Title: Quantizing and Dequantizing Reference Frames

Date: Jul 12, 2004 03:00 PM

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Abstract: Quantum Information Workshop

Outline

The coherence as fact vs. coherence as fiction controversy

A resolution: Classical reference frames and quantum reference frames as alternative paradigms of description

The lessons I wish to draw from this:
• Quantum states describe relations
• Many, if not all, superselection rules can be circumvented in principle

Coherence: Fact or fiction?

There are many contexts in which the debate arises:

Superconductors – for superpositions of charge eigenstates
BECs – for superpositions of atom number eigenstates
Lasers – for superpositions of photon number eigenstates

We discuss the optical case, although the discussion would be similar for the others.

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But if we quantize the atoms in the gain medium, and:

•assume the gain medium is in an energy eigenstate,

•apply energy conservation

$$|e\rangle|n\rangle \rightarrow \alpha(t)|e\rangle|n\rangle + \beta(t)|g\rangle|n+1\rangle$$

$\rightarrow$ atoms and field evolve to an entangled state

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$$\rho = \sum_{n=0}^{\infty} p_n |n\rangle\langle n|$$

$$p_n = \frac{e^{-|\alpha|^2}|\alpha|^{2n}}{n!}$$

Thus, coherence is a fiction!

The ensuing controversy

• T. Rudolph and B. C. Sanders, Phys. Rev. Lett. 87, 077903 (2001)
• H. M. Wiseman, J. Mod. Opt. 50, 1797 (2003); arXiv:quant-ph/0104004
• S. J. van Enk and C. A. Fuchs, Phys. Rev. Lett. 88, 027902 (2002)
• S. J. van Enk and C. A. Fuchs, Quantum Information and Computation 2, 151 (2002)
• T. Rudolph and B. C. Sanders, quant-ph/0112020 (2001)
• K. Nemoto and S. L. Braunstein, quant-ph/0207135 (2002)
• H. M. Wiseman, J. Mod. Opt. 50, 1797 (2003)
• B. C. Sanders, S. D. Bartlett, T. Rudolph, P. L. Knight, Phys. Rev. A 68, 042329 (2003)
• J. Smolin, quant-ph/0407009
• …

A possible dialogue

C: The reduced density operator should be interpreted as a mixture of coherent states

$$\rho = \sum_{n=0}^{\infty} p_n |n\rangle\langle n| = \int_0^{2\pi} \frac{d\phi}{2\pi} |\alpha\rangle\langle\alpha|$$

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C: This is a proper mixture, the PEF only applies to improper mixtures

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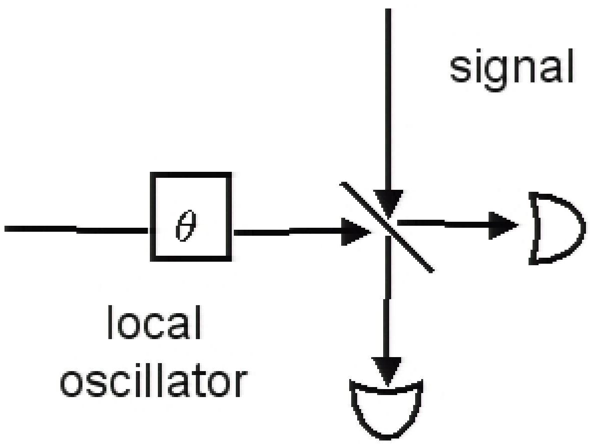
Example: Homodyne detection



$$|\beta\rangle|\alpha\rangle$$

$$\langle b^\dagger a + b a^\dagger \rangle \neq 0$$

C: Experiments have shown that lasers have a well-defined phase

NC: No they haven't

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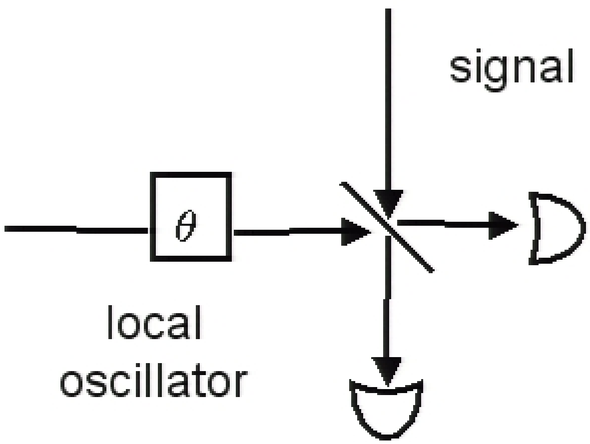

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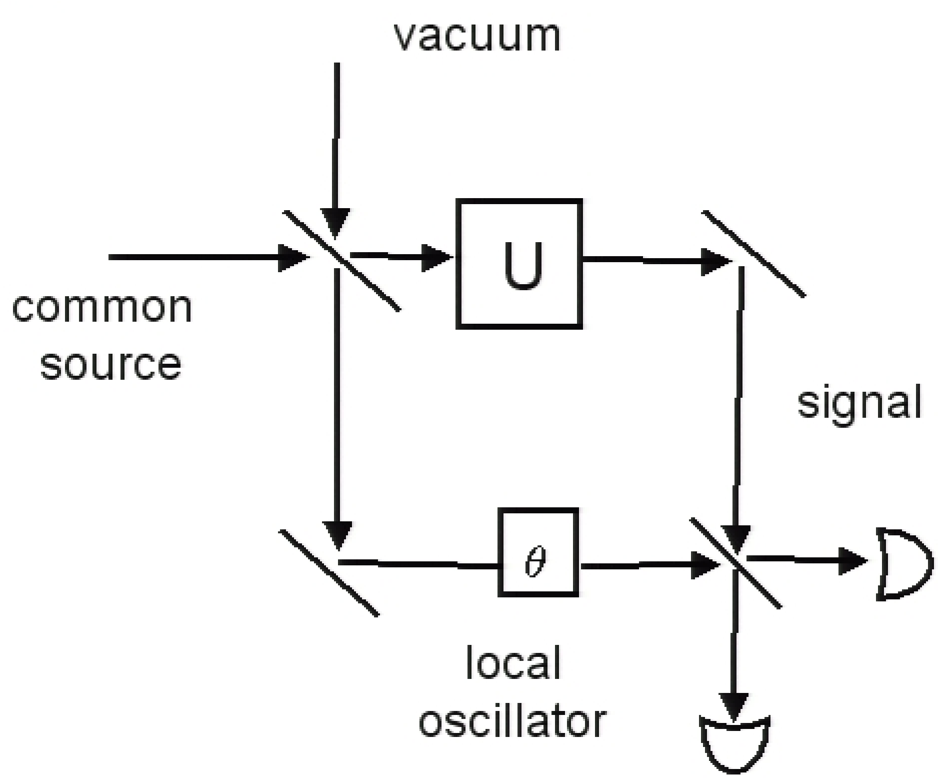

Demonstrates coherence between states of different relative number

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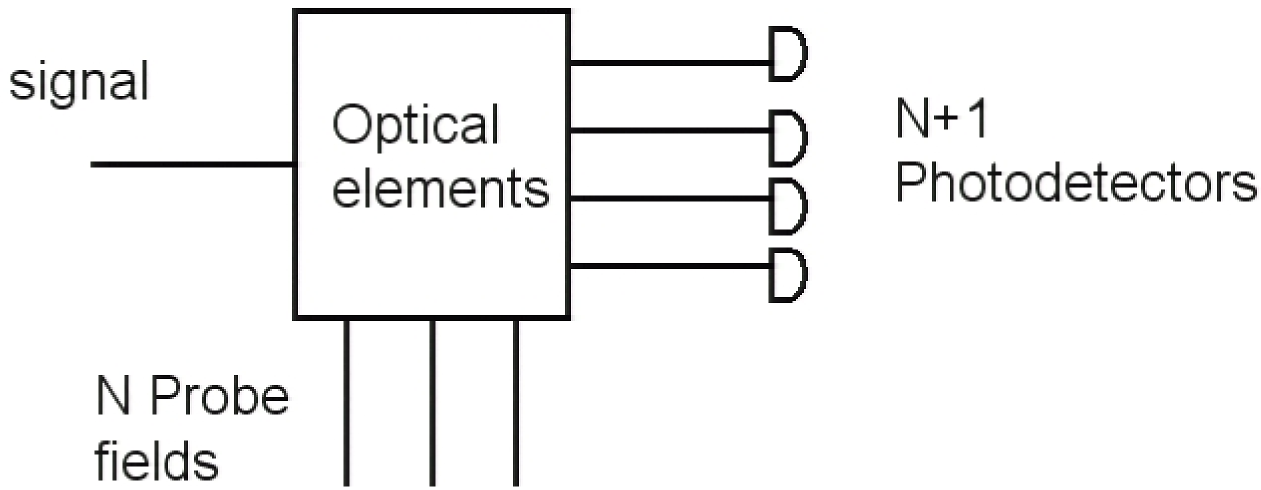
No.



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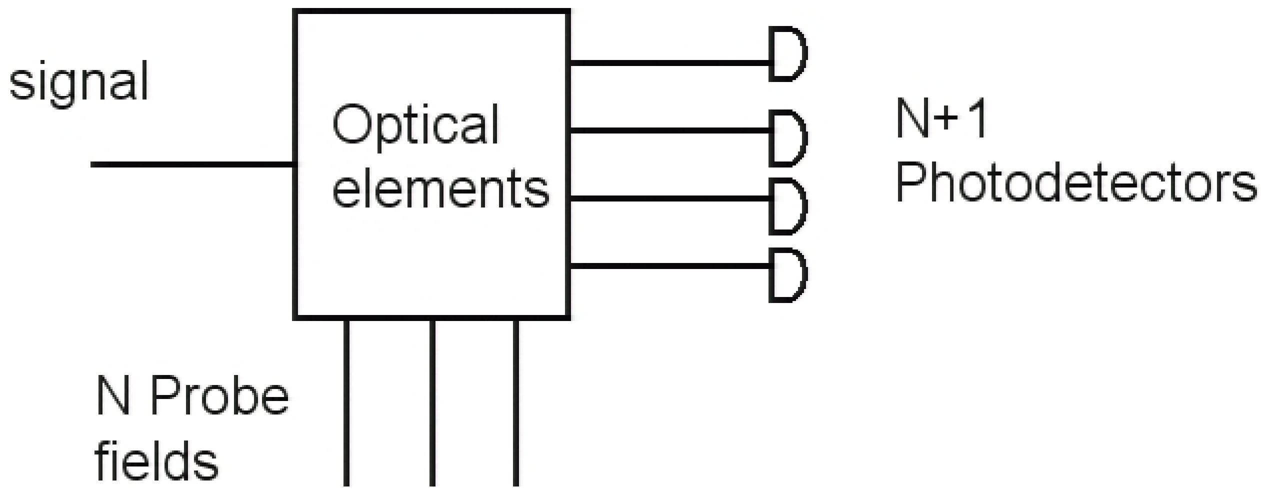

This cannot distinguish $\rho = \sum_{n,m} p_{nm} |n\rangle\langle m|$ from $\rho = \sum_{n} p_{nn} |n\rangle\langle n|$

The coherence has no operational significance!

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Generating coherence relative to a classical clock

Ex: classical oscillating current

$$U(t,0) = \exp(\alpha(\mathrm{t})\mathrm{a}^\dagger - \alpha(\mathrm{t})^*\mathrm{a})$$

$$U(t,0)|\mathrm{vac}\rangle = |\alpha(\mathrm{t})\rangle$$

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Detecting coherence relative to a classical clock

Ex: In homodyne detection, if the local oscillator is treated classically, then the interference term is

$$\langle \beta^* a + \beta a^\dagger \rangle$$

So, both descriptions are empirically adequate!

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Our suggestion: there are really only relations between systems and the quantum state describes these. In this case, the two descriptions can be consistent.

Relational view of quantum states

The quantum state describes the relation between the system and the reference frame

Coherence paradigm = classical RF paradigm

No coherence paradigm = quantum RF paradigm

See: Aharonov and Susskind, Phys. Rev. **155**, 1428 (1967).

Non-eigenstate of	Classical RF	Group
linear momentum	spatial frame (e.g. GPS satellites)	HW
angular momentum	orientation frame (e.g. gyroscopes)	SU(2)
photon number	clock	U(1)
atom number	BEC phase	U(1)
charge	Superconducting phase	U(1)

We shall consider a general framework that works for all these cases

G = group of transformations for the relevant d.o.f.

No classical RF for G $\Rightarrow$ Operations and observables must be invariant under collective action of G (Superselection rule)

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Suppose T:G $\rightarrow$ GL(H) is a collective representation of G

A G-invariant CP map $\mathcal{O}$ satisfies

$$\mathcal{O}[T(g)\rho T^\dagger(g)] = T(g)\mathcal{O}[\rho]T^\dagger(g) \qquad \forall\ g \in G$$

A G-invariant POVM $\{E_k\}$ satisfies

$$T(g)E_k T^\dagger(g) = E_k \qquad \forall\ g \in G$$

Equivalence classes of states:

$$\rho \equiv \rho' \quad \text{if} \quad \begin{array}{l}\mathsf{Tr}[A\rho] = \mathsf{Tr}[A\rho'] \\ \text{for all } G\text{-invariant } A\end{array}$$

or

$$\mathcal{G}(\rho) = \mathcal{G}(\rho')$$

where

$$\mathcal{G}[\rho] \equiv \begin{cases} \frac{1}{|G|}\sum_{g\in G} T(g)\rho T^{\dagger}(g)\,, & \text{finite groups} \\ \int_G \mathrm{d}v(g)\, T(g)\rho T^{\dagger}(g)\,, & \text{Lie groups}\end{cases}$$

Convention: represent each equivalence class by the G-invariant state

$$\rho = \mathcal{G}(\rho)$$

Quantizing RFs

Suppose the system state w.r.t the classical RF is:
$|\psi\rangle \in H_S$

Quantize all physical objects that can serve as a RF.
Introduce a Hilbert space H_R

<u>Naïve approach</u>: assign $|\chi\rangle \otimes |\psi\rangle \in H_R \otimes H_S$

E.g. For optical case, one could take $|\chi\rangle$ to be a coherent state $|\alpha\rangle$

<u>Better approach</u>: Assign ρ on $H_R \otimes H_S$

$$\rho = \frac{1}{2\pi} \int_0^{2\pi} d\phi \, |\phi\rangle \langle\phi| \otimes T(\phi)|\psi\rangle\langle\psi|T^\dagger(\phi)$$

Problem with naïve approach to quantization:
There is no observational difference among states

$$U(g)|\chi\rangle \otimes U(g)|\psi\rangle$$

for different $g \in G$

There is no real difference associated with this distinction

The only real degree of freedom is in the relative orientation

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We must find a set of G-invariant states in $H_R \otimes H_S$
that encode the possible relations
Can these simulate the states in H_S? Yes.

See: Kitaev, Mayers, Preskill, quant-ph/0310088

Classical RF paradigm

States ρ

Measurements $\{E_k\}$

Transformations $\mathcal{O}$

defined on $\mathcal{H}_S$

Quantum RF paradigm

States $\tilde{\rho}$

Measurements $\{\tilde{E}_k\}$

Transformations $\tilde{\mathcal{O}}$

defined on $\mathcal{H}_R \otimes \mathcal{H}_S$

and G-invariant

Find a mapping

$$\rho \longrightarrow \tilde{\rho}$$

$$E_k \longrightarrow \tilde{E}_k$$

$$\mathcal{O} \longrightarrow \tilde{\mathcal{O}}$$

such that

$$\boxed{\mathrm{Tr}_S[\mathcal{O}(\rho)E_k] = \mathrm{Tr}_{RS}[\tilde{\mathcal{O}}(\tilde{\rho})\tilde{E}_k]}$$

Define $$\tilde{\rho} = \$(\rho)$$

$$\tilde{E}_k = \$(E_k)$$
$$\tilde{A}_\mu = \$(A_\mu)$$

where

$$\$: A \mapsto \int_G d\nu(g)\, |g\rangle\, \langle g| \otimes T(g) A T^\dagger(g)$$

with $T(g')\,|g\rangle = |g' \circ g\rangle\,,$ for all $g, g' \in G$

and $\langle g|g'\rangle = \delta(g, g')$

Property 1: $\boxed{\$(A) \text{ is } G\text{-invariant}}$

Proof: $(T(g') \otimes T(g'))\$(A)(T^\dagger(g') \otimes T^\dagger(g'))$

$$\begin{aligned}
&= \int_G d\mu(g) T(g') \left|g\right\rangle \left\langle g\right| T^\dagger(g') \otimes T(g')T(g)AT^\dagger(g)T^\dagger(g') \\
&= \int_G d\mu(g) \left|g' \circ g\right\rangle \left\langle g' \circ g\right| \otimes T(g' \circ g)AT^\dagger(g' \circ g) \\
&= \$(A).
\end{aligned}$$

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Property 2: $\boxed{\begin{array}{c} \$(A+B) = \$(A) + \$(B) \\ \text{and} \\ \$(AB) = \$(A)\$(B) \end{array}}$

Proof: $\int_G d\nu(g) |g\rangle \langle g| \otimes T(g)AT^\dagger(g) \int_G d\nu(g') |g'\rangle \langle g'| \otimes T(g')BT^\dagger(g')$

$$\begin{aligned} &= \int_G d\mu(g) |g\rangle \langle g| \otimes T(g)AT^\dagger(g)T(g)BT^\dagger(g) \\ &= \int_G d\mu(g) |g\rangle \langle g| \otimes T(g)ABT^\dagger(g) \end{aligned}$$

Property 3: $\boxed{\mathsf{Tr}_{RS}(\$(A)) = \mathsf{Tr}_S(A)}$

Property 4: $\boxed{\text{if } A > 0 \text{ then } \$(A) > 0}$

Property 5: $\boxed{\$(I_S) = I_{RS}}$

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3,4 $\longrightarrow$ if ρ is a density operator, so is $\tilde{\rho}$

2,4,5 $\rightarrow$ if $\{E_k\}$ is a POVM, so is $\{\tilde{E}_k\}$

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$$\begin{aligned}
\text{Tr}_{RS}[\tilde{\mathcal{O}}(\tilde{\rho})\tilde{E}_k] &= \text{Tr}_{RS}[\textstyle\sum_k \$(A_\mu)\$(\rho)\$(A_\mu^\dagger)\$(E_k)] \\
&= \text{Tr}_{RS}[\$(\textstyle\sum_k A_\mu \rho A_\mu^\dagger E_k)] \\
&= \text{Tr}_S[\mathcal{O}(\rho)E_k]
\end{aligned}$$

Example: Superpositions of charge eigenstates

Consider a coherent superposition of charge eigenstates on H_S

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

This is simulated by the U(1)-invariant state

$$\rho = \frac{1}{2\pi}\int_0^{2\theta} d\theta\, |\theta\rangle\,\langle\theta| \otimes T(\theta)|\psi\rangle\langle\psi|T^\dagger(\theta)$$

$$|\theta\rangle = \frac{1}{\sqrt{2\pi}}\sum_{q=-\infty}^{\infty} e^{-iq\theta}|q\rangle$$

$$T(\theta) = e^{-i\theta\hat{Q}}$$

which may be written as

$$\rho = \sum_{q=-\infty}^{\infty} |\psi_q\rangle\,\langle\psi_q|$$

where $|\psi_q\rangle = \alpha|q+1\rangle|0\rangle + \beta|q\rangle|1\rangle$

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Writing $\mathcal{H} = \bigoplus_j \mathcal{H}_j^{\text{glob}} \otimes \mathcal{H}_j^{\text{rel}}$

Carrier space of jth irrep of G

Hilbert space for the multiplicity of the jth irrep of G

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Carrier space of jth irrep of G

Hilbert space for the multiplicity of the jth irrep of G

We have, by Schur's lemma,

$$\mathcal{G}(A) = \sum_j \mathcal{D}_j^{\mathrm{glob}} \otimes \mathcal{I}_j^{\mathrm{rel}}(P_j A P_j).$$

Decoherence-full subsystem

Decoherence-free subsystem

Dequantizing RFs

Wrong approach: Trace over reference frame

$$\rho_S = \mathrm{Tr}_{\mathrm{R}}(\rho_{\mathrm{RS}})$$

Right approach: Project into an irrep and trace over the decoherence-full subsystem
i.e. keep only the decoherence-free subsystem

$$\rho_S = \mathrm{Tr}_{\mathrm{glob}}(\mathrm{P_j}\rho_{\mathrm{RS}}\mathrm{P_j})$$

Conclusions

- Quantum states describe the relation of a system to a reference frame

- One can break superselection rules given appropriate resources

Future research

- Quantizing and dequantizing finite RFs
- Degradation of finite RFs (see poster by P. Turner)
- Possibility of condensates for novel degrees of freedom
- Connection to relationalism in quantum gravity (work with E. Livine and F. Girelli)