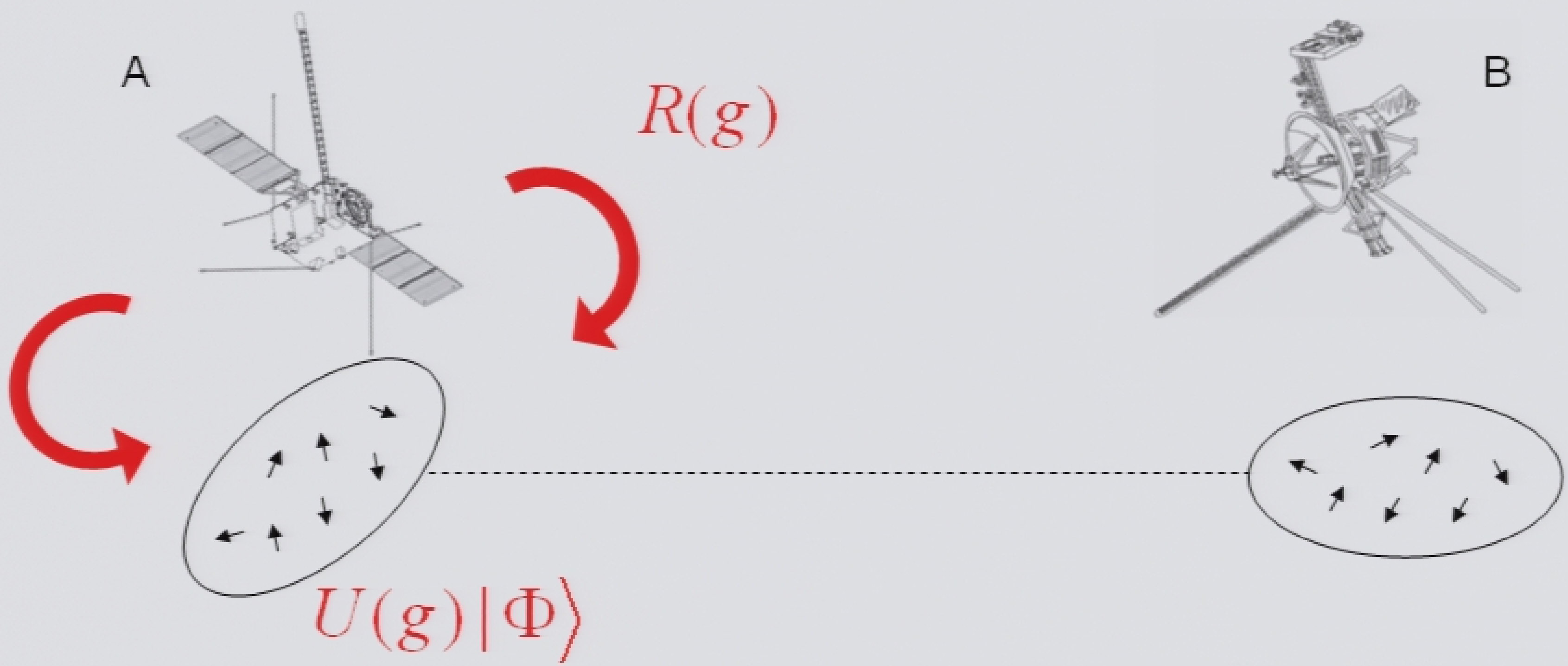


Journey

$$|\Phi\rangle \rightarrow U(g)|\Phi\rangle = u^{\otimes N}(g) \otimes I \quad |\Phi\rangle$$



“Journey”

$$|\Phi\rangle \rightarrow U(g)|\Phi\rangle = u^{\otimes N}(g) \otimes I \quad |\Phi\rangle$$

"Journey"

$$|\Phi\rangle \rightarrow U(g)|\Phi\rangle = u_A^{\otimes N}(g) \otimes I_B |\Phi\rangle$$

Measurement

$$O_x = c_x u^{\otimes N}(g_x) \otimes I \quad |\Psi\rangle\langle\Psi| \quad \left[u^{\otimes N}(g_x)\right]^{+} \otimes I$$

$$|\Psi\rangle = \sum_j \sqrt{d_j} \sum_{m=-j}^{j} |jm\rangle \; |jm\rangle$$

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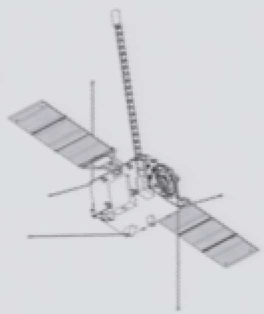

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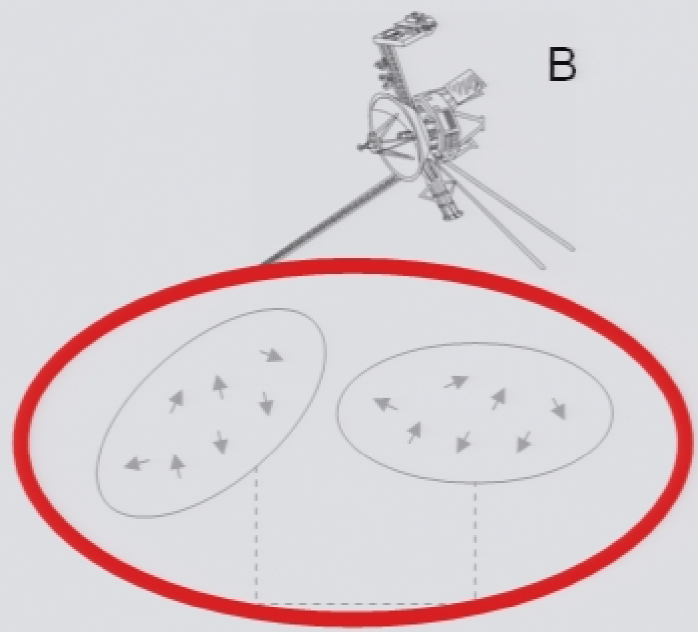

$\{O_x\}$

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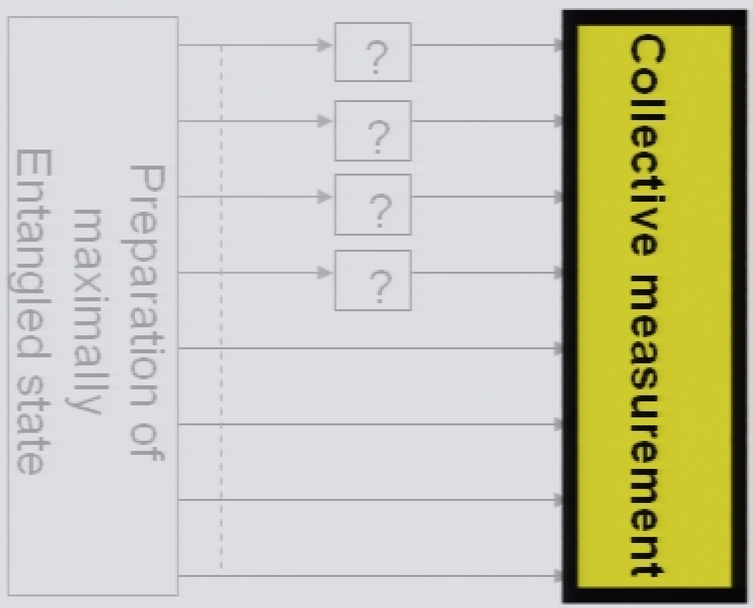

Measurement

$$O_x = c_x u_A^{\otimes N}(g_x)\otimes I_B \, |\Psi\rangle\langle\Psi| \, \left[u_A^{\otimes N}(g_x)\right]^{+}\otimes I_B$$

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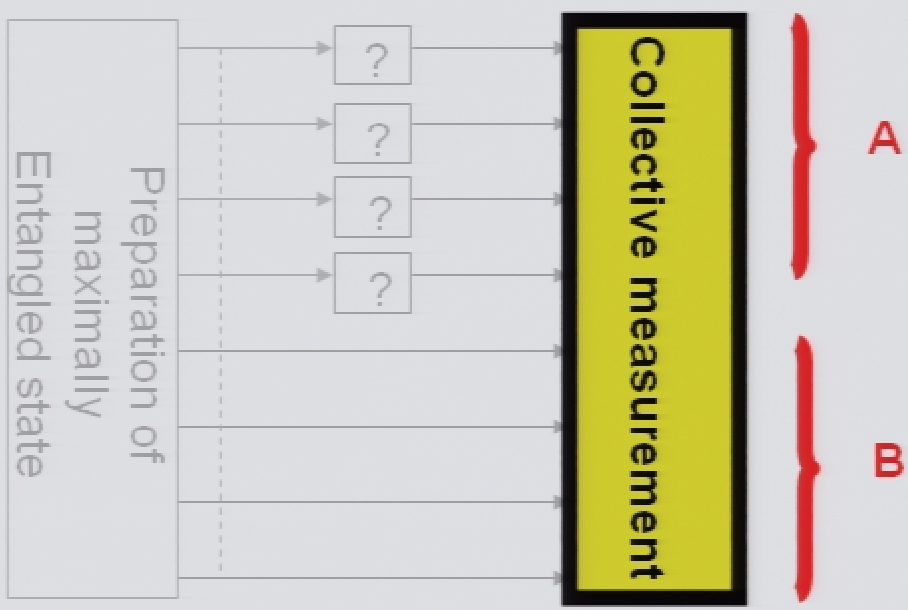

Measurement

A particular instance of

$$O_x = c_x u^{\otimes N}(g_x) \otimes I \quad |\Psi\rangle\langle\Psi| \quad \left[u^{\otimes N}(g_x)\right]^{+} \otimes I$$

Is the continuous POVM

$$O(g) = u^{\otimes N}(g) \otimes I \quad |\Psi\rangle\langle\Psi| \quad \left[u^{\otimes N}(g)\right]^{+} \otimes I$$

$$|\Psi\rangle = \sum_j \sqrt{d_j} \sum_{m=-j}^{j} |jm\rangle \ |jm\rangle$$

Some technical details

N qubit states belong to $\left(\tfrac{1}{2}\right)^{\otimes N}$

Clebsch-Gordan series for SU(2) $\quad \left(\tfrac{1}{2}\right)^{\otimes N} \to \bigoplus_{j=0,\,1/2}^{N/2} n_j \, \mathbf{j}$

where

$$n_j = \frac{2j+1}{N/2 + j + 1} \binom{N}{N/2 + j}$$

$n_j \geq d_j$ if $j < N/2$ whereas $n_j = 1$ if $j = N/2$

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N qubit states belong to $\left(\tfrac{1}{2}\right)^{\otimes N}$

Clebsch-Gordan series for SU(2) $\quad \left(\tfrac{1}{2}\right)^{\otimes N} \rightarrow \bigoplus_{j=0,\frac{1}{2}}^{N/2} n_j \, \mathbf{j}$

Acin, Jane & Vidal, PRA 64, 050302 (2001)

$$|\Phi\rangle = \sum_j a_j |\Phi^j\rangle = \sum_j \frac{a_j}{\sqrt{d_j n_j}} \sum_{m=-j}^{j} \sum_{\alpha=1}^{n_j} |jm;\alpha\rangle \, |jm;\alpha\rangle$$

But $< U^{(j)}(g) \otimes I |\Phi^j\rangle >_{\forall g}$ has dimension d_j (rather than $n_j d_j$)

We can change bases so than $< U^{(j)}(g) \otimes I |\Phi^j\rangle >_{\forall g}$ is in just 1 irreducible representation **j**.

Holevo error/Fidelity

$$\langle \chi_1 \rangle_{\max}$$

$$\langle \chi_1 \rangle_{\max} = 1 + 2\cos\frac{2\pi}{N+3}$$

$$a_j = \frac{2}{\sqrt{N+3}} \sin\frac{(2j+1)\pi}{N+3}$$

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Main ingredients of the proof:

- Schur's lemma
- Schwarz inequality
- $\langle \chi_1 \rangle \leq 1 + a^t M a$ $\quad$ *M* tridiagonal
- Characteristic polynomial is a combination of Chevishev's polynomials
- Continuous POVM saturates bound

Holevo error/Fidelity

$$\left\langle \chi_1 \right\rangle_{\max}$$

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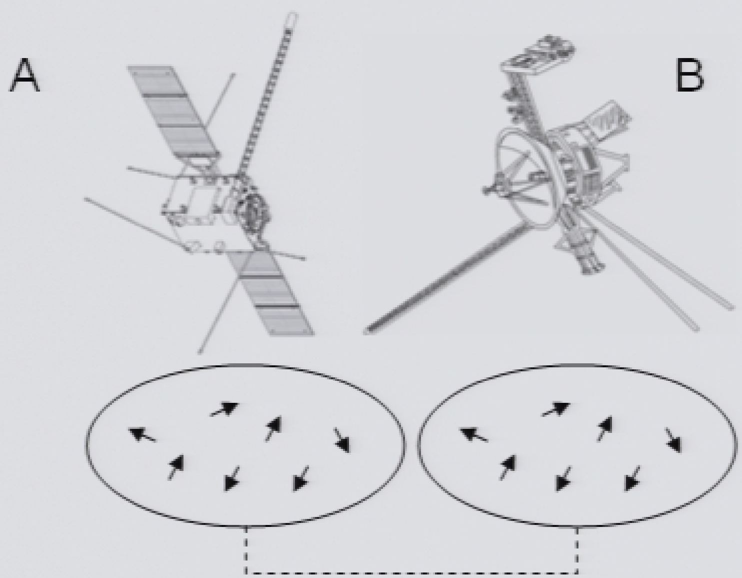
For large *N* we have

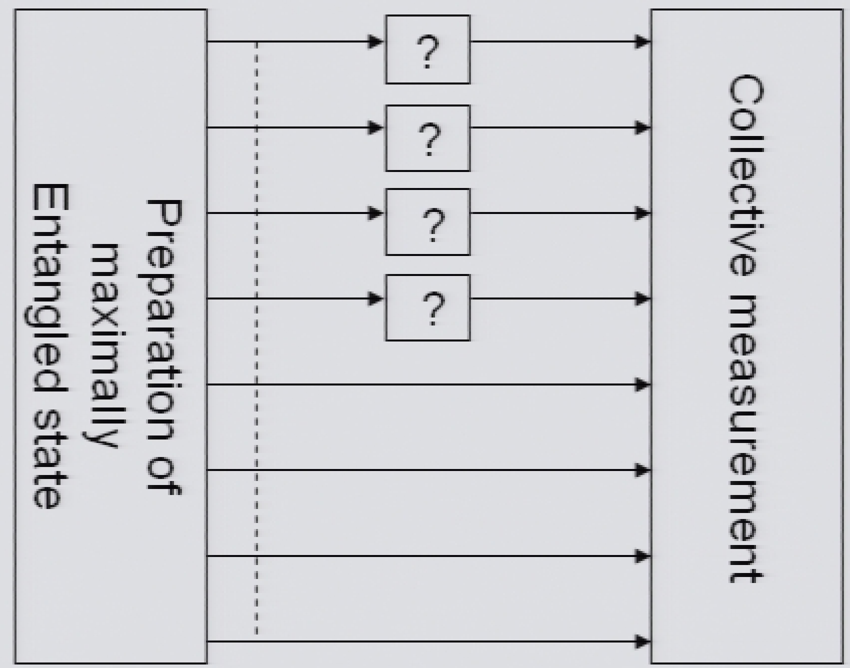
$$\left\langle \chi_1 \right\rangle_{\max} = 3 - \frac{4\pi^2}{N^2} + \ldots$$

$$\left\langle h \right\rangle_{\min} = \frac{8\pi^2}{N^2} + \ldots$$

$$\left\langle F \right\rangle_{\max} = 1 - \frac{\pi^2}{N^2} + \ldots$$

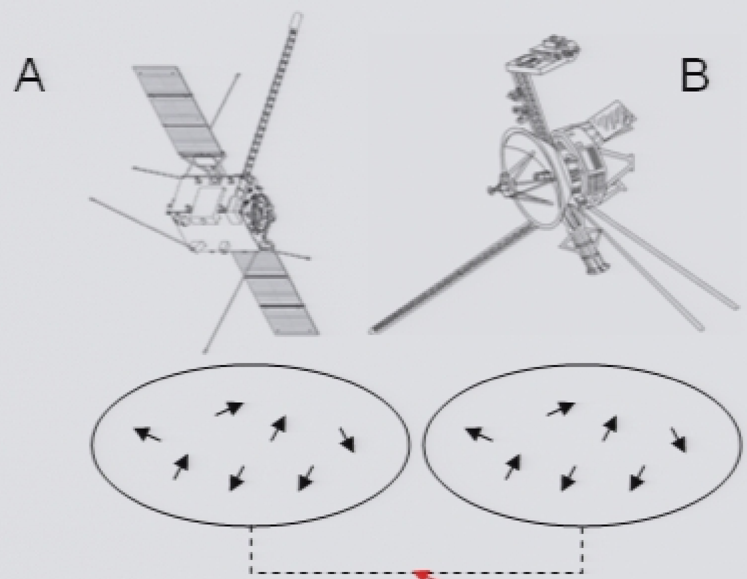
Saving resources

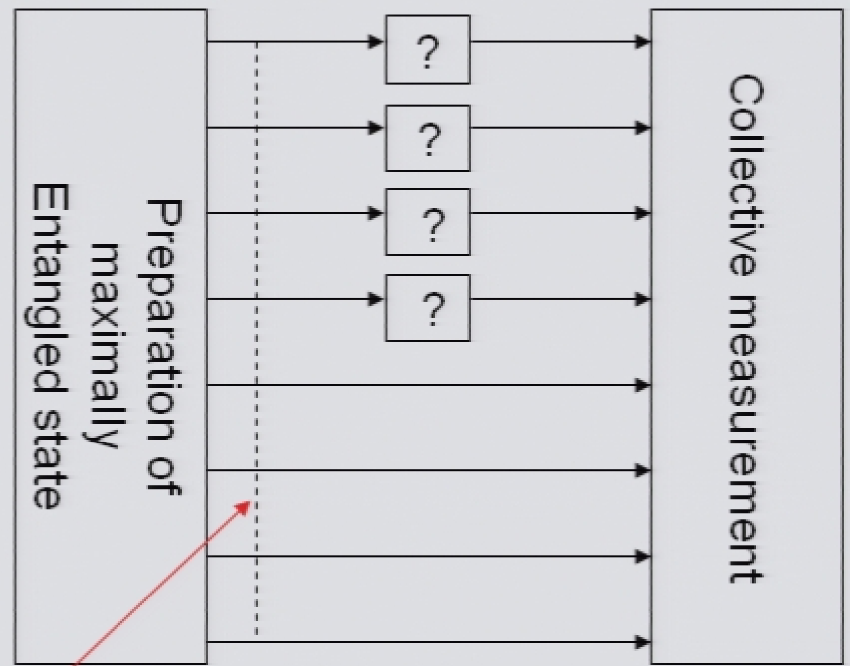




Shared entanglement + N extra spins

Saving resources





Shared entanglement + N extra spins

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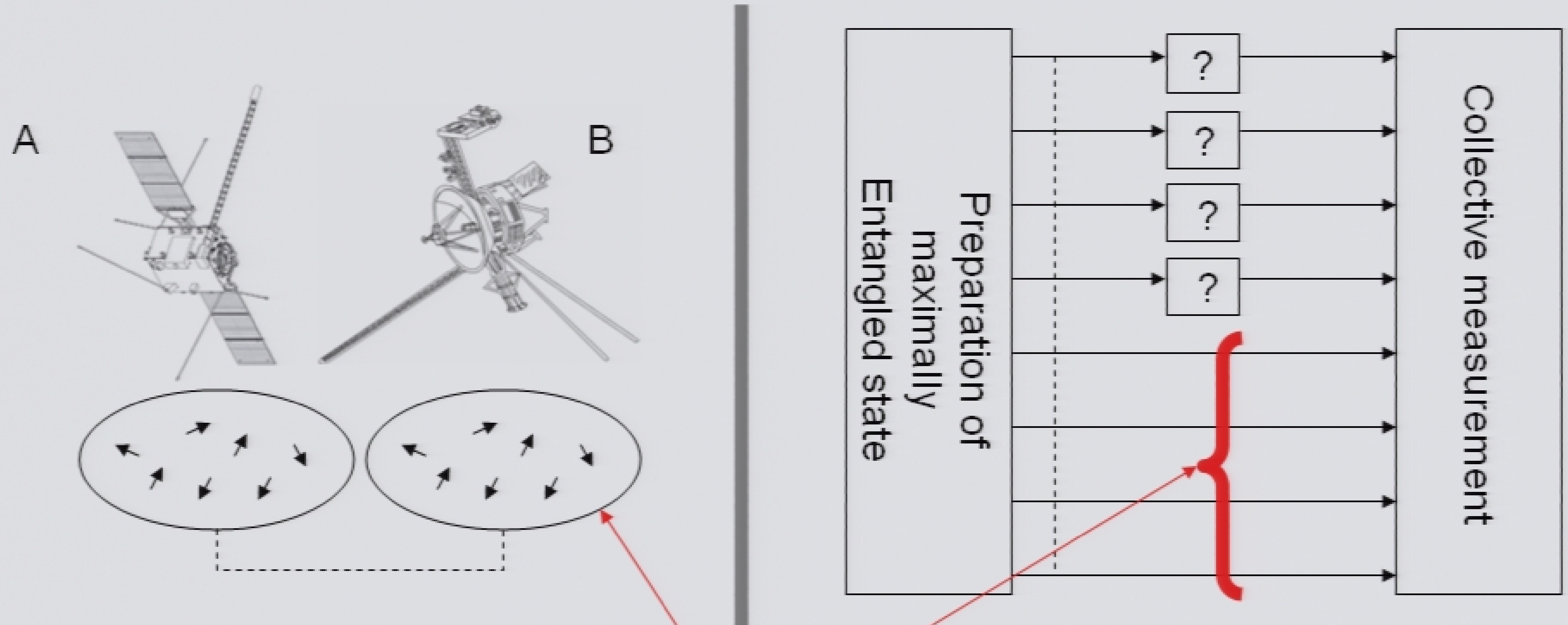

Shared entanglement + N extra spins

Saving resources

Saving resources

- We can cut number of spins down to N

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Saving resources

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- How !!??

Saving resources

Recall

Clebsch-Gordan series for SU(2) $\left(\tfrac{1}{2}\right)^{\otimes N} \rightarrow \bigoplus_{j=0,\frac{1}{2}}^{N/2} n_j \, \mathbf{j}$

where

$n_j \geq d_j$ if $j < N/2$ whereas $n_j = 1$ if $j = N/2$

$$|\phi\rangle = \sum_j |\phi^j\rangle = \sum_j \sum_{m=-j}^{j} \sum_{\alpha=1}^{n_j} \phi^j_{m\alpha} |jm;\alpha\rangle$$

Saving resources

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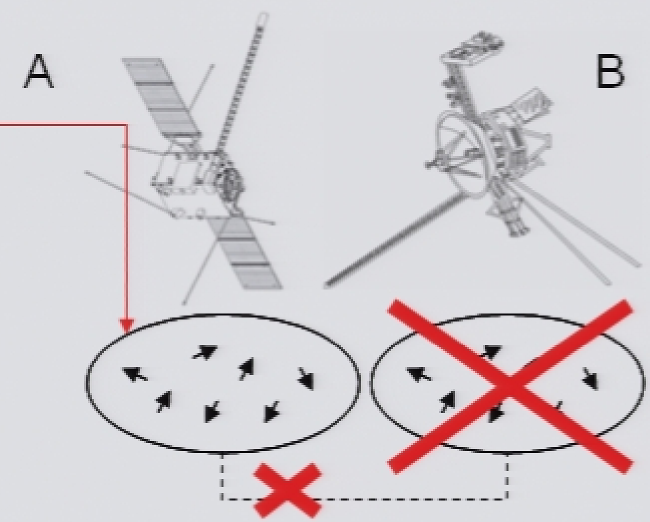

Saving resources

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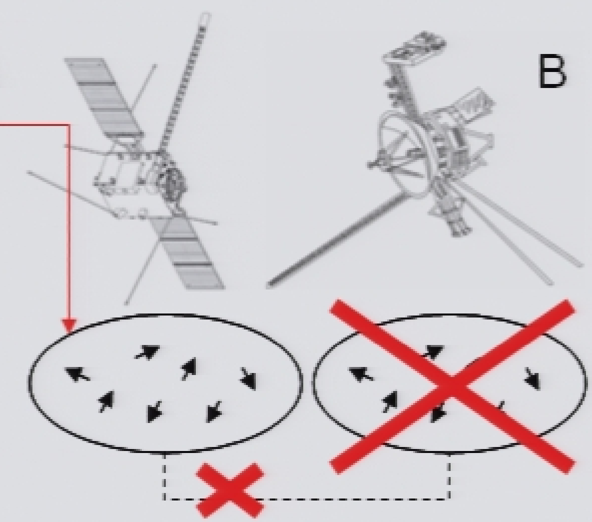

But $< U^{(j)}(g) \left|\phi^j\right\rangle >_{\forall g}$

belongs to d_j representations **j** *at most.*

Saving resources

preparation

$$|\Phi\rangle = \sum_j \frac{a_j}{\sqrt{d_j}} \sum_{m=-j}^{j} |jm\rangle_A \, |jm\rangle_B \;\Rightarrow\; |\phi\rangle = \sum_j \frac{a_j}{\sqrt{d_j}} \sum_{m=-j}^{j} |\overset{A}{jm};\overset{B}{\alpha_m}\rangle$$

α_m 1-1 map from

$$\{-j, -j+1, \ldots, j\} \longrightarrow \{1, \ldots, d_j\}$$

Rather than entangling two different parties, we entangle different degrees of freedom

Saving resources

journey

$$|\Phi\rangle = \sum_j \frac{a_j}{\sqrt{d_j}} \sum_{m=-j}^{j} |jm\rangle\ |jm\rangle \;\Rightarrow\; |\phi\rangle = \sum_j \frac{a_j}{\sqrt{d_j}} \sum_{m=-j}^{j} |jm;\alpha_m\rangle$$

Just like **j**, α is a scalar under SU(2).

$$|\Phi^j\rangle \to U_A^{(j)}(g) \otimes I_B |\Phi^j\rangle \;\Rightarrow\; |\phi^j\rangle \to U^{(j)}(g) \otimes I^{(\alpha)}\ |\phi^j\rangle$$

Saving resources

journey

$$|\Phi\rangle = \sum_j \frac{a_j}{\sqrt{d_j}} \sum_{m=-j}^{j} |jm\rangle\,|jm\rangle \quad \Rightarrow \quad |\phi\rangle = \sum_j \frac{a_j}{\sqrt{d_j}} \sum_{m=-j}^{j} |jm;\alpha_m\rangle$$

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Saving resources

measurement

$$|\Phi\rangle = \sum_j \frac{a_j}{\sqrt{d_j}} \sum_{m=-j}^{j} |jm\rangle\ |jm\rangle \;\Rightarrow\; |\phi\rangle = \sum_j \frac{a_j}{\sqrt{d_j}} \sum_{m=-j}^{j} |jm;\alpha_m\rangle$$

$$|\Phi^j\rangle \to U_A^{(j)}(g) \otimes I_B |\Phi^j\rangle \;\Rightarrow\; |\phi^j\rangle \to U^{(j)}(g) \otimes I^{(\alpha)}\ |\phi^j\rangle$$

$$|\Psi\rangle = \sum_j \sqrt{d_j} \sum_{m=-j}^{j} |jm\rangle\ |jm\rangle \;\Rightarrow\; |\psi\rangle = \sum_j \sqrt{d_j} \sum_{m=-j}^{j} |jm;\alpha_m\rangle$$

Saving resources

but…..

$$n_j = 1 < d_j \qquad \text{if} \qquad j = {}^{N}\!/_{2}$$

$$|\Phi\rangle = \sum_j \frac{a_j}{\sqrt{d_j}} \sum_{m=-j}^{j} |jm\rangle \; |jm\rangle \;\Rightarrow\; |\phi\rangle = \sum_j \frac{a_j}{\sqrt{d_j}} \sum_{m=-j}^{j} |jm;\alpha_m\rangle$$

Saving resources

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$$\downarrow$$

$$|\phi\rangle = \sum_{j<N/2} \frac{a_j}{\sqrt{d_j}} \sum_{m=-j}^{j} |jm;\alpha_m\rangle$$

Saving resources

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$$|\phi\rangle = \sum_{j<N/2} \frac{a_j}{\sqrt{d_j}} \sum_{m=-j}^{j} |jm;\alpha_m\rangle + a_{N/2} |N/2\ N/2\rangle$$

Bagan, Baig & Muñoz-Tapia, PRL 87 257903 (2001)

Saving resources

as to the measurement…..

$$|\psi\rangle = \sum_{j<N/2} \sqrt{d_j} \sum_{m=-j}^{j} |jm;\alpha_m\rangle + \sqrt{d_{N/2}} \left|N/2\; N/2\right\rangle$$

Saving resources

as to the measurement…..

$$|\psi\rangle = \sum_{j<N/2} \sqrt{d_j} \sum_{m=-j}^{j} |jm;\alpha_m\rangle + \sqrt{d_{N/2}}\,|\tfrac{N}{2}\,\tfrac{N}{2}\rangle$$

This scheme is **not** optimal but saturates de optimal bound **asymptotically**

$$\langle \chi_1 \rangle_{\max} = 3 - \frac{4\pi^2}{N^2} + \ldots$$

$$\langle h \rangle_{\min} = \frac{8\pi^2}{N^2} + \ldots$$

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Bagan, Baig & Muñoz-Tapia, quant-ph/0405082

Chiribella et al., quant-ph/0405095

Summary & conclusions

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- **Quantum systems can**
  - transmit spacial reference frames
  - be used for *reverse-engineering*

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Peres & Scudo, PRL 87, 167901 (2001); Bagan, Baig & Muñoz-Tapia, PRL 87, 257903 (2001)

- **Quantum systems can**
  - transmit spacial reference frames
  - be used for *reverse-engineering*