

Title: Elliptic Rydberg States as Direction Indicators

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Abstract:

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Elliptic Rydberg states as direction indicators

Perimeter Institute, July 2004
Netanel Lindner
Asher Peres Daniel R. Terno

N.L. et al, PRA 68, 042308 (2003)

Outline Slides

- 1 Perimeter Institute, July 2004
Netanel Lindner
Asher Peres Daniel R. Terno
- 2 Alice wants to indicate a direction in space to Bob.
They don't share a joint reference frame.
- 3 A physical object needs to be sent.
Classical object: gyroscopes.
Quantum mechanical objects: Spins, hydrogen atom.
- 4 A quantum game – the rules
Alice picks a direction
Alice sends a quantum state to Bob. For example, angular momentum eigenstates
Bob measures the state and guesses a direction
Bob gets a score:

Click to add notes

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English (U.K.)

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Elliptic Rydberg states as direction indicators



Perimeter Institute, July 2004

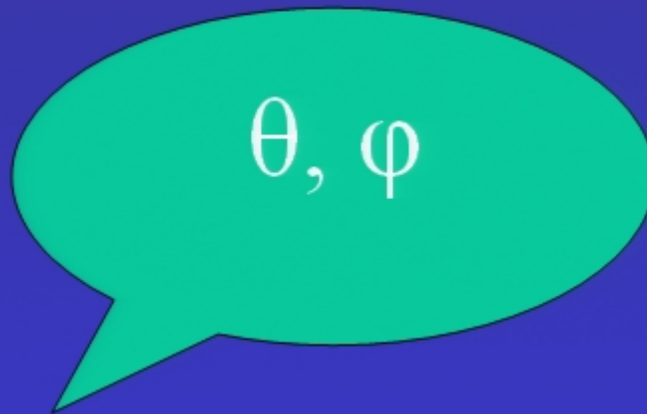
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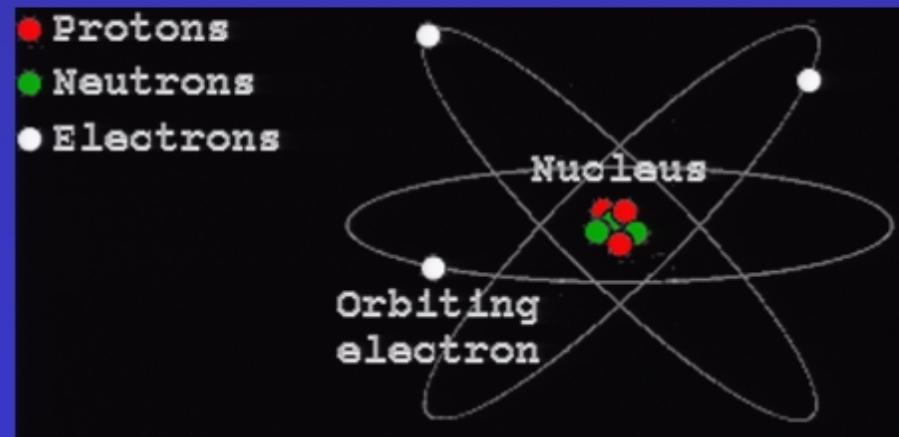
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- A physical object needs to be sent.
 - Classical object: gyroscopes.
 - Quantum mechanical objects: Spins, hydrogen atom.



A quantum game – the rules

- Alice picks a direction $\hat{\mathbf{n}}_A$
- Alice sends a quantum state to Bob. For example, angular momentum eigenstates $\hat{\mathbf{n}}_A \cdot \mathbf{J} |\psi\rangle = j |\psi\rangle$
- Bob measures the state and guesses a direction $\hat{\mathbf{n}}_B$
- Bob gets a score: $\frac{1 + \hat{\mathbf{n}}_A \cdot \hat{\mathbf{n}}_B}{2} \equiv \cos^2(\omega / 2)$



- Objective: Maximize fidelity $F = \langle \cos^2(\omega/2) \rangle$

Case study: Alice has two spins

$$|\uparrow\uparrow\rangle$$

$$|\uparrow\downarrow\rangle$$



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Case study: Alice has two spins

$$|\uparrow\uparrow\rangle$$

$$F = 0.750$$

$$|\uparrow\downarrow\rangle$$

$$F = 0.785$$



- Objective: Maximize fidelity $F = \langle \cos^2(\omega/2) \rangle$

Case study: Alice has two spins

$$|\uparrow\uparrow\rangle$$

$$F = 0.750$$

$$= |J = 1, M = 1\rangle$$

$$|\uparrow\downarrow\rangle$$

$$F = 0.785$$

$$= \frac{1}{\sqrt{2}} |J = 0, M = 0\rangle$$

$$+ \frac{1}{\sqrt{2}} |J = 1, M = 0\rangle$$



Aligning a Cartesian frame

- Find the rotation that brings Bob's frame to Alice's.
- Same rules .
- Three directions:

$$F = \sum_{k=1,3} \langle \cos^2(\omega_k/2) \rangle$$



Bob's measurement

- Bob's detectors have labels $\psi\theta\phi$ which indicate the unknown *Euler angles* relating his Cartesian axes to those used by Alice.
- The mathematical representation of his apparatus is a **POVM**:

$$\int d_{\psi\theta\phi} E_{\psi\theta\phi} = 1 \quad d_{\psi\theta\phi} = \sin\theta d\theta d\psi d\phi / 8\pi^2$$

$$E_{\psi\theta\phi} = |\psi\theta\phi\rangle\langle\psi\theta\phi|$$



$$|B\rangle = \sum_{l=0}^{n-1} \sqrt{2l+1} \sum_{m=-l}^l b_{lm} |lm\rangle$$

$$\sum_{m=-l}^l |b_{lm}|^2 = 1$$

$$|\psi\theta\phi\rangle = U(\psi\theta\phi) |B\rangle$$

By Schur's lemma, this construction gives a resolution of identity,



Bob's measurement

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- Suppose Alice sends a hydrogen atom in the n th energy level.
- We wish to find feasible atomic state which would minimize the transmission error.
- Alice's signal, written in Alice's notation is

$$|A\rangle = \sum_{l=0}^{n-1} \sum_{m=-l}^l a_{lm} |lm\rangle.$$



- **Caution:** $|A\rangle$ is written in Alice's notation, $|B\rangle$ is written in Bob's.
- Define the Euler angles $\alpha\beta\gamma$ that rotate Bob's Cartesian frame into his **estimate** of Alice's frame and then rotate back by the **true angles** from Alice's to Bob's frame.

$$U(\alpha\beta\gamma) = U(\text{true})^\dagger U(\alpha\beta\gamma)$$

- $\alpha\beta\gamma$ indicate Bob's measurement error. The probability of that error is:

$$dP(\alpha\beta\gamma) = \left| \langle A | U(\alpha\beta\gamma) | B \rangle \right|^2 d_{\alpha\beta\gamma}.$$



Optimality

- One direction: A. Peres and P. Scudo PRL **86** (2001)

$$|A\rangle = \sum_{j=0, n-1} a_j |j0\rangle \quad F = 1 - \frac{2.4}{(2n+1)^2}$$

- Reference frame: A. Peres and P. Scudo, PRL **87** (2001)
Bagan et al, PRL **87** 257903 (2001)

$$|A\rangle = \sum_{j=0, n-1} a_j |jj\rangle \quad F = 1 - \frac{1}{n}$$

$$b_{jm} = a_{jm} \left(\sum_n |a_{jn}|^2 \right)^{-1/2}$$



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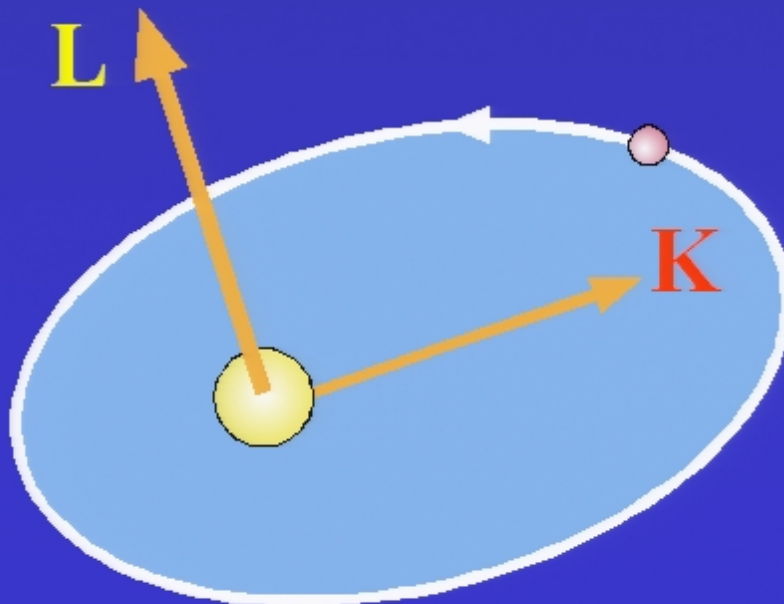
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Classical Mechanics

- Vectorial constants of motion in a classical Keplerian orbit:
 - Angular momentum.
 - Laplace-Runge-Lenz vector.



$$\mathbf{L} \cdot \mathbf{K} = 0$$



Quantum elliptic states

Atomic states with the following properties:

- Eigenstates of the Hamiltonian.
- $|\psi|^2$ maximally localized on a classical elliptic trajectory. (elliptical “torus”).
- minimal fluctuations on $\mathbf{L}$ and $\mathbf{K}$, i.e.
minimal $(\Delta\mathbf{L})^2 + (\Delta\mathbf{K})^2$.



The Laplace-Runge-Lenz (LRL) operator is defined by

$$\mathbf{K} = (-2H)^{-1/2} [(\mathbf{p} \times \mathbf{L} - \mathbf{L} \times \mathbf{p})/2 - \mathbf{r}/r]$$

where $\mathbf{L}$ is the orbital angular momentum, and H is the Hamiltonian.

$\mathbf{L}$ and $\mathbf{K}$ have the commutation relations of infinitesimal rotation generators in four dimensional Euclidian space:

$$[L_i, L_j] = i\epsilon_{ijk} L_k$$

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$$[K_i, K_j] = i\epsilon_{ijk} L_k.$$

$\mathbf{L}, \mathbf{K} \rightarrow$ Generators of
 $\text{SO}(4)$ rotation group.



- As in a classical Keplerian orbit: $\mathbf{L} \cdot \mathbf{K} = \mathbf{K} \cdot \mathbf{L} = 0$.
- For energy eigenstates: $\mathbf{L}^2 + \mathbf{K}^2 = 1 - 1/2H = n^2 - 1$.
- Define: $\mathbf{J}_1 = (\mathbf{L} - \mathbf{K})/2, \quad \mathbf{J}_2 = (\mathbf{L} + \mathbf{K})/2$.
- $\mathbf{J}_1, \mathbf{J}_2$ have the commutation relations of two independent three dimensional angular momentum operators.

$$\text{SO}(4) = \text{SO}(3) \otimes \text{SO}(3)$$
- For energy eigenstates: $\mathbf{J}_1^2 = \mathbf{J}_2^2 = (n^2 - 1)/4$.



Coherent States of SO(3)

Coherent states of 3D angular momentum obey:

$$\mathbf{u} \cdot \mathbf{J} |J, \mathbf{u}\rangle = j |J, \mathbf{u}\rangle$$

These states have **minimal dispersion**: $\Delta \mathbf{J} = \sqrt{\langle \mathbf{J}^2 \rangle - \langle \mathbf{J} \rangle^2} = \sqrt{j}$.

Coherent states of SO(3) are obtained by a rotation of a fiducial coherent state, which is an eigenstate of J_z

$$|J, \mathbf{u}(\theta, \phi)\rangle = e^{-iL_z\phi} e^{-iL_z\theta} |jj\rangle.$$



Quantum elliptic states = coherent states of SO(4)

- SO(4) coherent states are obtained as a direct product of two SO(3) subgroups coherent states

$$|n, \mathbf{u}_1 \mathbf{u}_2\rangle = |J_1, \mathbf{u}_1\rangle \otimes |J_2, \mathbf{u}_2\rangle, \quad j_1 = j_2 = (n-1)/2.$$

- For these states, the dispersion of $L^2 + K^2$ is minimal:

$$(\Delta \mathbf{L})^2 + (\Delta \mathbf{K})^2 = (\Delta \mathbf{J}_1)^2 + (\Delta \mathbf{J}_2)^2 = 2(n-1).$$



Corresponding classical orbit

Define the following unit vectors:

$$\mathbf{l} = \frac{\mathbf{u}_1 + \mathbf{u}_2}{|\mathbf{u}_1 + \mathbf{u}_2|}, \quad \mathbf{k} = \frac{\mathbf{u}_1 - \mathbf{u}_2}{|\mathbf{u}_1 - \mathbf{u}_2|}, \quad \mathbf{w} = \mathbf{k} \times \mathbf{l}.$$

The expectation values of the components of $\mathbf{K}$ and $\mathbf{L}$ along the directions of $\mathbf{k}$, $\mathbf{l}$ and $\mathbf{w}$ are

$$\langle K_k \rangle = (n-1) \sin \zeta, \quad \text{where } \langle K_k \rangle = \mathbf{k} \cdot \mathbf{K}, \text{ etc.}$$

$$\langle L_l \rangle = (n-1) \cos \zeta.$$

ζ = Half the angle between $\mathbf{u}_1$ and $\mathbf{u}_2$



- In the perpendicular directions the expectation values vanish:

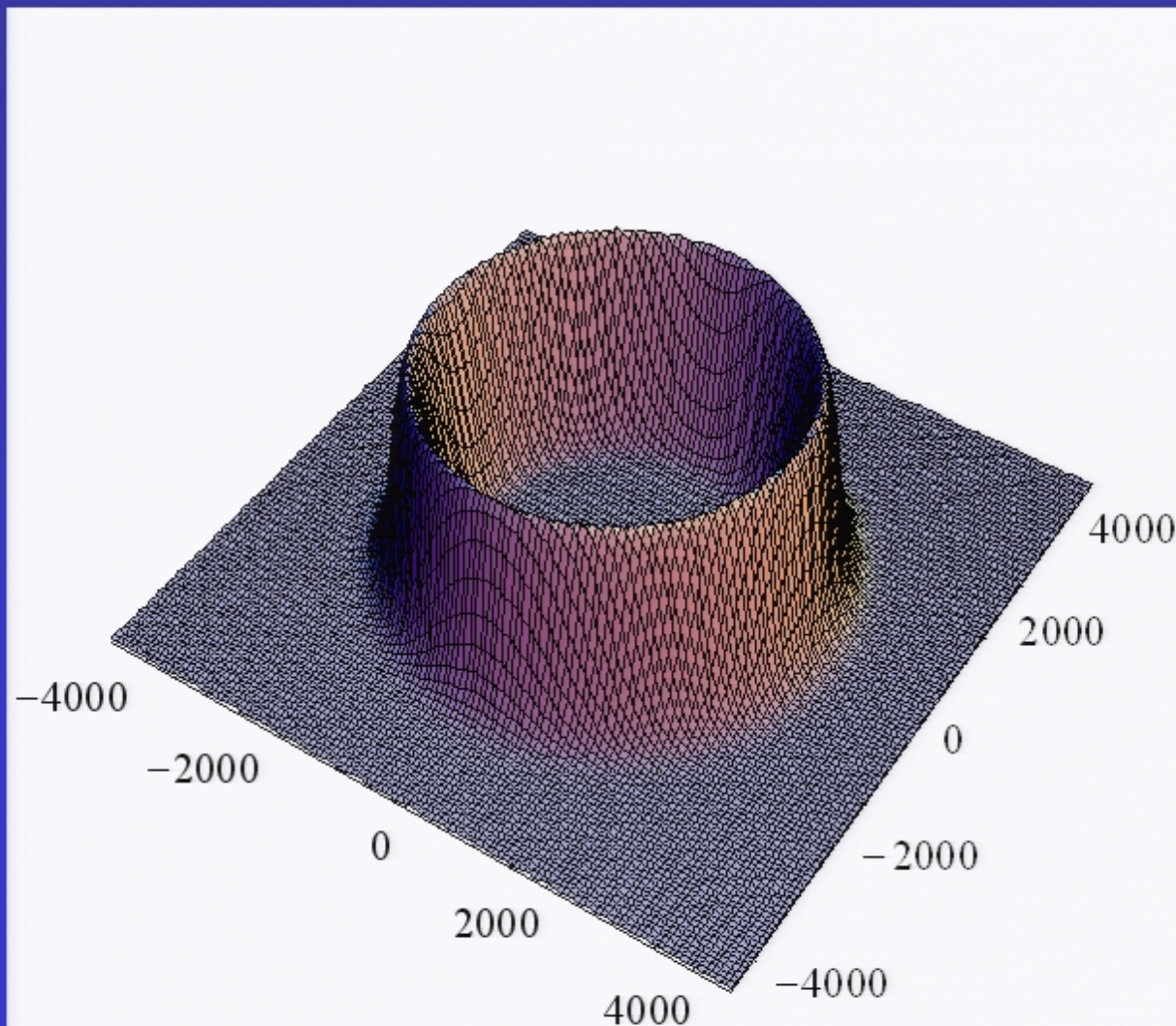
$$\langle K_l \rangle = \langle L_k \rangle = \langle K_w \rangle = \langle L_w \rangle = 0.$$

- The coherent state $|n, \mathbf{u}_1 \mathbf{u}_2\rangle$ corresponds to a classical orbit in the $\mathbf{k}\mathbf{w}$ plane with
 - major axis in the $\mathbf{k}$ direction,
 - angular momentum in the l direction,
 - eccentricity $e = \sin \zeta$.



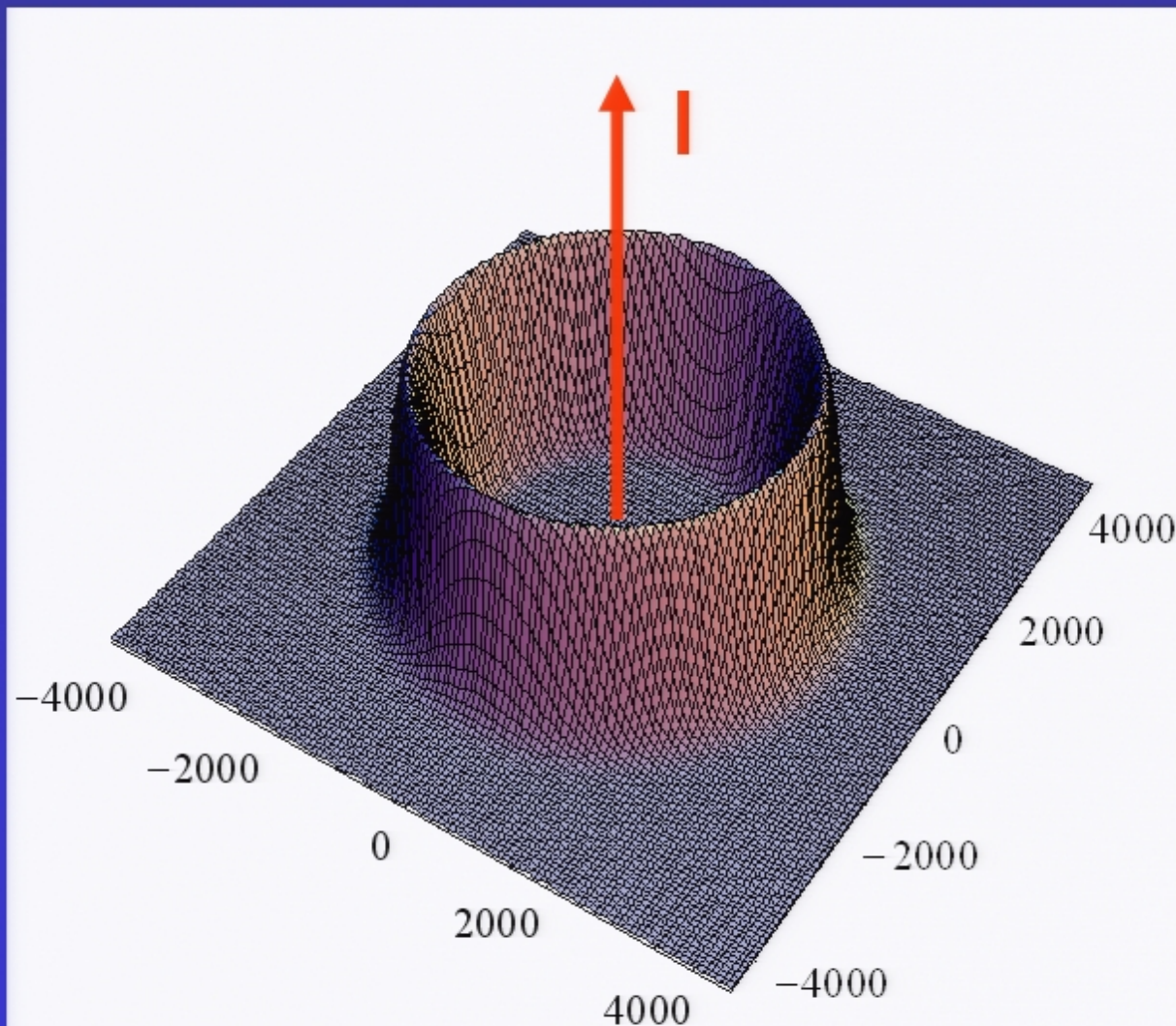
Circular state, xy -plane

$|\psi|^2$ for $n=50, e=0, l=z$



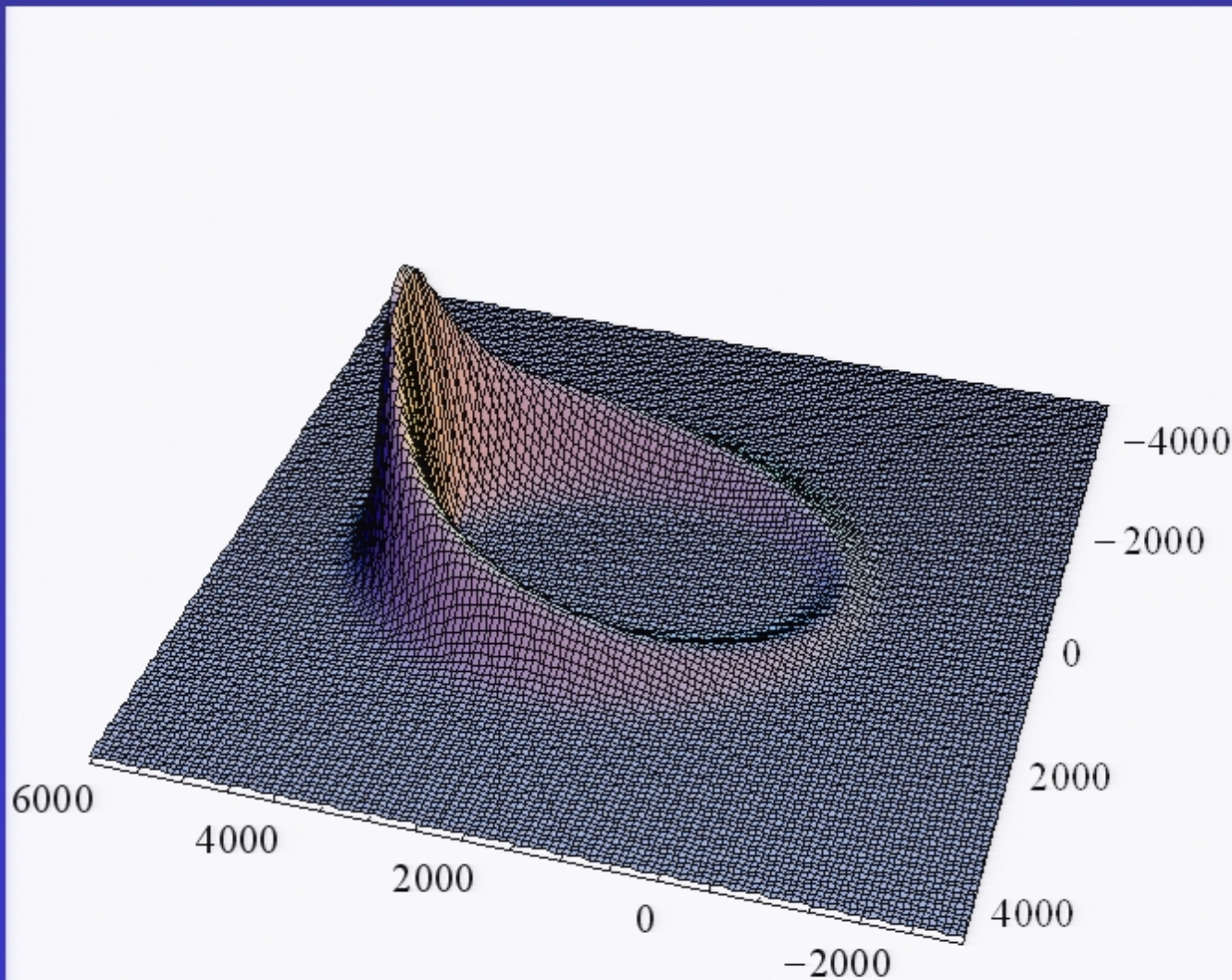
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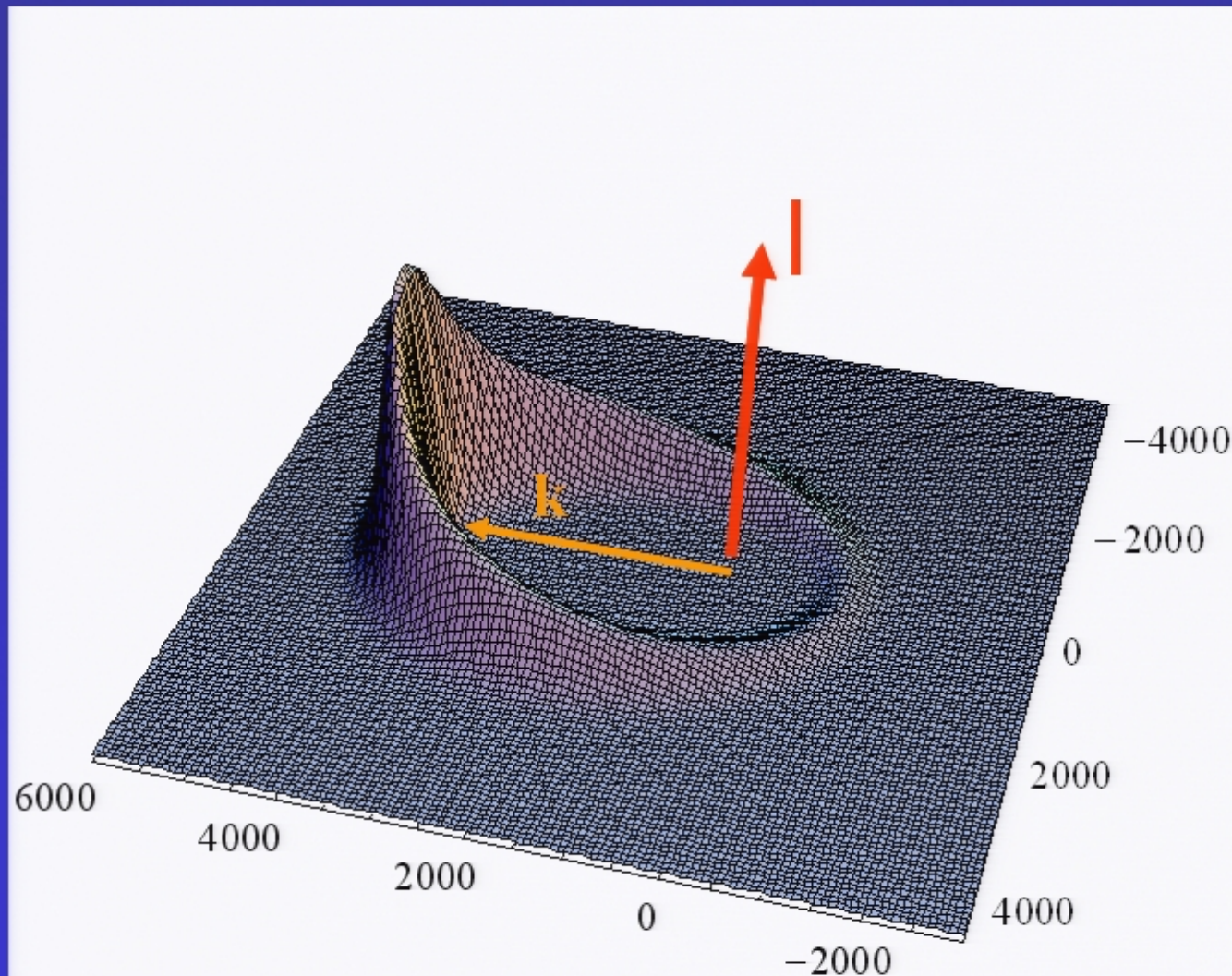
Generic elliptic state, xy -plane

$|\psi|^2$ for $n=50$, $e=0.7$, $k=x$, $l=z$



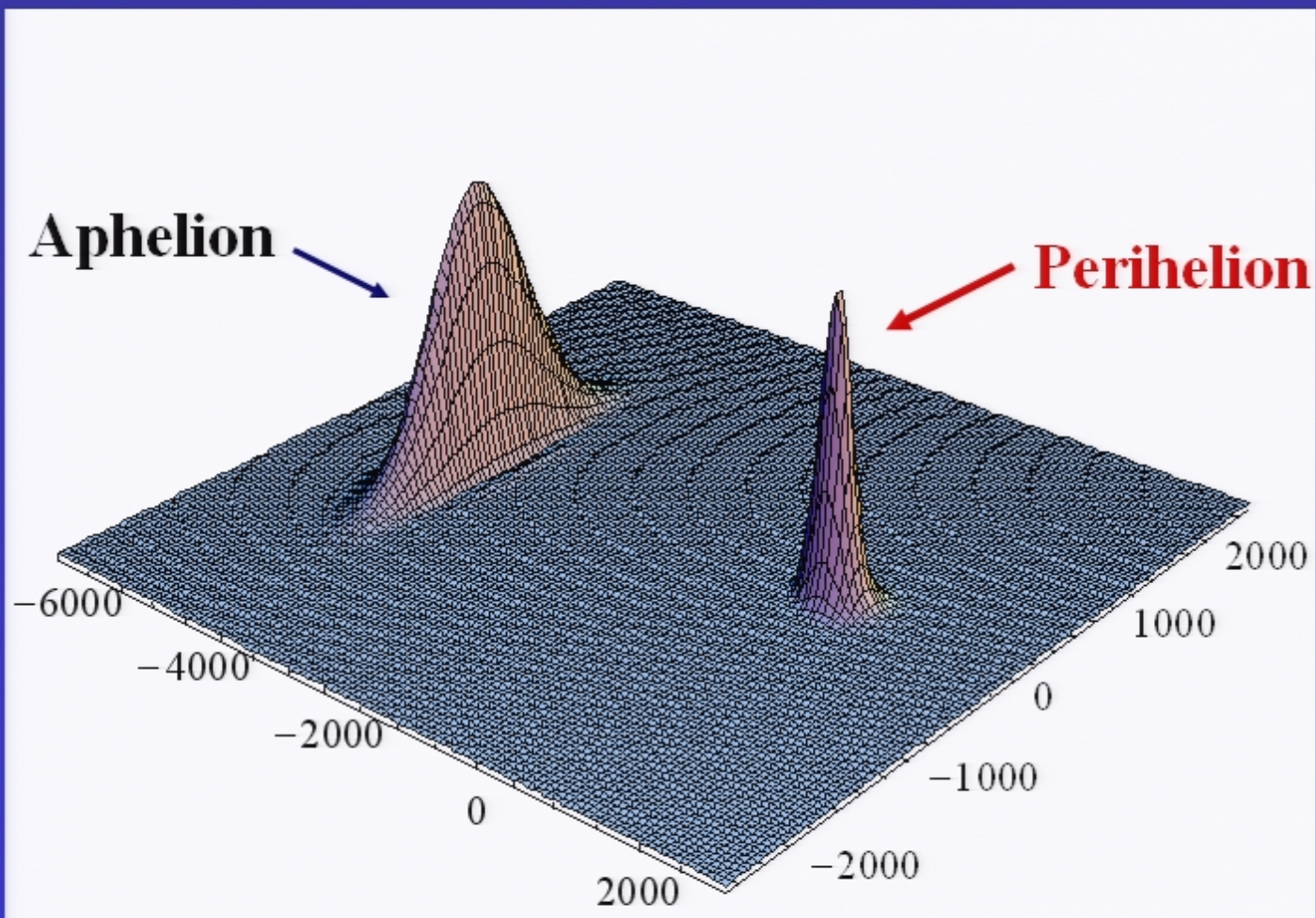
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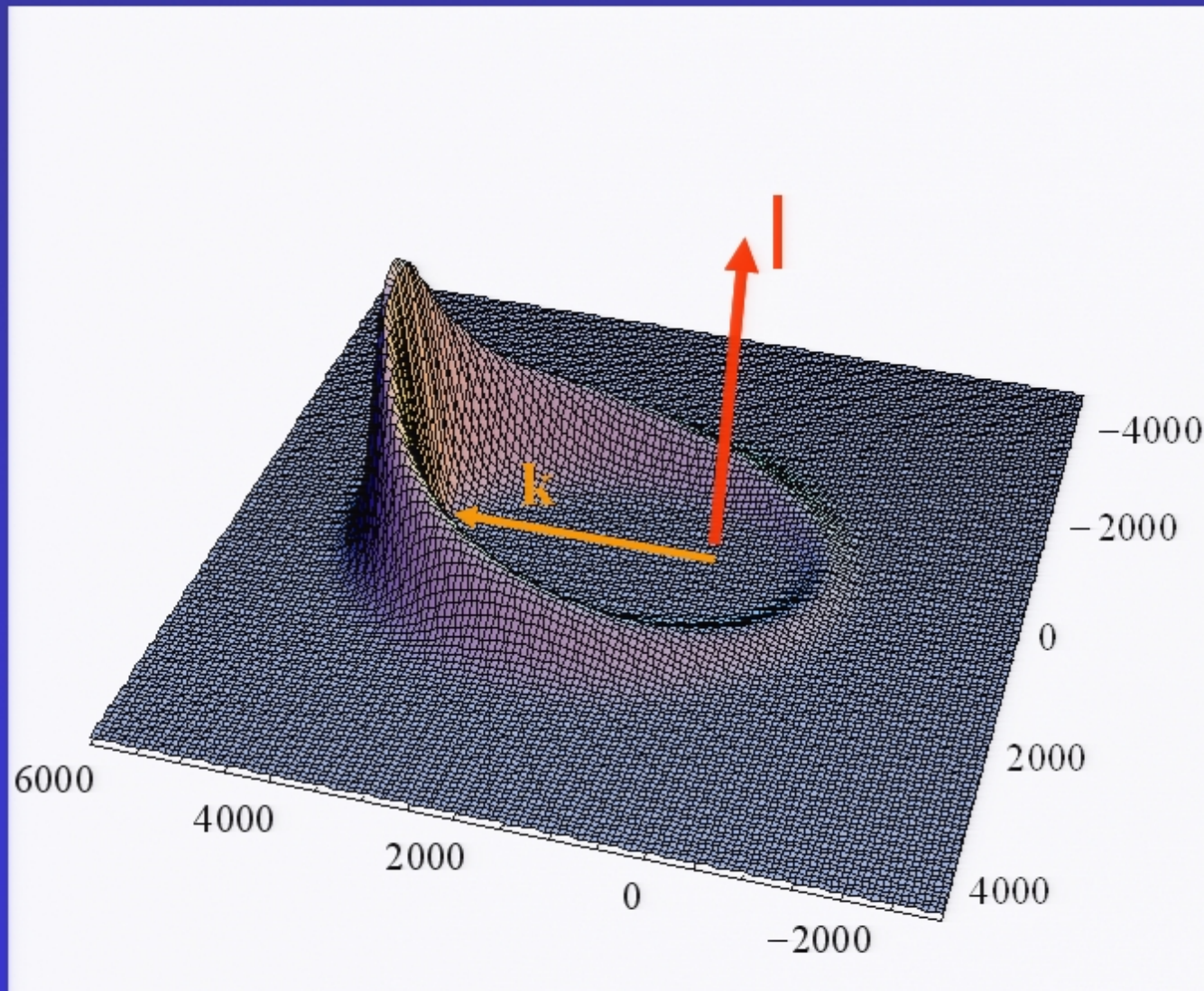
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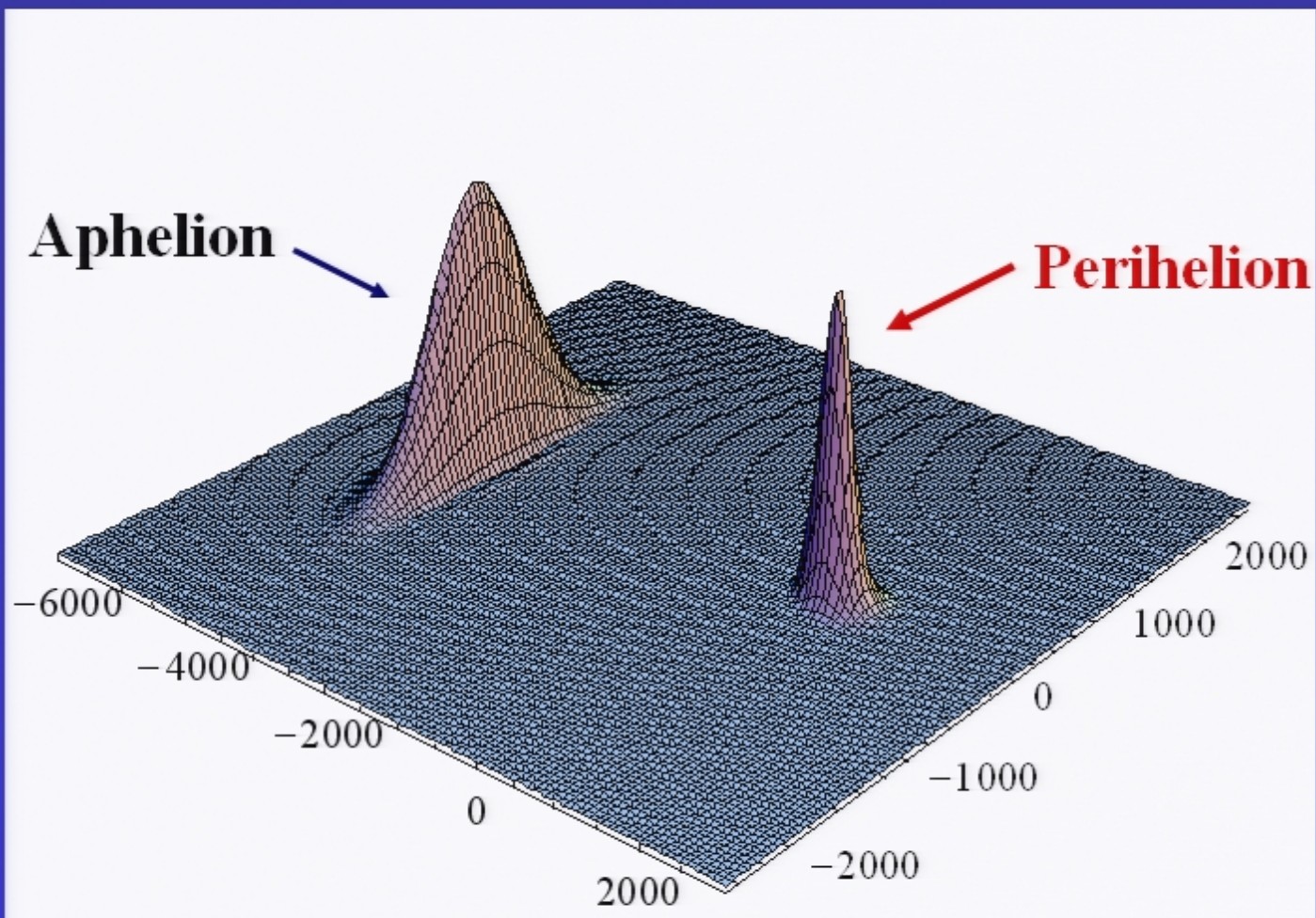
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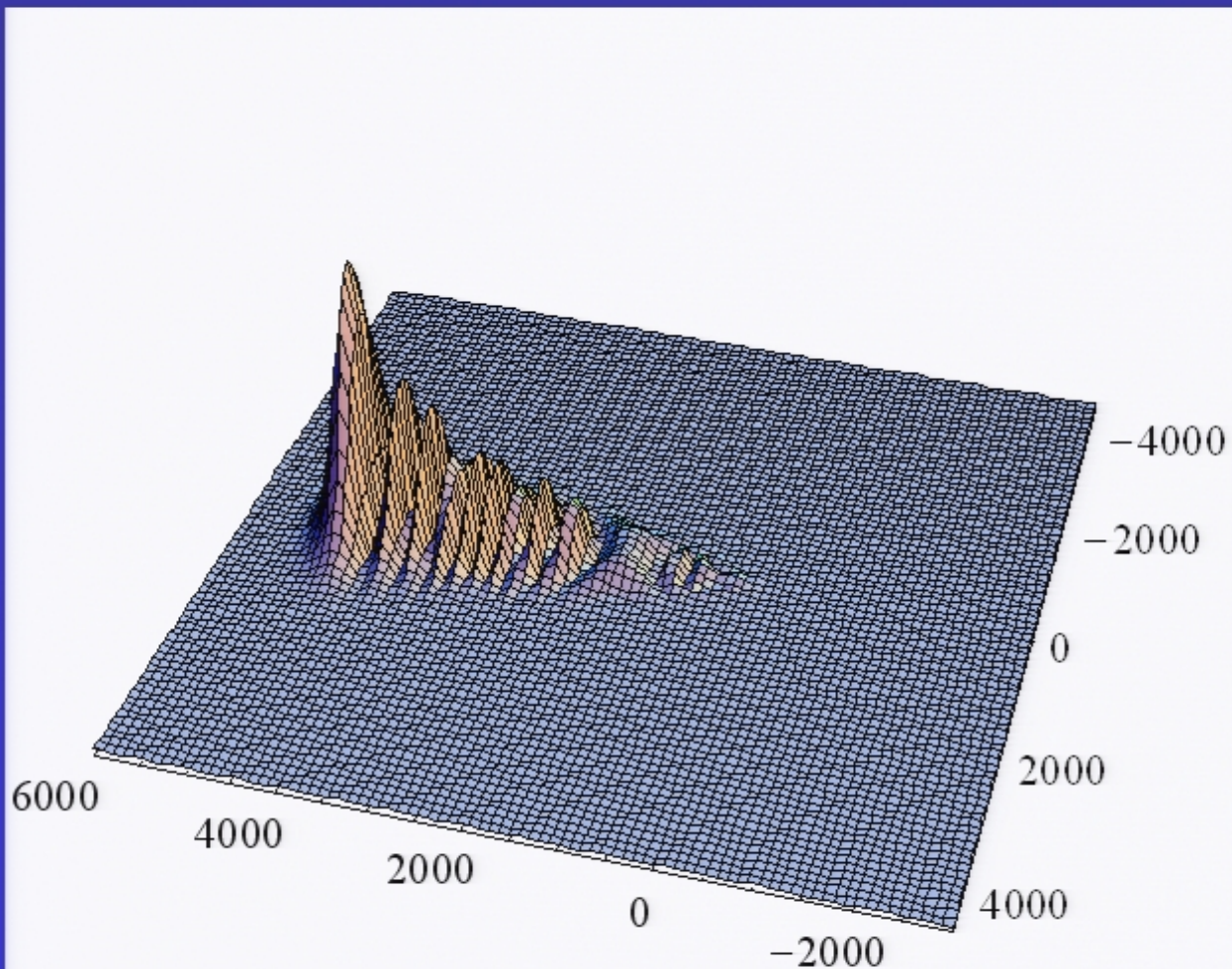
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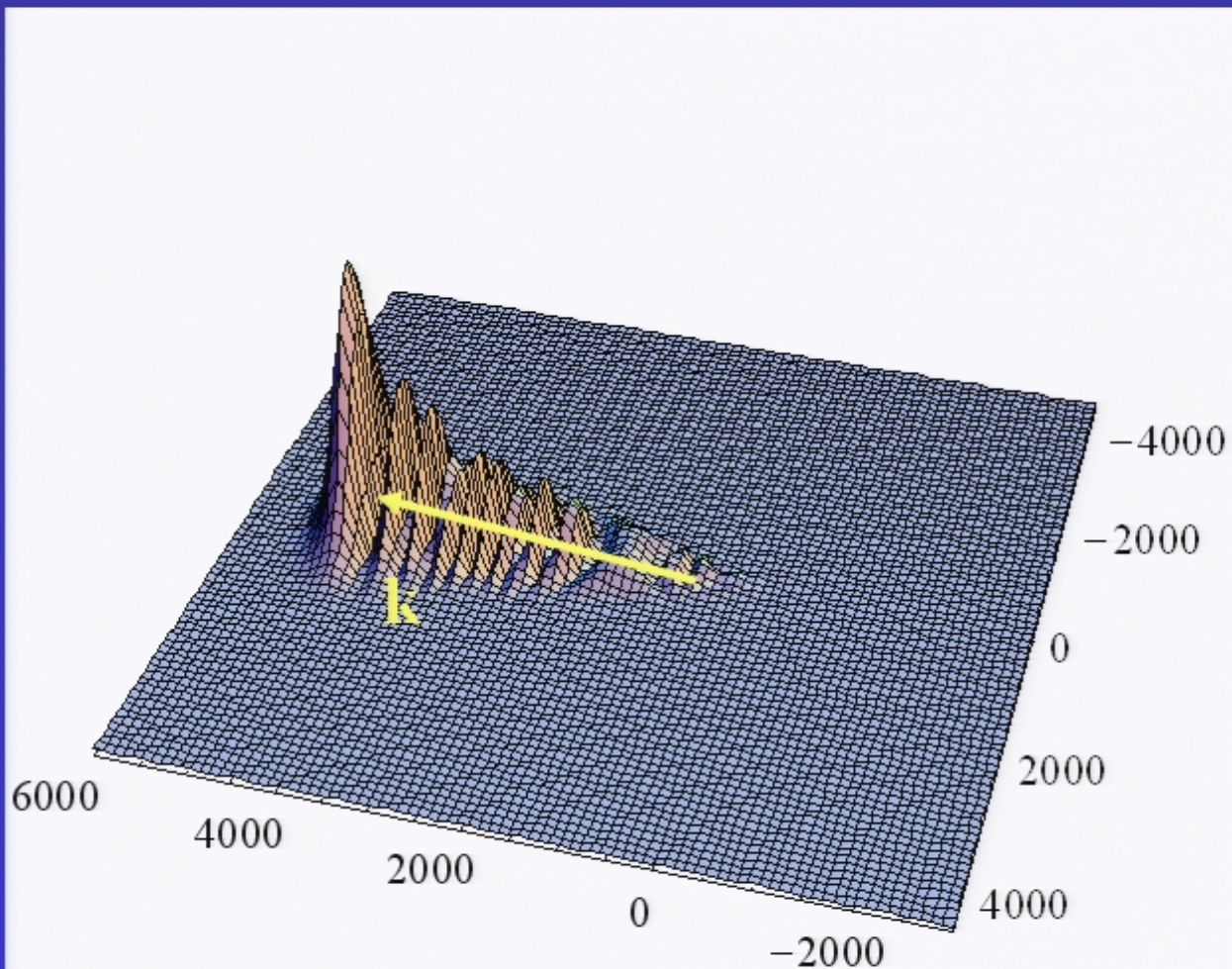
Extreme Stark state, xy -plane

$|\psi|^2$ for $e=1$, $k=x$



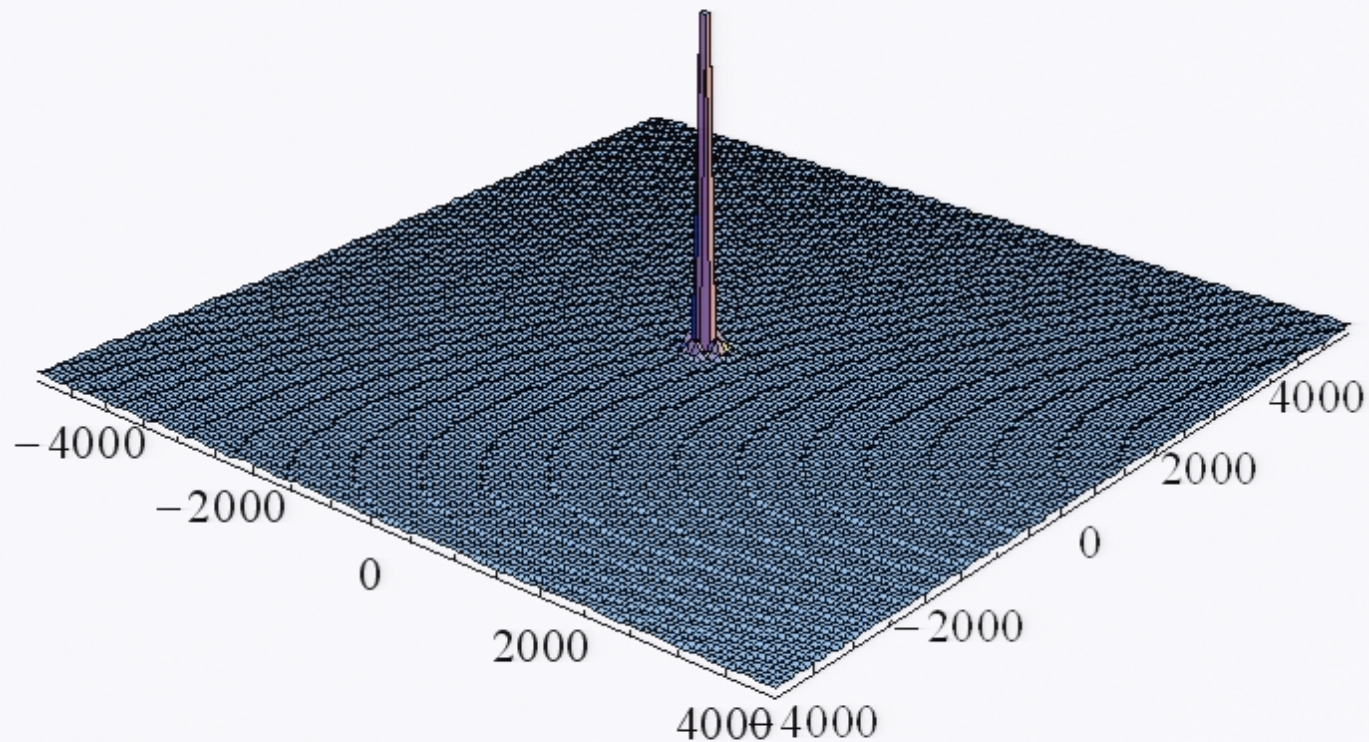
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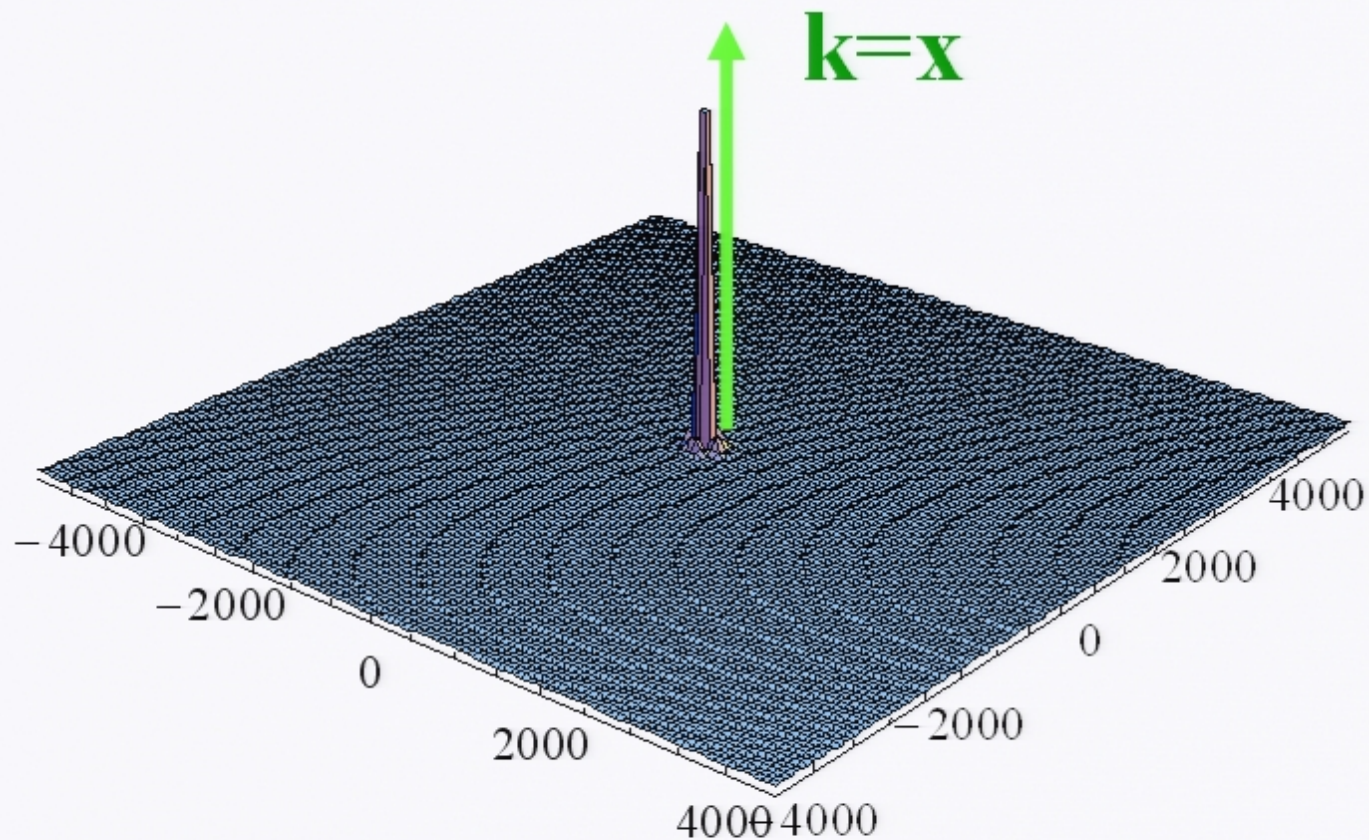
Extreme Stark state, zy -plane

$|\psi|^2$ for $n=50, e=1, \mathbf{k}=\mathbf{x}$



Extreme Stark state, zy -plane

$|\psi|^2$ for $n=50$, $e=1$, $\mathbf{k}=\mathbf{x}$



Transmission of One Direction

- Taking the transmitted direction to be Alice's z-axis

$$\langle \cos \omega_z \rangle = \int d_{\alpha\beta\gamma} \left| \langle A | U(\alpha\beta\gamma) | B \rangle \right|^2 \cos \beta.$$

- Consider the following cases:

- Circular State: $e = 0$, $l = z$, $\langle \mathbf{K} \rangle = 0$

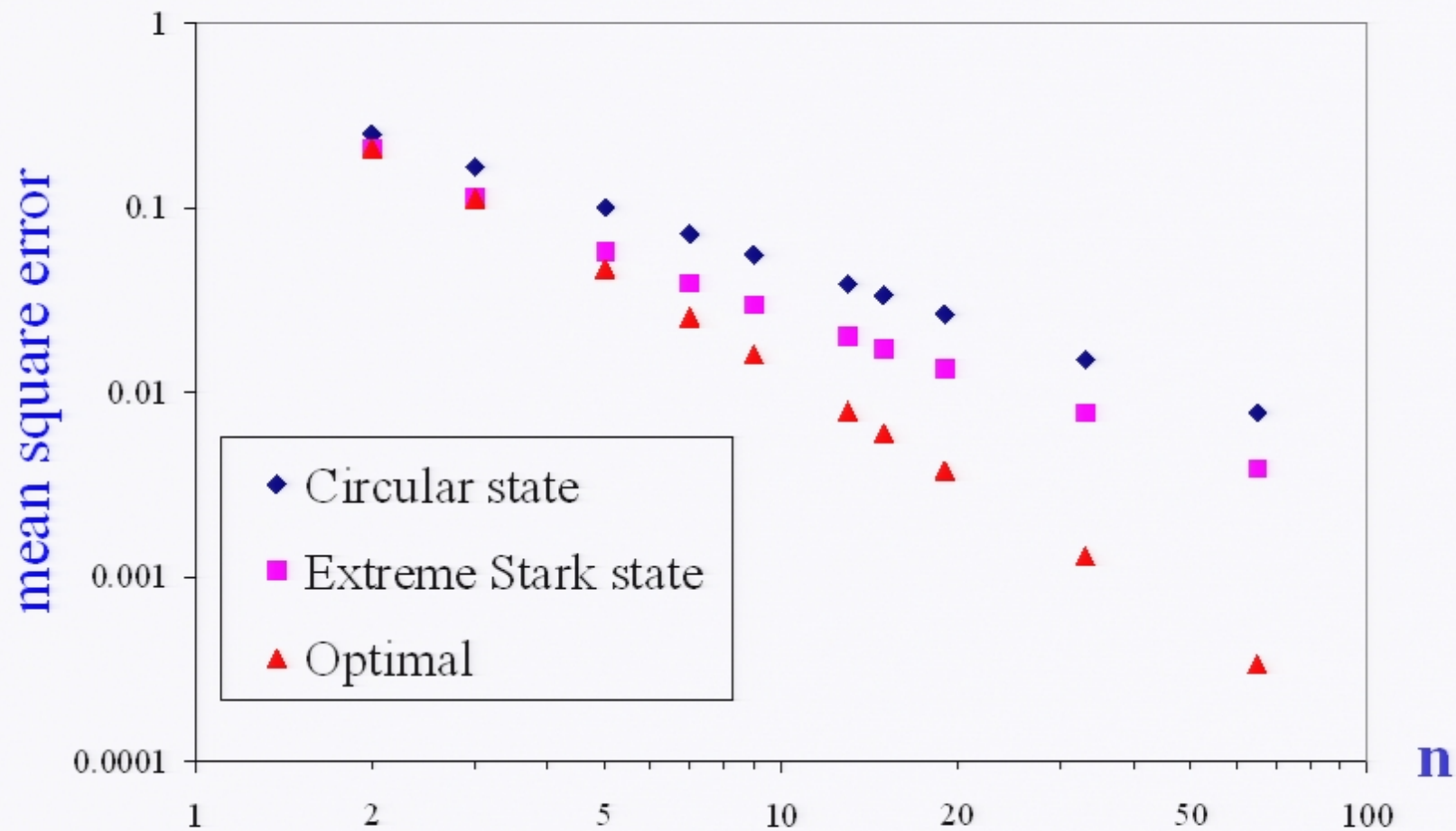
$$|A\rangle = |ll\rangle = |jj\rangle \otimes |jj\rangle$$

- Extreme Stark State: $e = 1$, $\mathbf{k} = z$, $\langle \mathbf{L} \rangle = 0$

$$|A\rangle = |j, -j\rangle \otimes |jj\rangle$$



Comparison of the mean square error $1-F$ to the optimal one



Transmission of two directions

- We choose for Alice an elliptic state with $\mathbf{k}=\mathbf{x}$, $\mathbf{l}=\mathbf{y}$, and eccentricity $0 < e < 1$. The optimal eccentricity should be

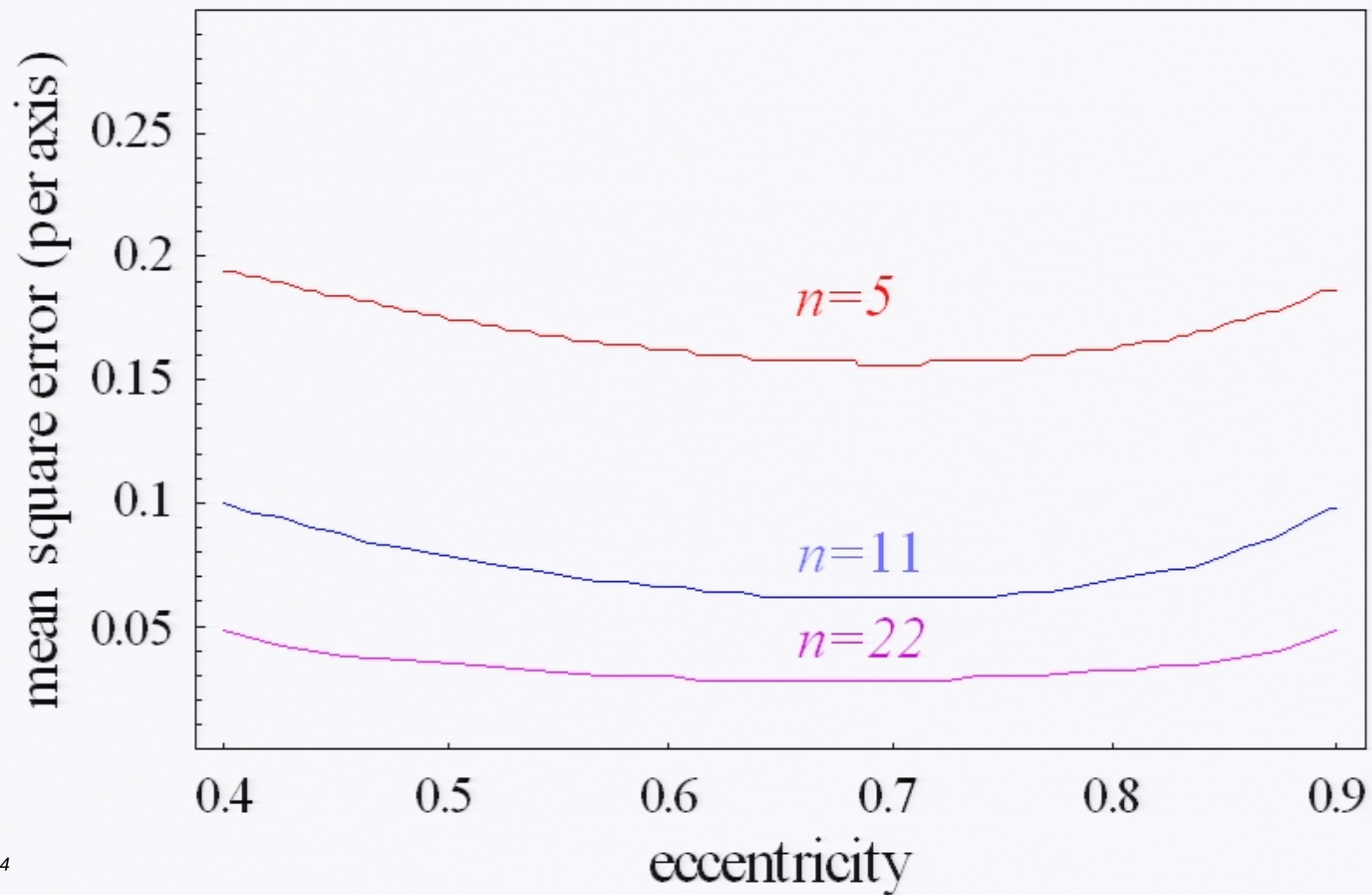
$$\frac{\Delta K_{\perp}}{\langle K_k \rangle} = \frac{\Delta L_{\perp}}{\langle L_l \rangle} \Rightarrow e \approx 1/\sqrt{2}.$$

- To calculate the fidelity for the transmission of the x and y axes we need to calculate

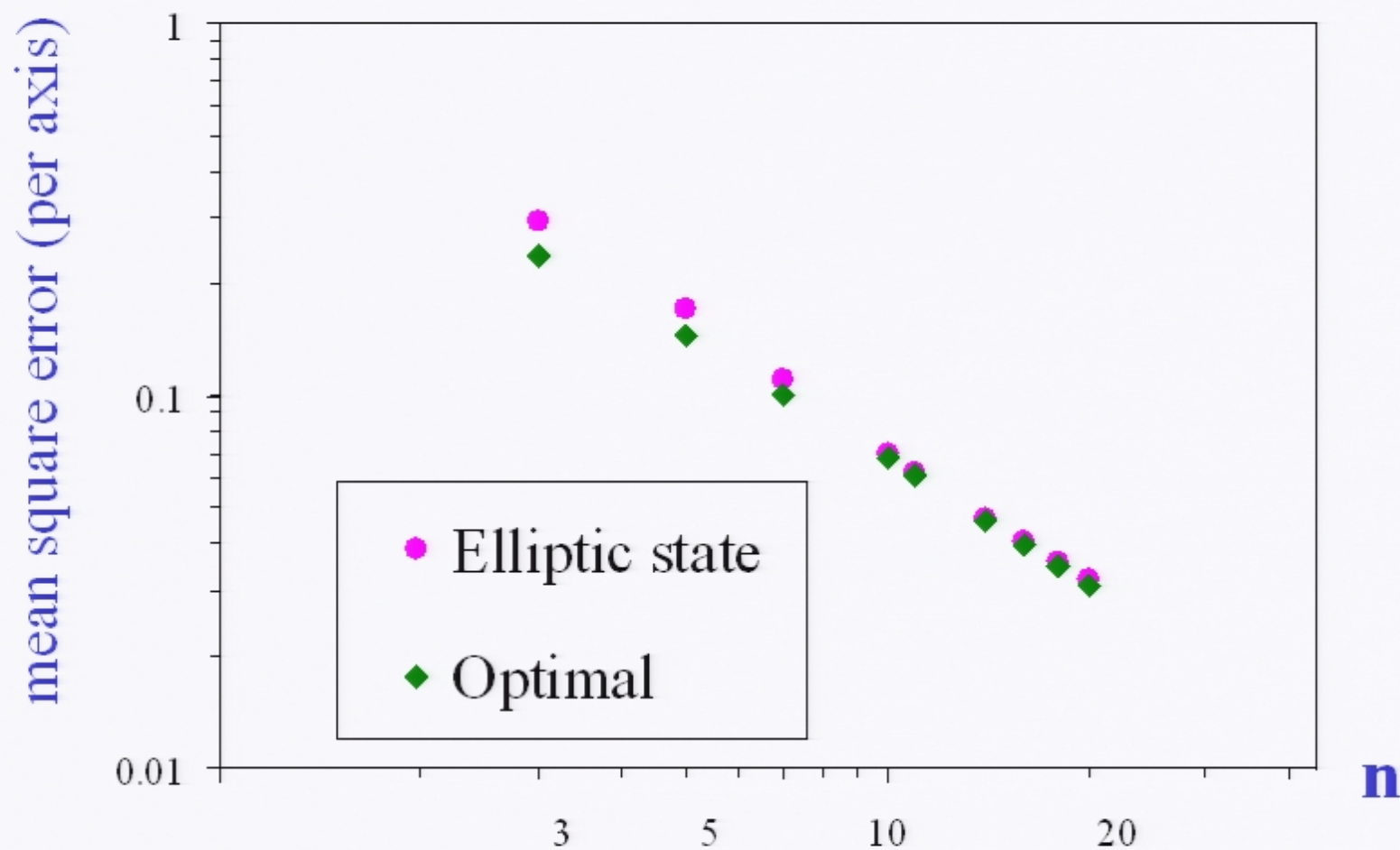
$$\langle \cos \omega_x + \cos \omega_y \rangle = \langle (1 + \cos \beta)(\cos \alpha + \cos \gamma) \rangle.$$



Mean square error vs. eccentricity for $n=5$, $n=11$, $n=22$

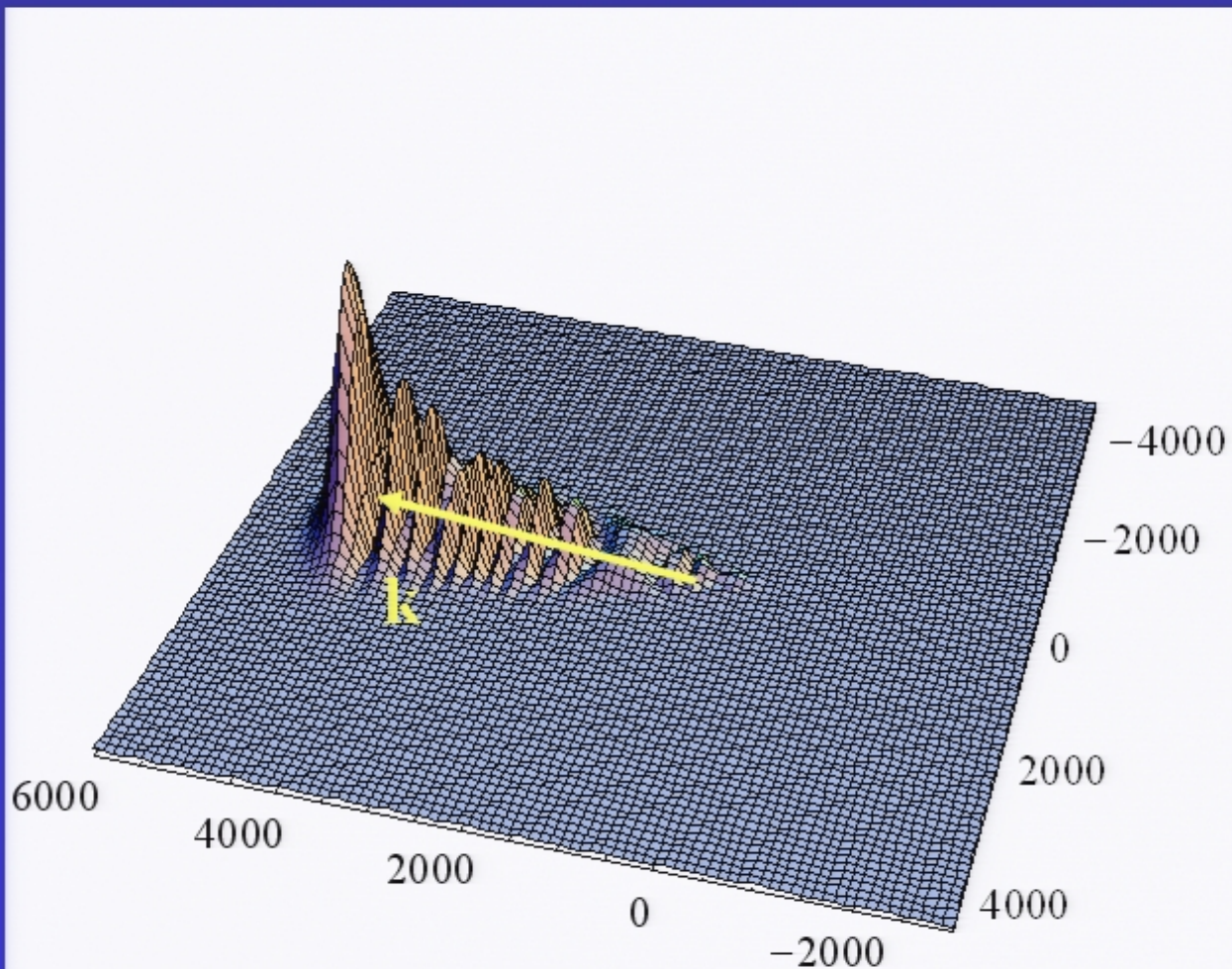


Comparison with the optimal transmission



Extreme Stark state, xy -plane

$|\psi|^2$ for $e=1$, $k=x$





The experimental situation

- Elliptic Rydberg states can be produced by exciting atoms in external orthogonal electric and magnetic field. J.C. Day et al, PRL. 72 (1994)
- Relativistic effects:
Energy depends on l , causes precession of the elliptic state. P. Rozmej et al, J. Phys A. 35 (2002)

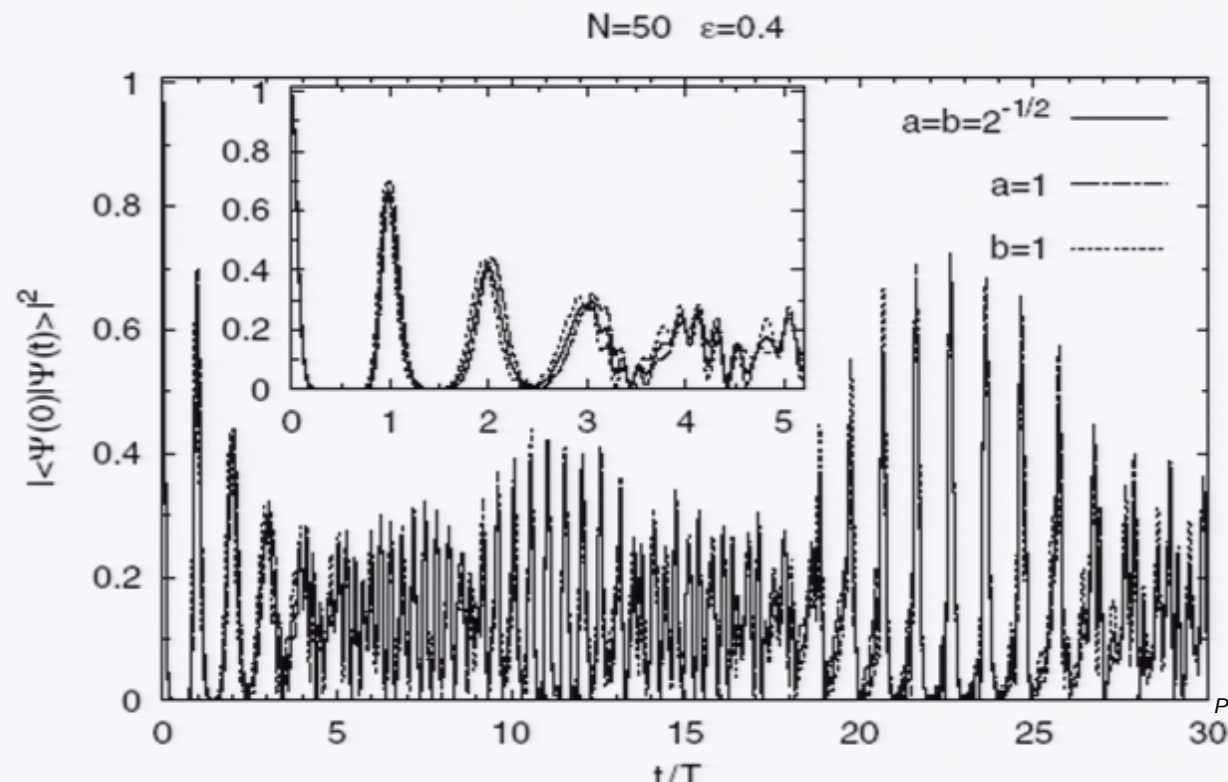


$$E_{nlj} = E - \frac{Z^4 \alpha^2}{2n^3(l+s+1/2)} \quad j = l + s$$

$$T_p = \frac{2\pi}{\left\langle \frac{dE_{nlj}}{dl} \right\rangle}$$

$$T_p = \frac{4\pi n^3 [(n-1)\cos\zeta]^2}{Z^4 \alpha^2}$$

P. Rozmej et al, J. Phys. A **35**, 7803 (2002)

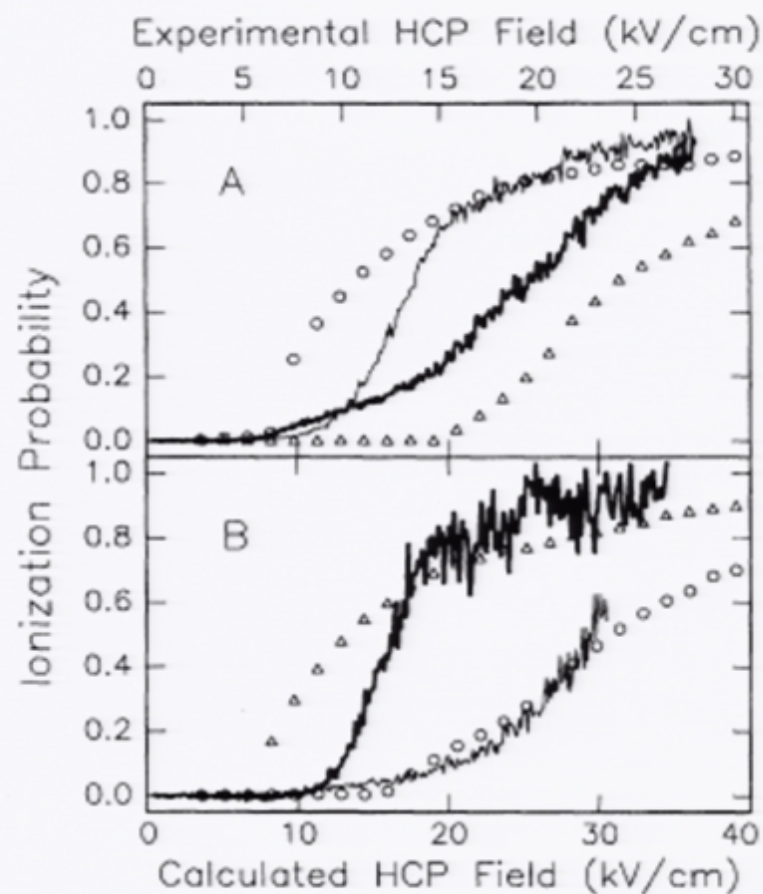


Characteristic Time Scales

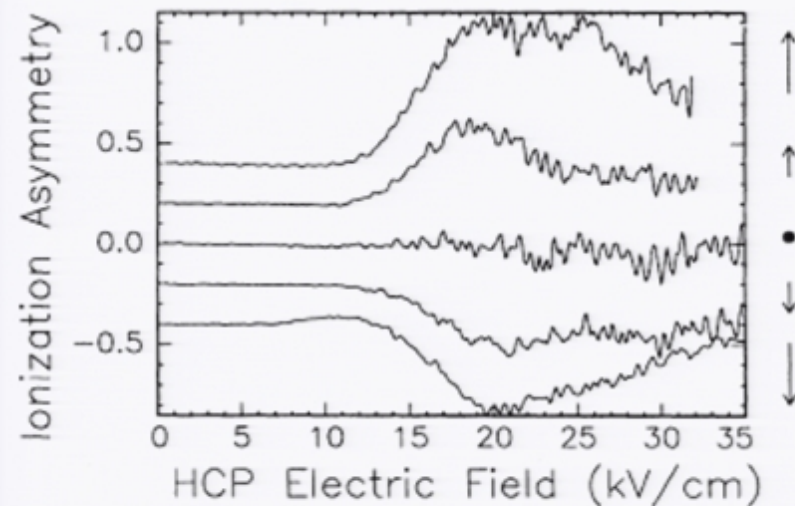
- For $Z=1$: $T_p \approx 1.4 \times 10^{-3} \text{ sec}$
- Spontaneous decay: $\frac{T_{n,l}^{\text{rad}}}{T_p} = \frac{3}{8\pi\alpha} \approx 16.3$



Probing the orientation of Rydberg states by an ionization pulse

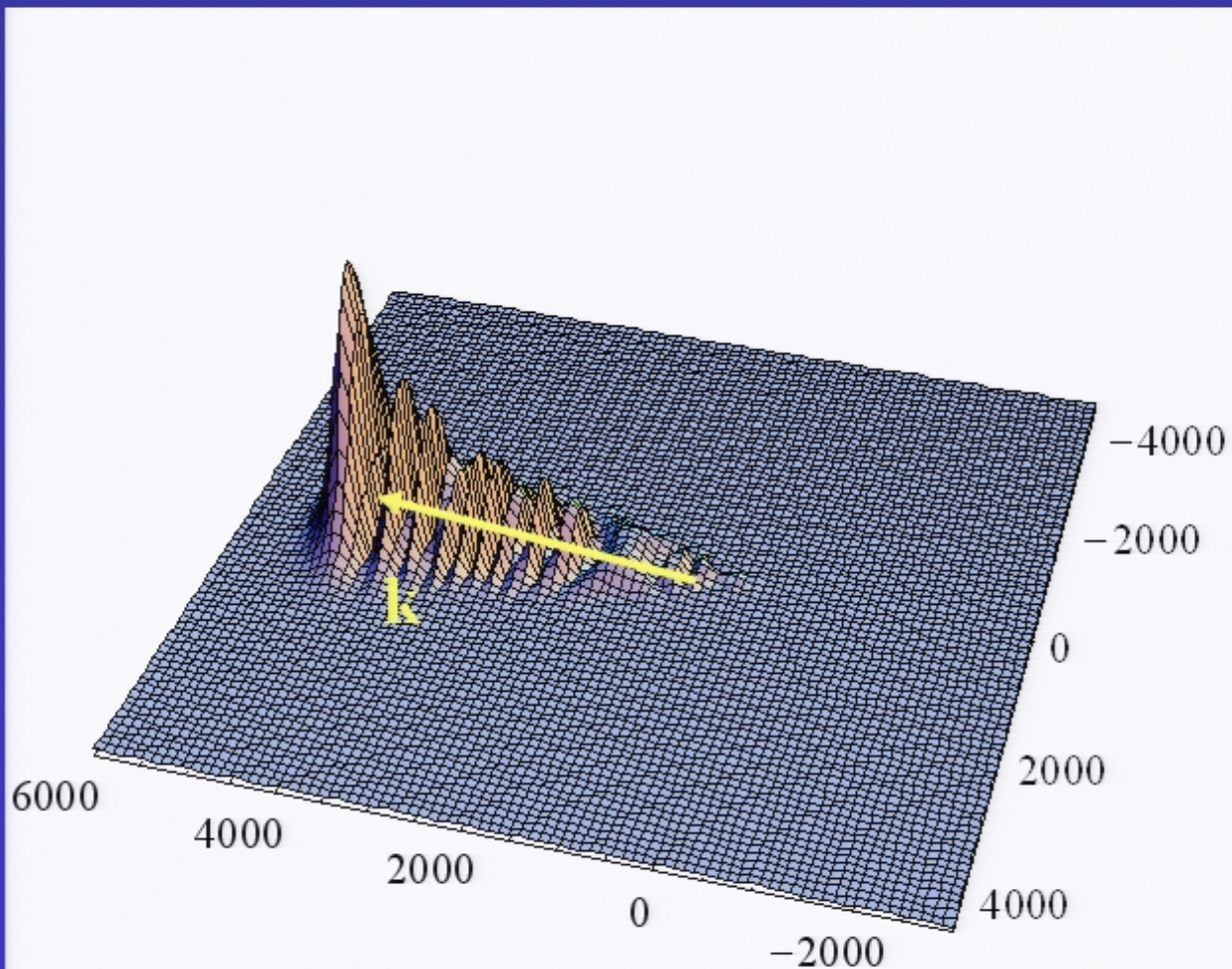


Jones et al, **PRA** 51 2687 (1995)

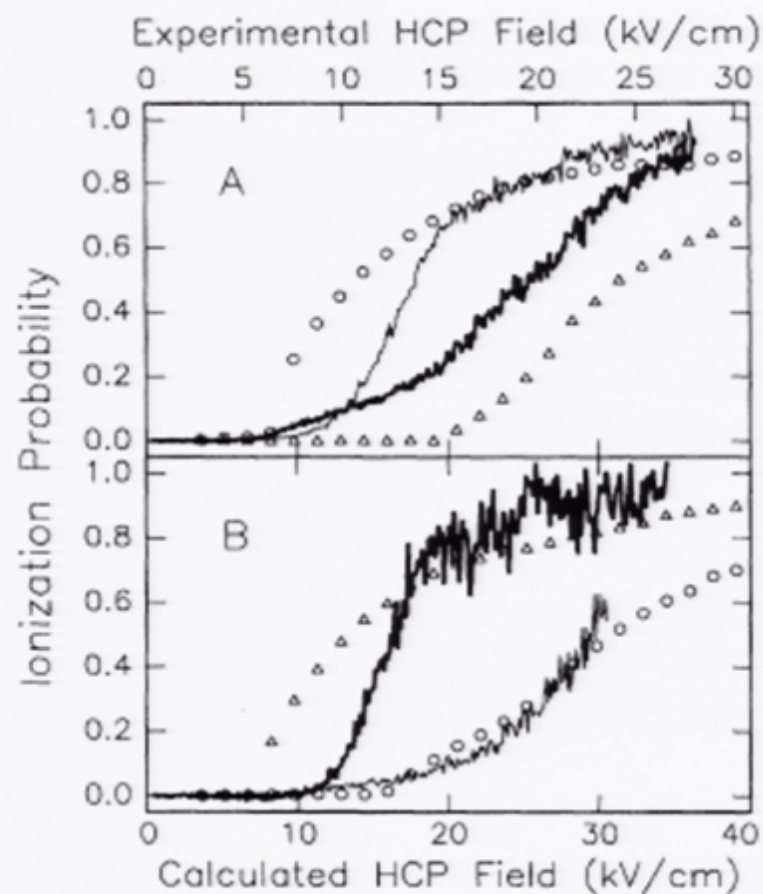


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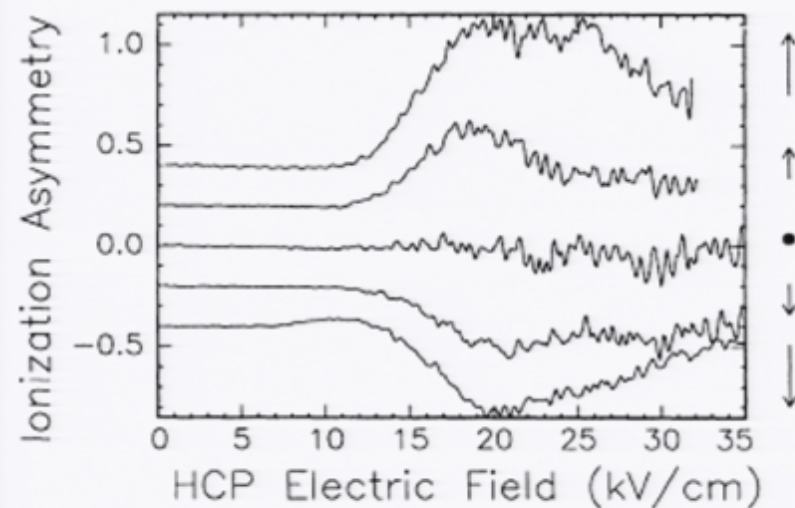
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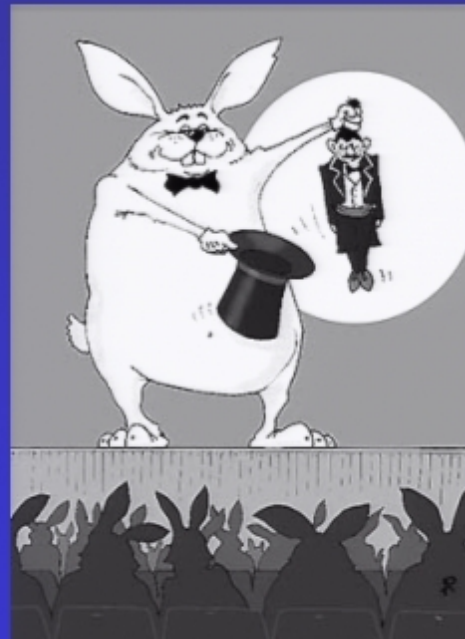
Summary

- For transmission of one direction, LRL vector is better than angular momentum.
- Elliptic states are not optimal for transmission of one axis, but are nearly optimal for frames.
- They are practical.



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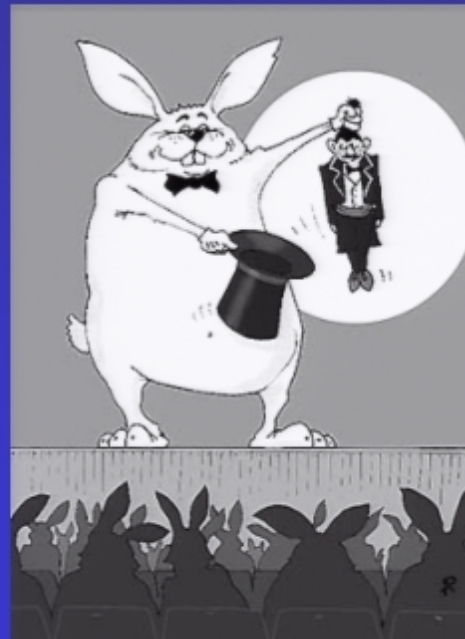
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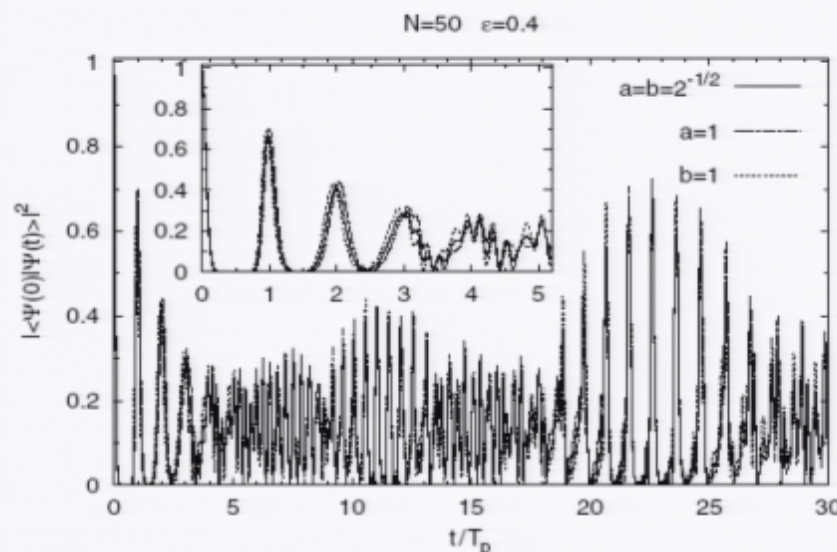
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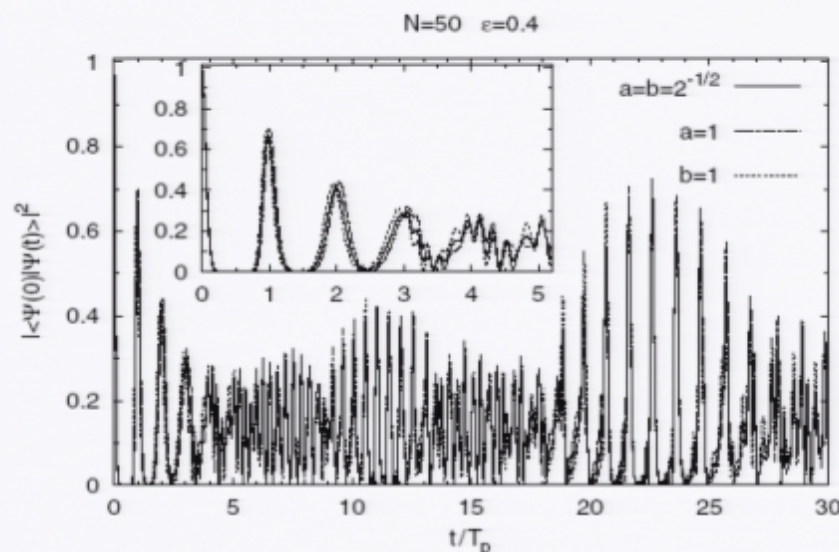
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- Spontaneous decay:

Probing the orientation of

33

Rydberg states

by an ionization pulse

Summary

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For transmission of one

- direction, LRL vector is better than angular momentum.

Elliptic states are not optimal

- for transmission of one axis, but are nearly optimal for frames.

They are practical.

-

Click to add notes

Probing the orientation of Rydberg states by an ionization pulse

Outline Slides

magnetic field. J.C. Day et al, PRL 72 (1994)

Relativistic effects:
 Energy depends on l , causes precession of the elliptic state.
 P. Rozmej et al, J. Phys A. 35 (2002)

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Characteristic Time Scales

32

For $Z=1$:
 Spontaneous decay:

Probing the orientation of Rydberg states

33

by an ionization pulse

Summary

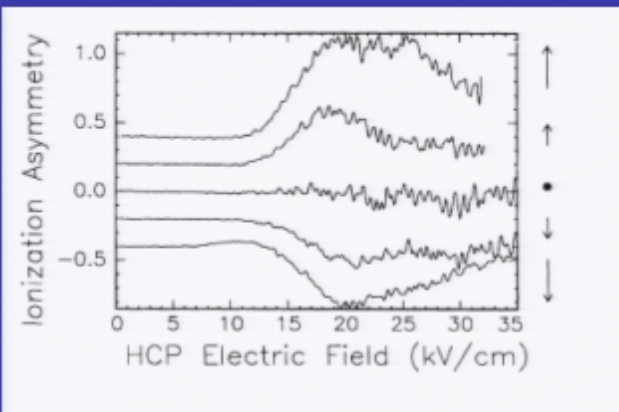
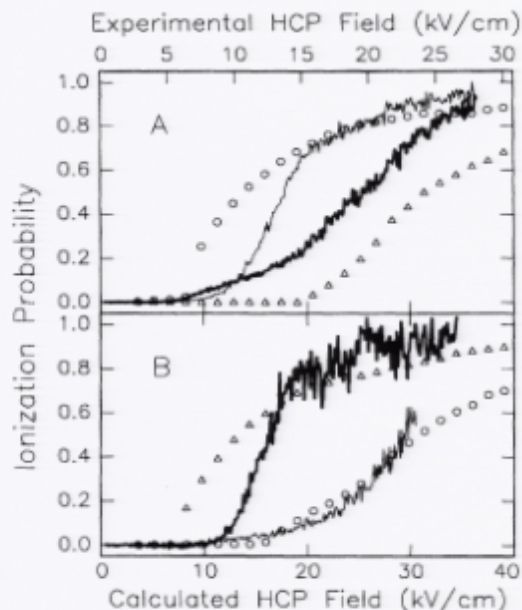
34

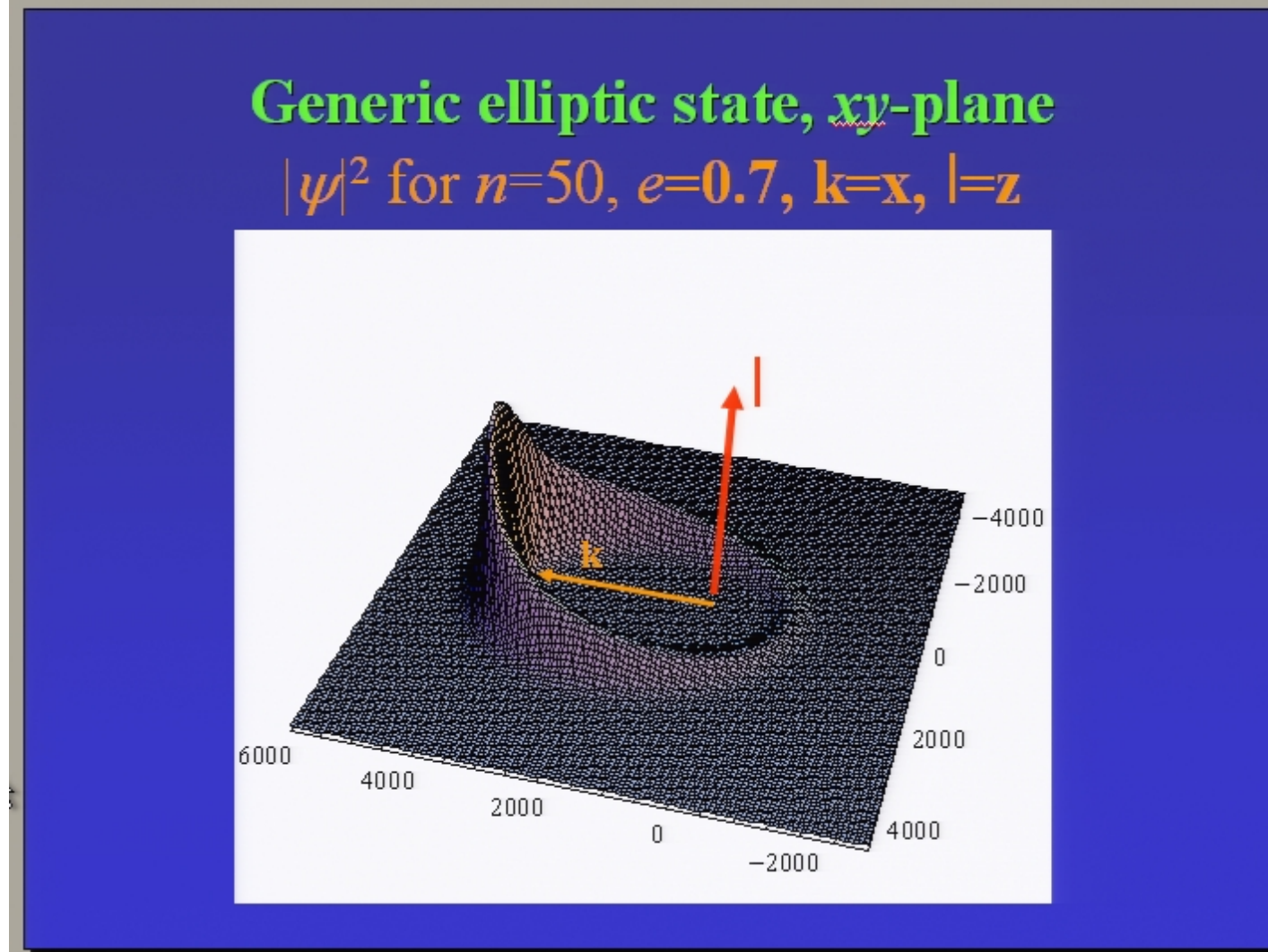
For transmission of one direction, LRL vector is better than angular momentum.

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Jones et al, PRA 51 2687 (1995)



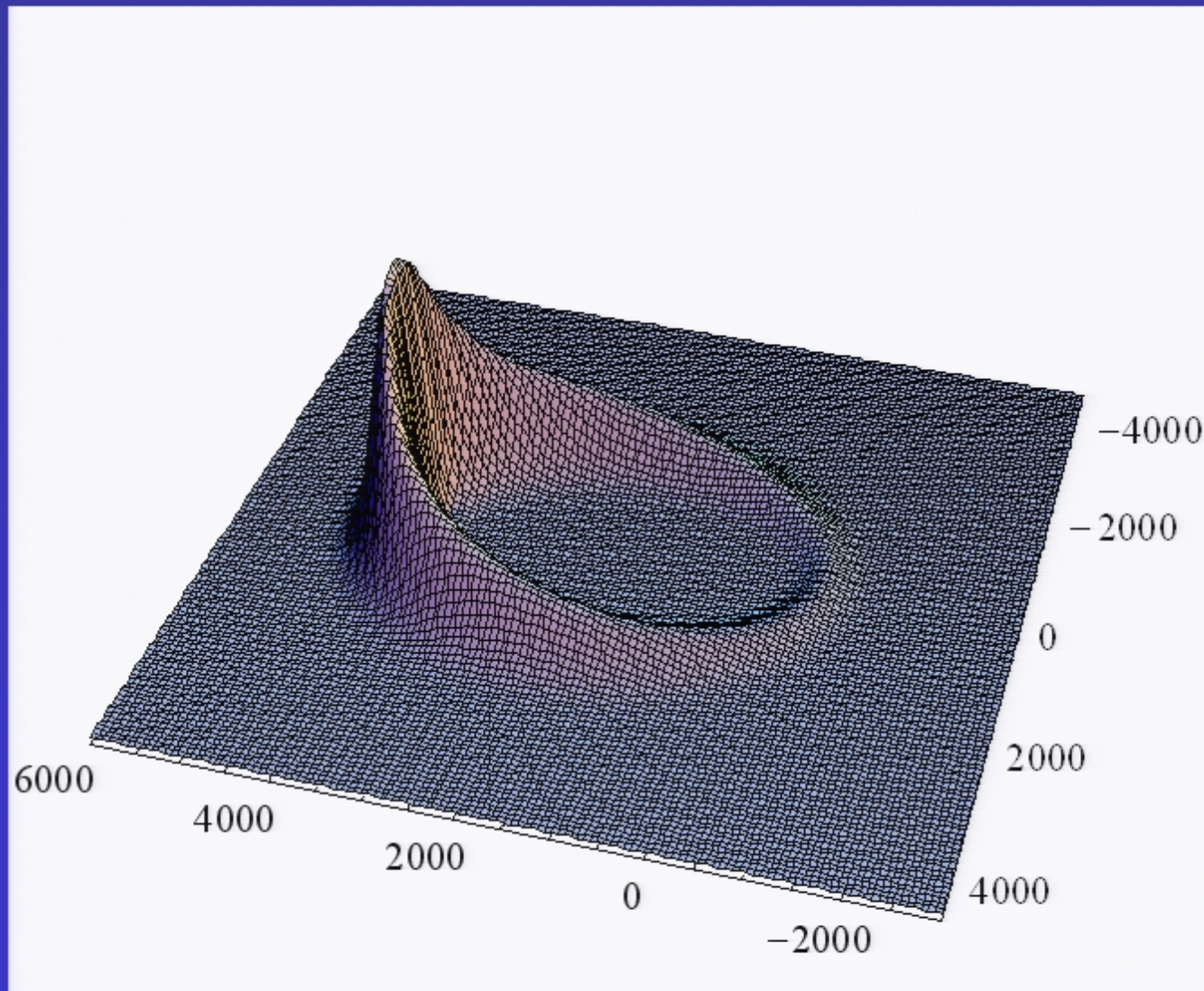


Outline Slides

- Generic elliptic state, xy -plane 21
 - $|\psi|^2$ for $n=50, e=0.7, k=x, l=z$
- Generic elliptic state, xz -plane 22
 - $|\psi|^2$ for $n=50, e=0.7, k=x, l=z$
- Extreme Stark state, xy -plane 23
 - $|\psi|^2$ for $e=1, k=x$
- Extreme Stark state, zy -plane 24
 - $|\psi|^2$ for $n=50, e=1, k=x$
- Transmission of One Direction 25
 - Taking the transmitted direction • to be Alice's z -axis
- Consider the following cases: •
- Comparison of the mean square error $1-F$ to the optimal one 26
- Transmission of two directions 27
- We choose for Alice an elliptic • state with $k=x, l=y$, and eccentricity $0 < e < 1$. The optimal eccentricity should be

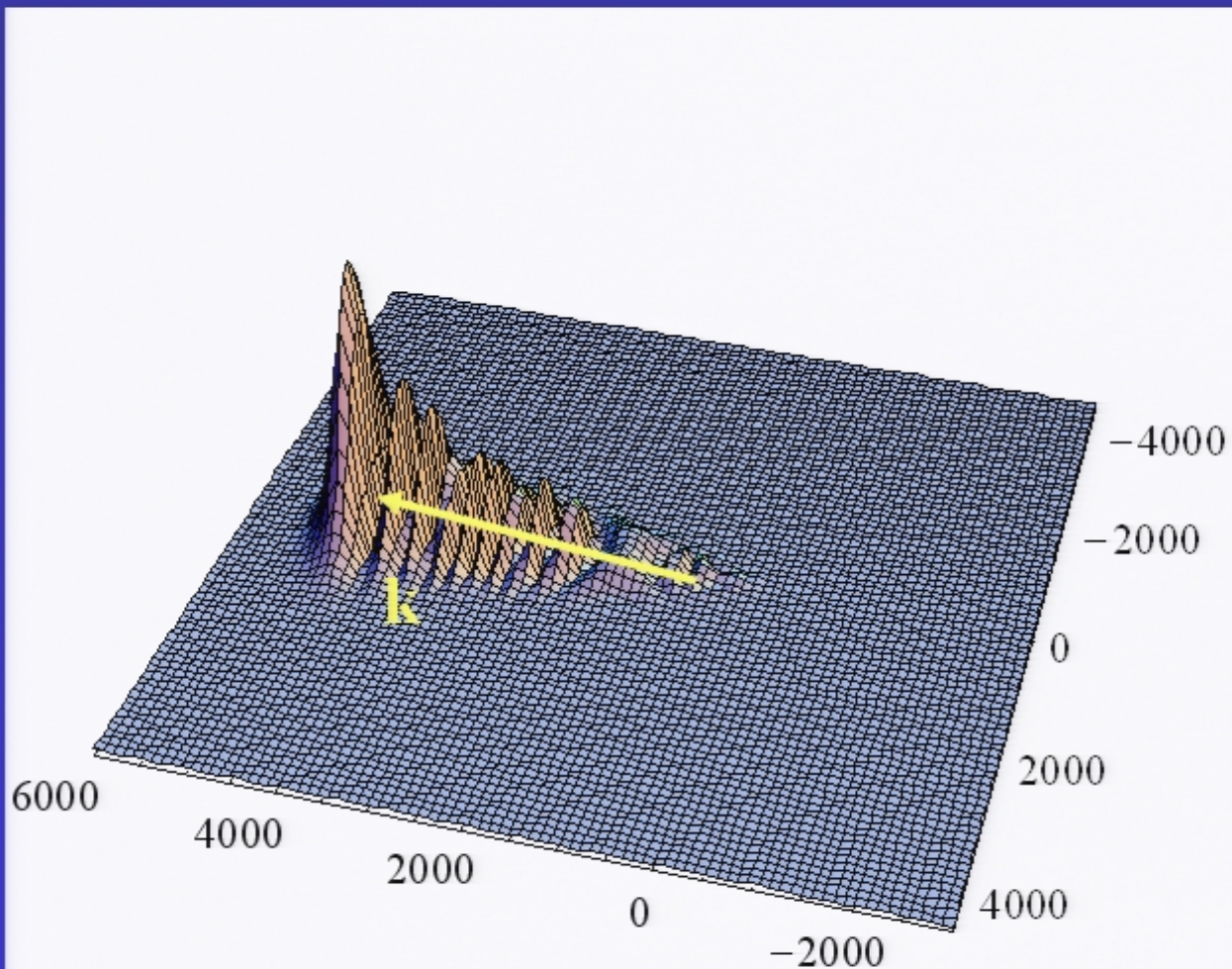
Generic elliptic state, xy -plane

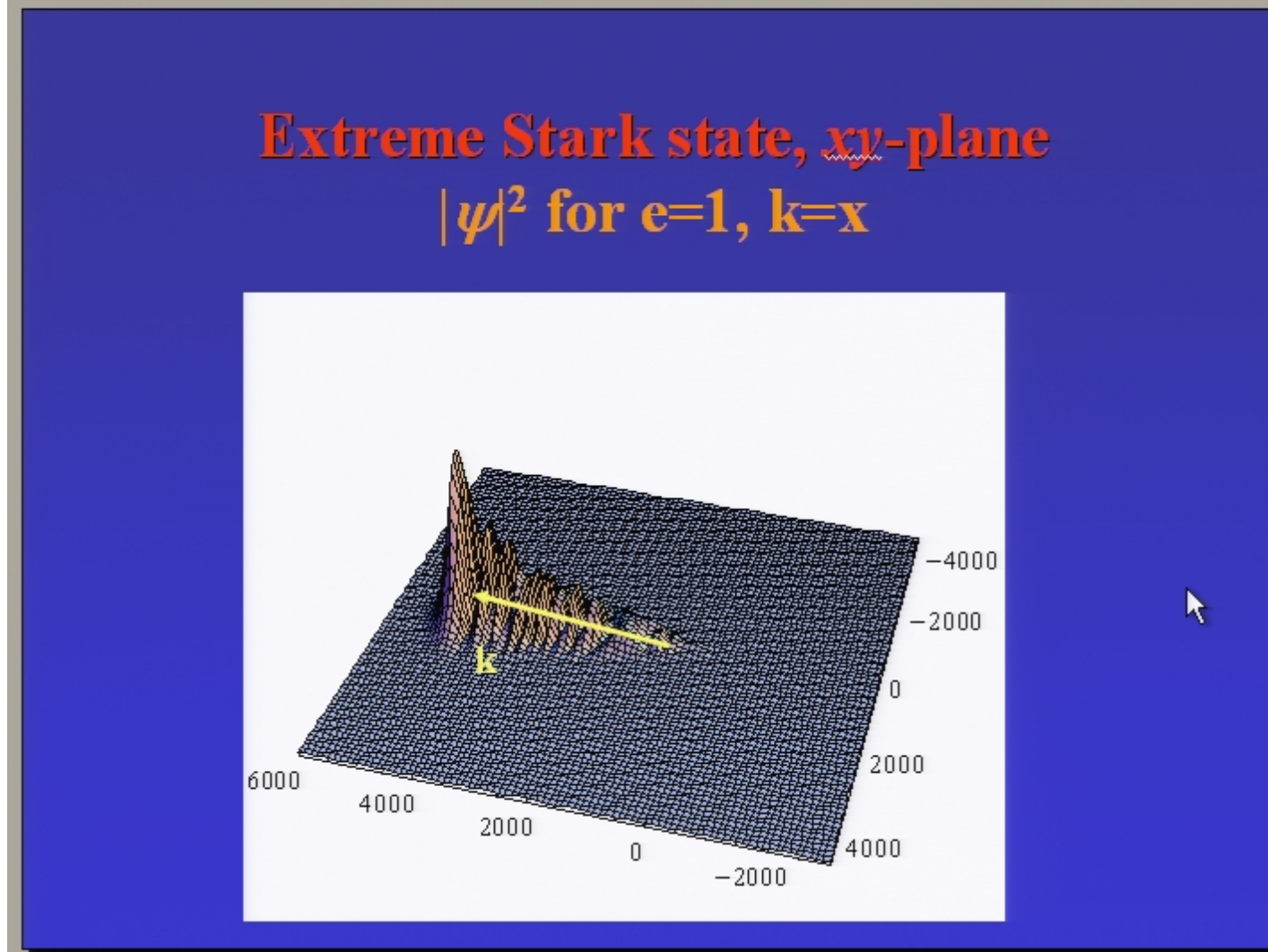
$|\psi|^2$ for $n=50$, $e=0.7$, $k=x$, $l=z$



Extreme Stark state, xy -plane

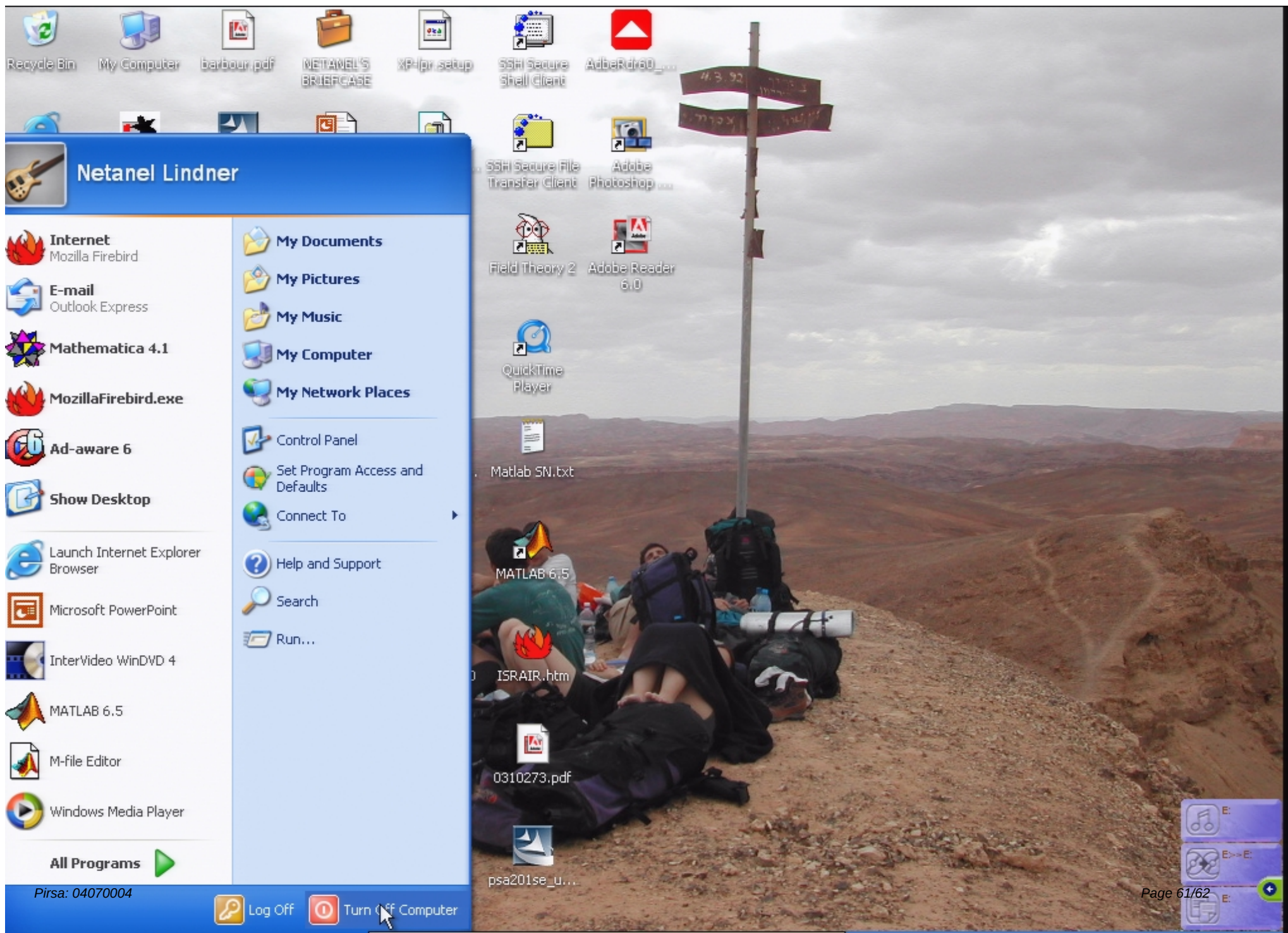
$|\psi|^2$ for $e=1$, $k=x$





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