Title: Quantum Reference Frames and Relative States

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Abstract:

Quantum reference frames and relative states

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Université de Genève

- The photon number picture of continuous variable QC
- A quantum reference pulse
- A toy model of a quantum reference frame
- Relative states
- Optimizations
- Parallel versus anti-parallel spins
- Is homodyne detection a quantum coherent measurement?
- To Q-teleport or not to Q-teleport

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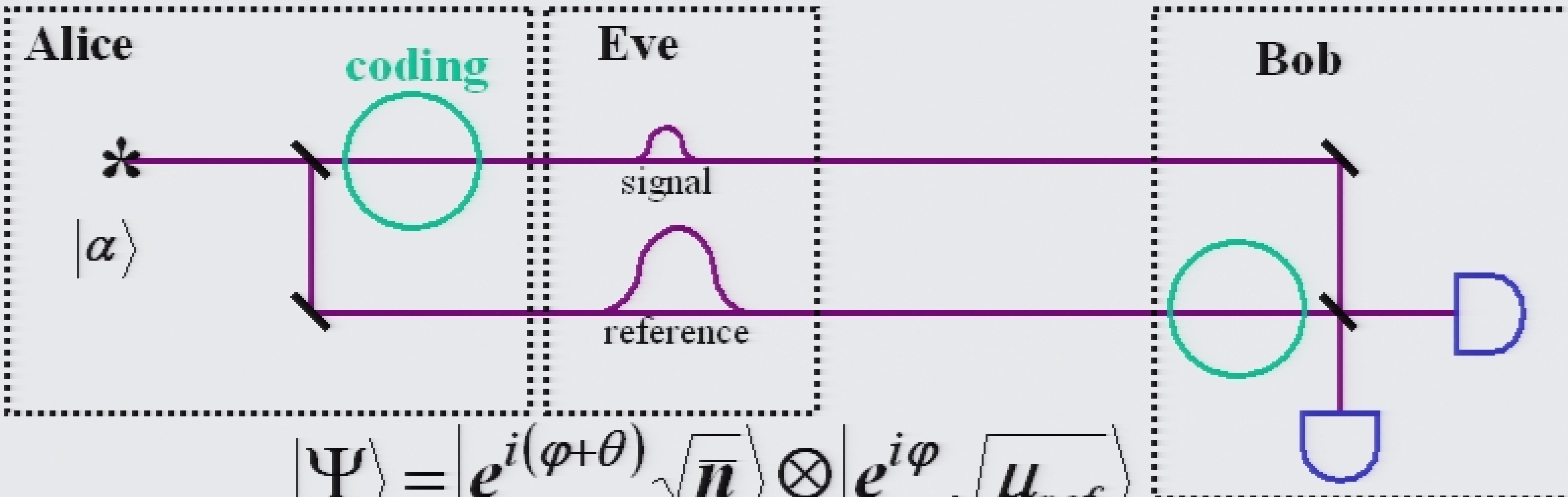
The photon number picture of continuous variable QC



$$|\Psi\rangle = \left|e^{i(\varphi+\theta)}\sqrt{\bar{n}}\right\rangle \otimes \left|e^{i\varphi}\sqrt{\mu_{ref}}\right\rangle$$

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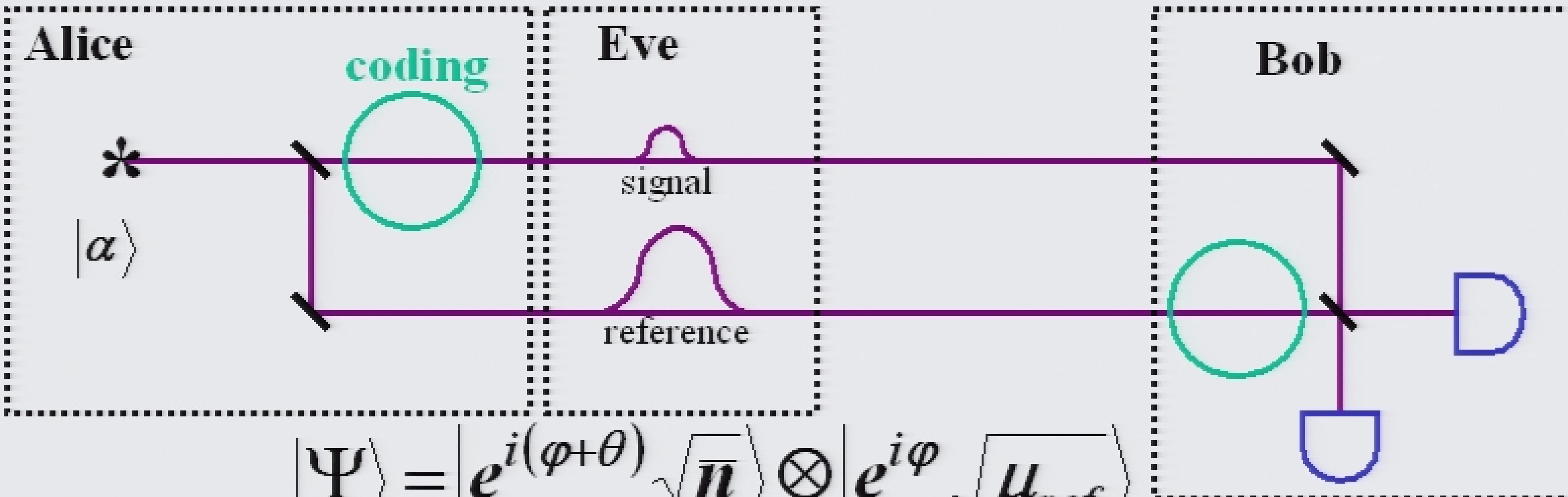

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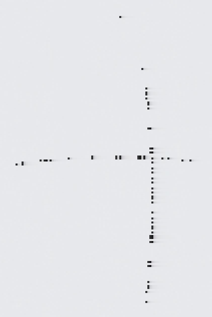
$$\int \frac{d\varphi}{2\pi} |\Psi\rangle\langle\Psi|$$



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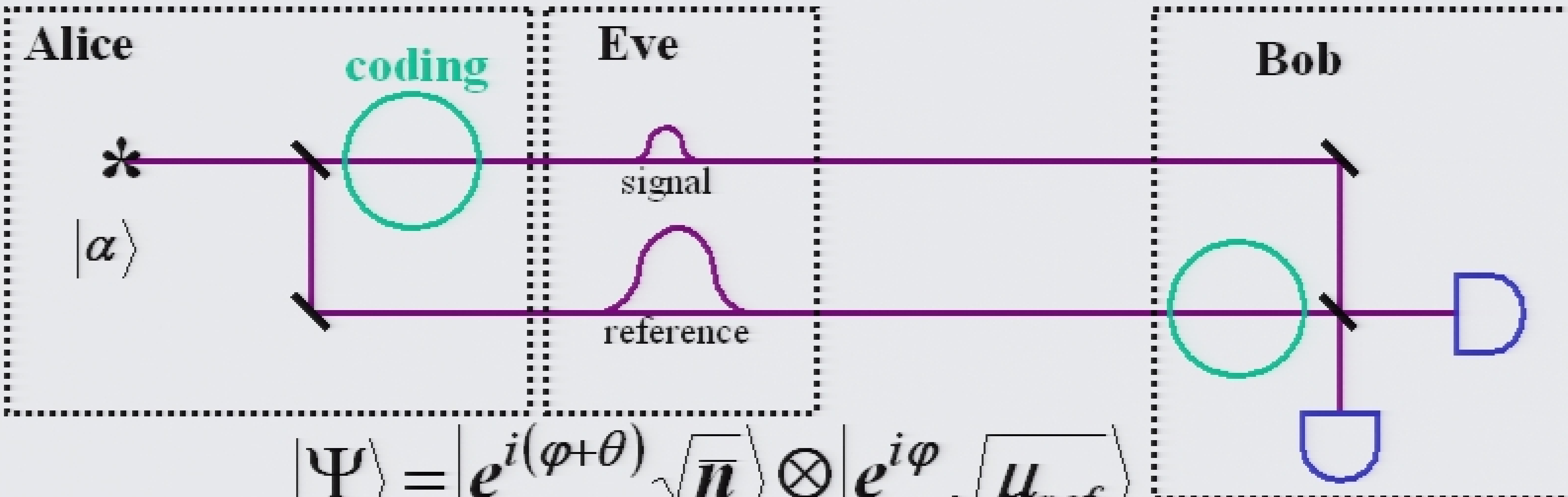

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$$\int \frac{d\varphi}{2\pi} |\Psi\rangle\langle\Psi| = \sum_n p(n\,|\,\bar{n}+\mu_{ref}) P_\psi^{\otimes n}$$

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The photon number picture of continuous variable QC

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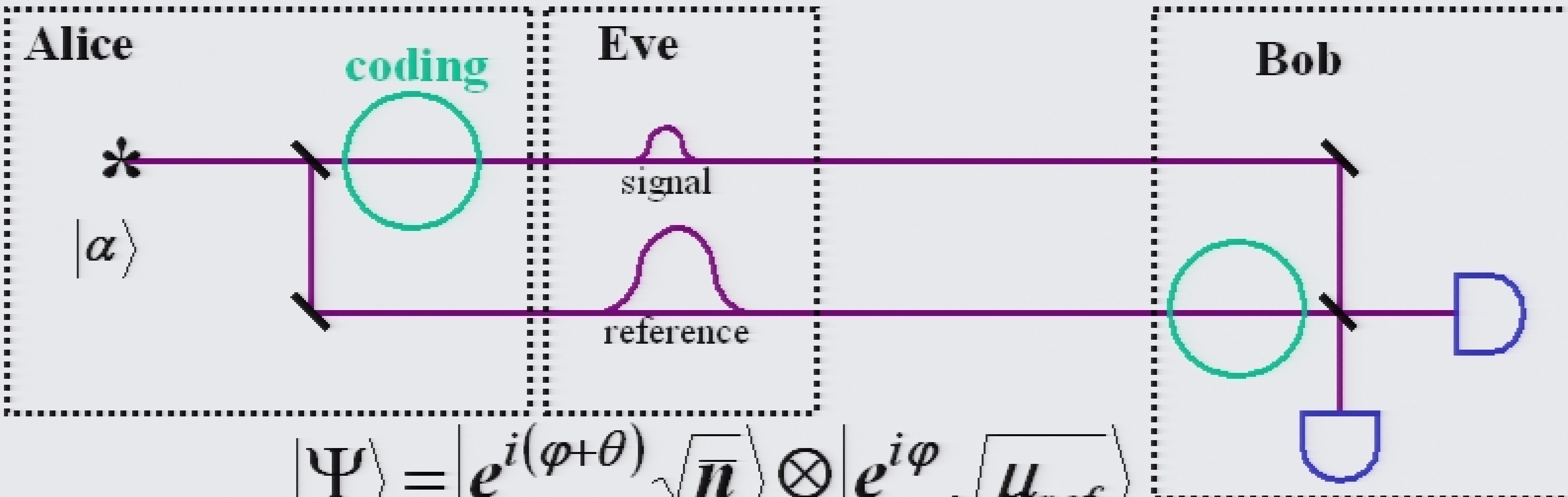

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where $\psi = \dfrac{1}{\sqrt{\bar{n}^2 + \mu_{ref}^2}} \left(e^{i\theta} \sqrt{\bar{n}}\,|0\rangle + \sqrt{\mu_{ref}}\,|1\rangle \right)$

The photon number picture of continuous variable QC

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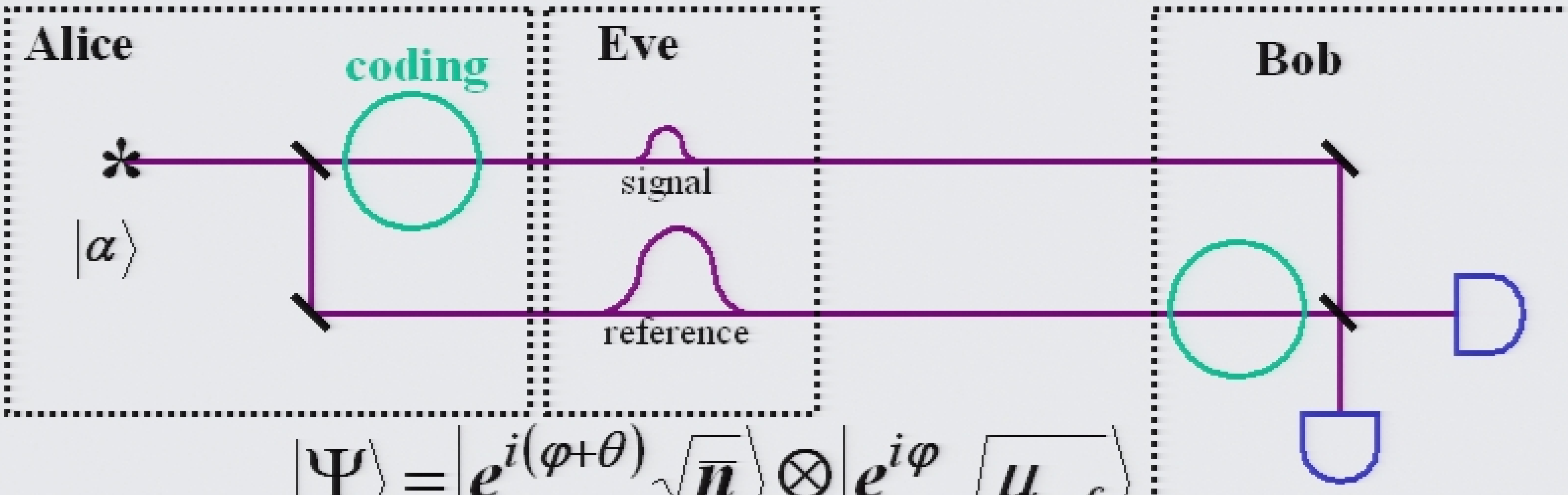

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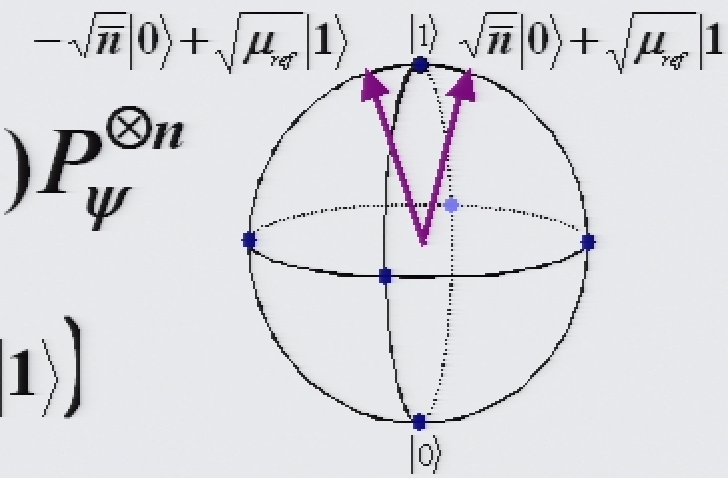

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$\Longrightarrow$ **continuous variables ≈ Poission distribution of n qubits with large mean υ-number**

Eve can perform photon-number-splitting attacks also on continuous variable quantum cryptography !

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Eve can perform photon-number-splitting attacks also on continuous variable quantum cryptography !

This hold also in the absence of the reference pulse :

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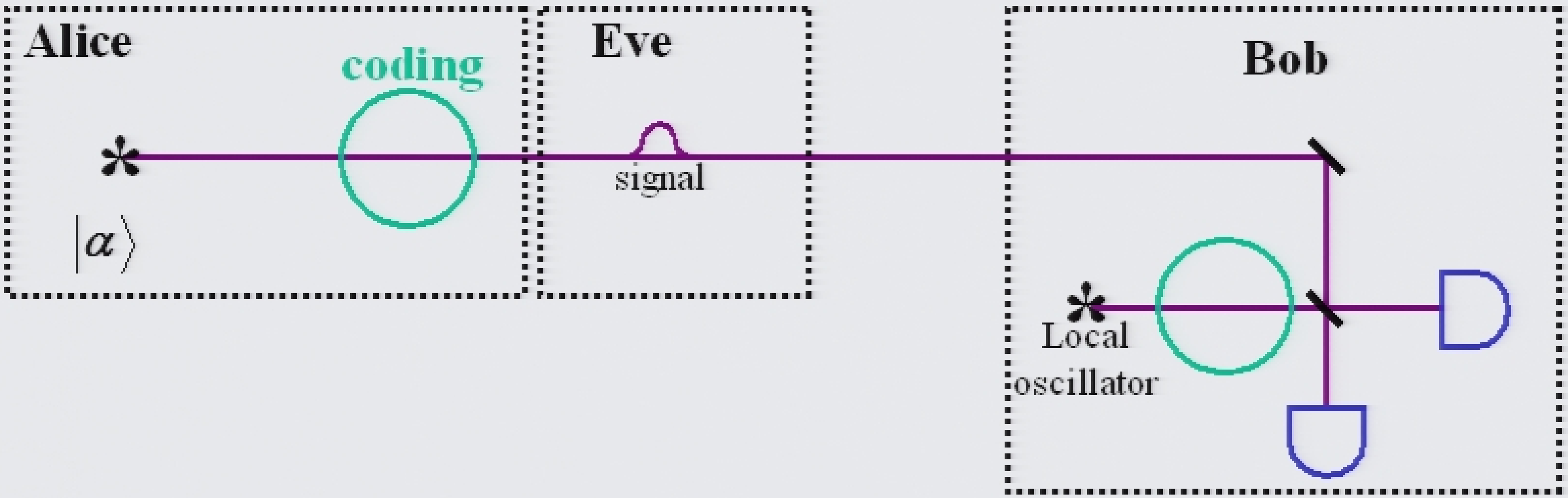

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The photon number picture of continuous variable QC

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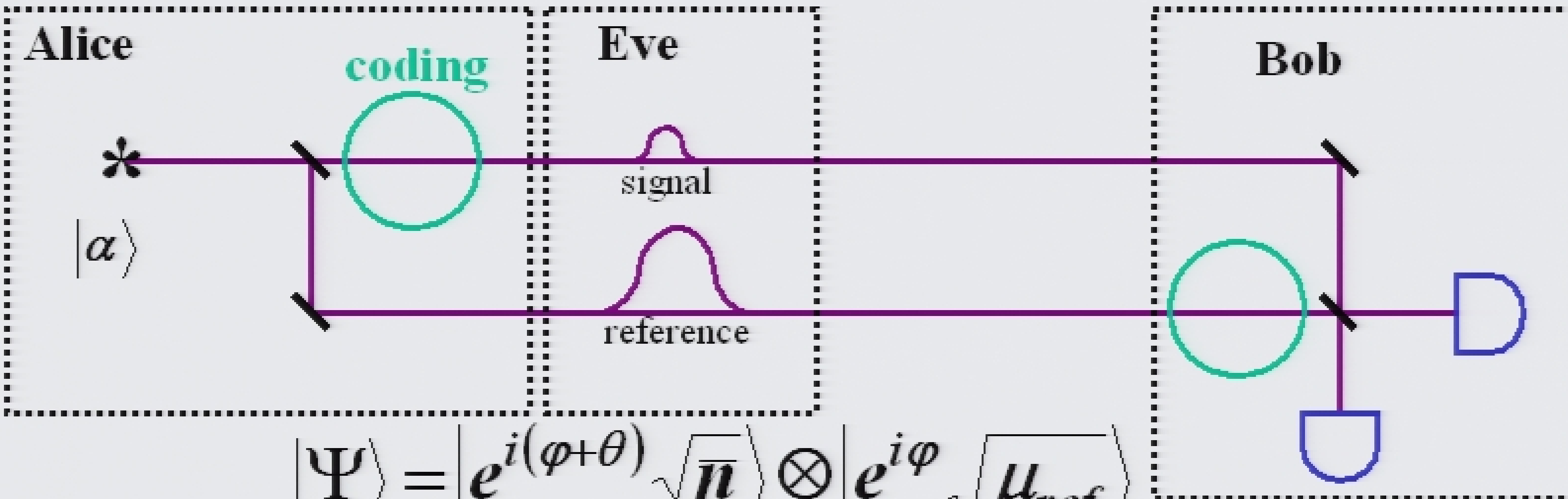

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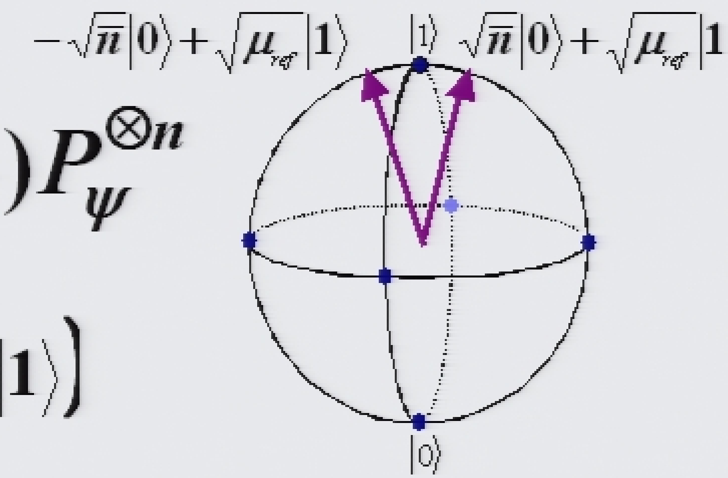

$\Longrightarrow$ continuous variables ≈ Poission distribution of n qubits with large mean [illegible] number

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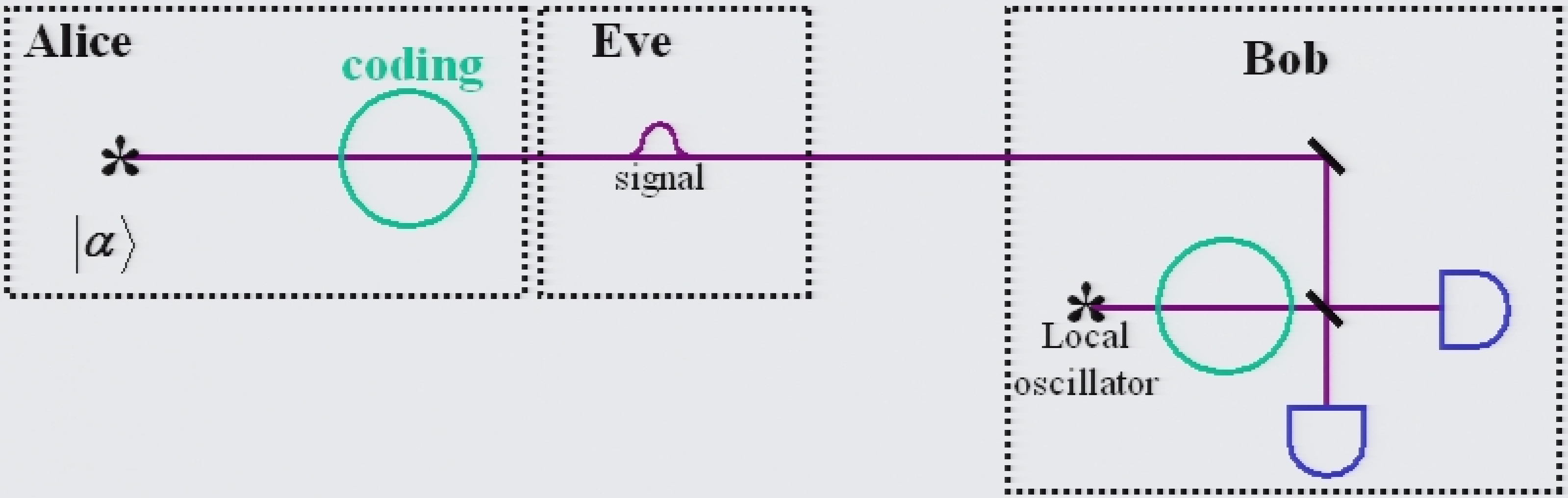

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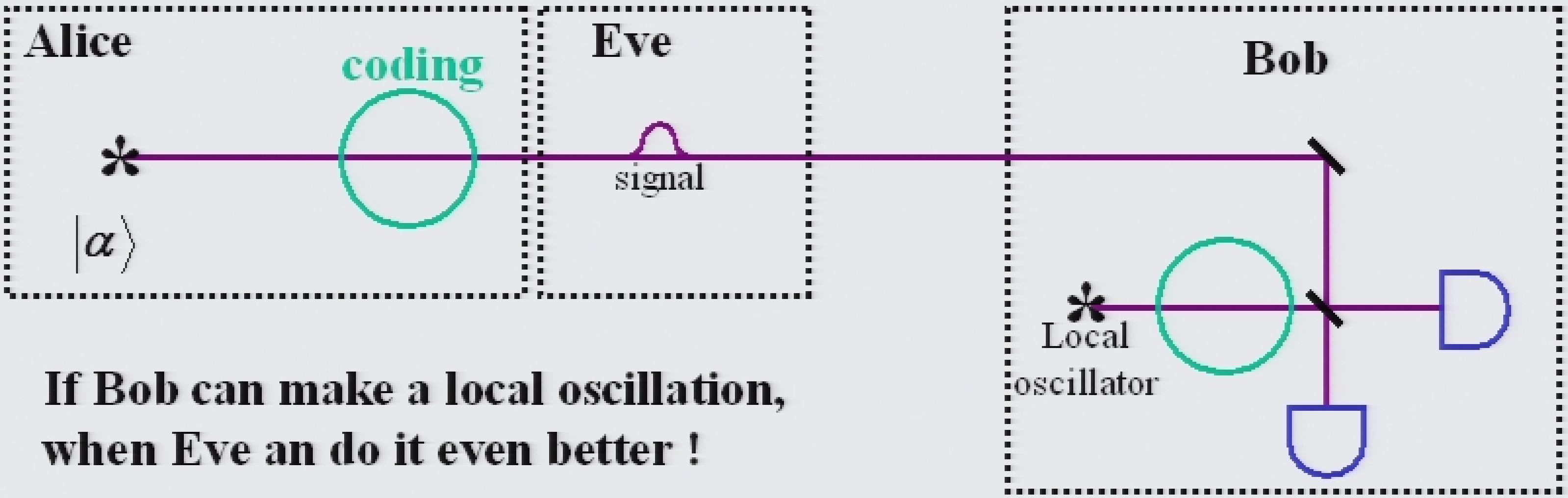

If Bob can make a local oscillation, when Eve an do it even better !

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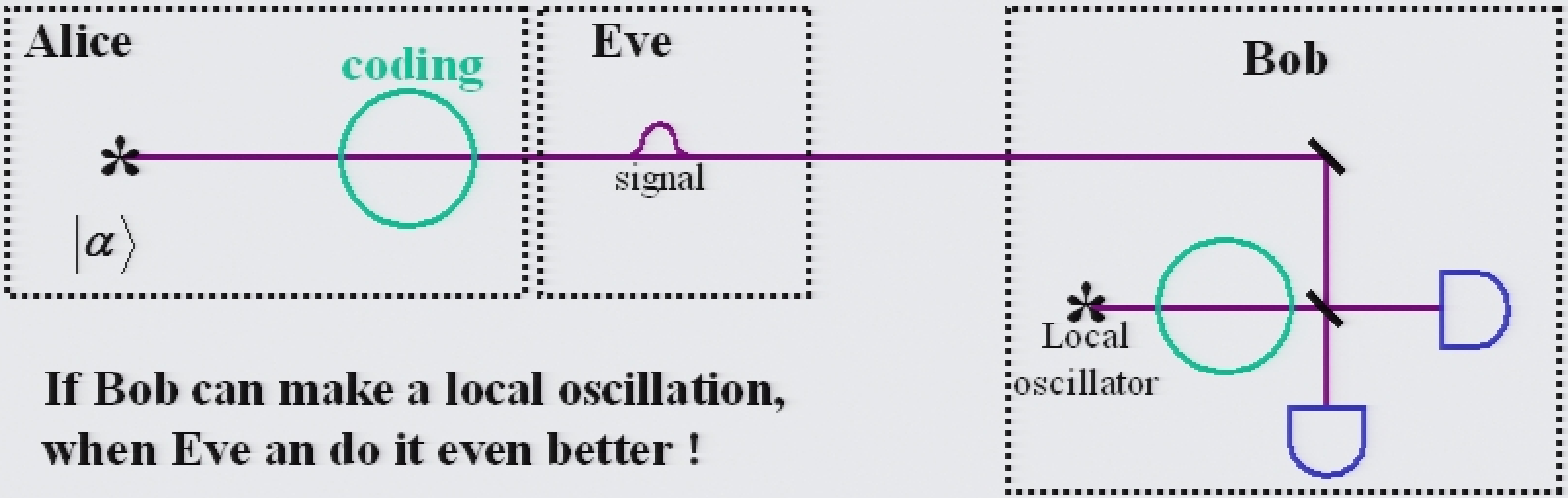
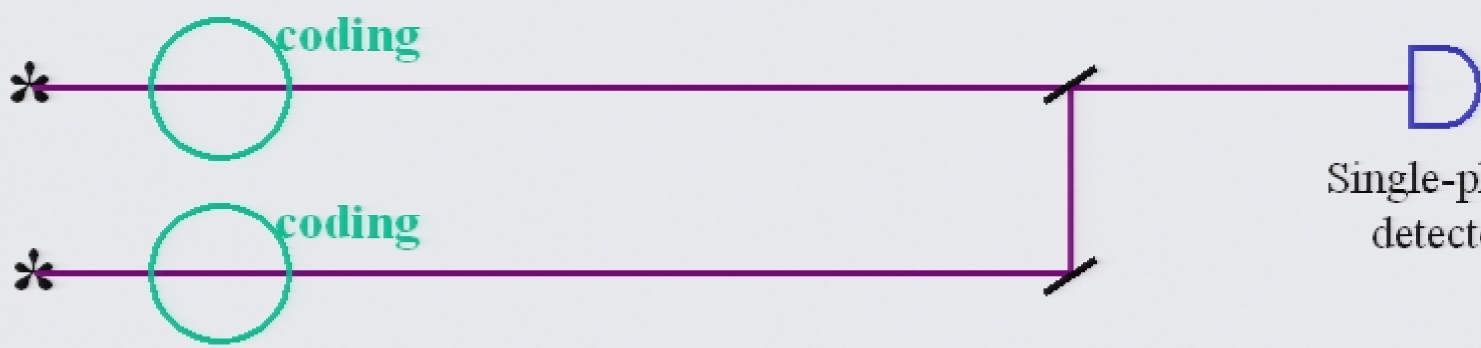


If Bob can make a local oscillation, when Eve an do it even better !

One photon can be produced by 2 independent lasers :



Bob's optimal analyzing strategy ?

- **Homodyne detection**

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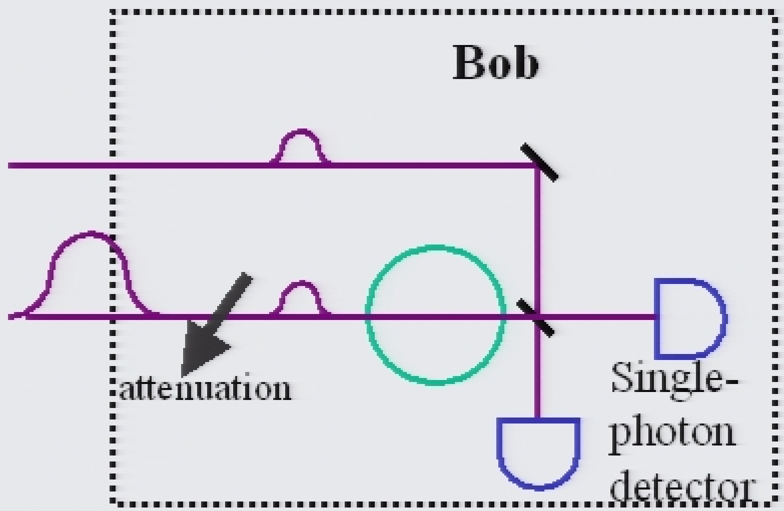
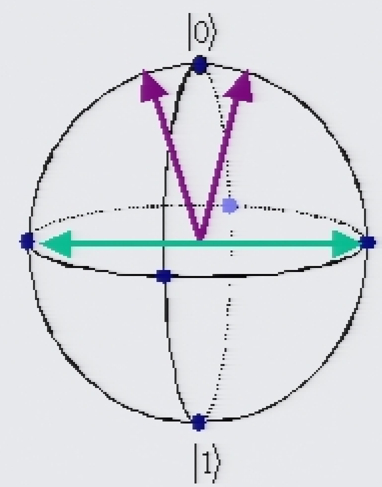
Bob's optimal analyzing strategy ?

- **Homodyne detection**
- **4+2 protocol**

(B. Huttner et al., Phys. Rev. A, 51, 1863, 1995)





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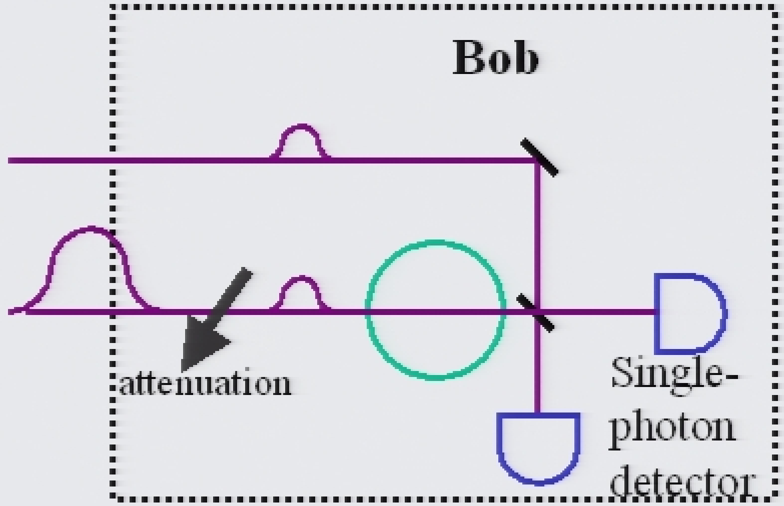

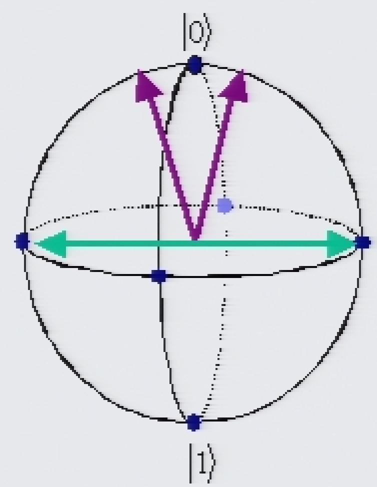

What is more efficient:

- **always have a noisy signal and process the data classically, or**
- **have few data, selected by Nature ?**

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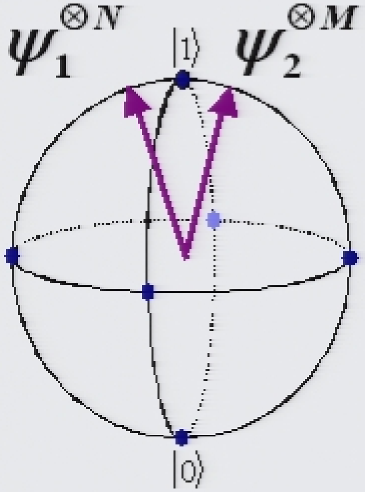
Relative states

$\psi_1^{\otimes N}$ $\psi_2^{\otimes M}$



Given two arbitrary directions defined by N and M spin 1/2, estimate

$$\left|\langle \psi_1 | \psi_2 \rangle\right|^2$$

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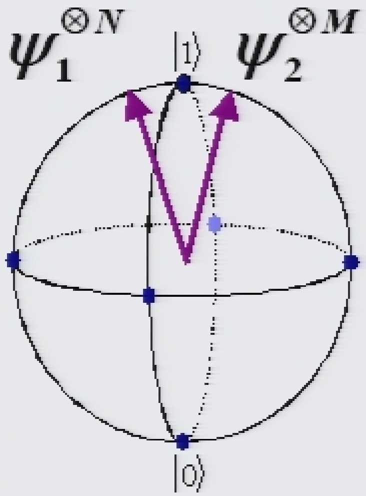
Relative states



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$$\left|\langle\psi_1|\psi_2\rangle\right|^2$$

Figure of merit: mean variance

$$\Delta = \int de \iint d\psi_1 d\psi_2 \, P(e\,|\,\psi_1\psi_2)\left(e - \left|\langle\psi_1|\psi_2\rangle\right|^2\right)^2$$

estimate

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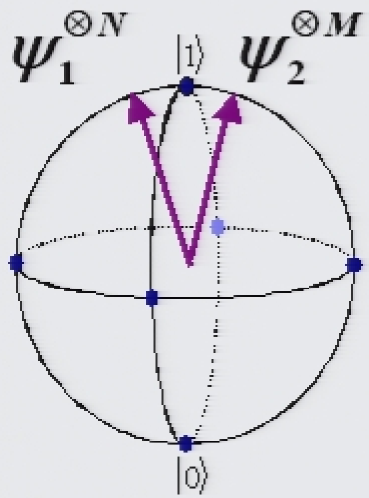
Relative states



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$$\left|\langle \psi_1 | \psi_2 \rangle\right|^2$$

Figure of merit: mean variance

$$\Delta = \int \underset{\text{estimate}}{de} \iint d\psi_1 d\psi_2 \, \underbrace{P(e\,|\,\psi_1\psi_2)}_{\left\langle \psi_1^{\otimes N} \otimes \psi_2^{\otimes M} \right| \underset{\text{POVM element}}{P_e} \left| \psi_1^{\otimes N} \otimes \psi_2^{\otimes M} \right\rangle} \left(e - \left|\langle \psi_1 | \psi_2 \rangle\right|^2\right)^2$$

estimate

POVM element

Case N=M=1

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Rotational invariance: $[P_e, U\otimes U]=0$ for all unitary U.

Hence, each POVM element P_e is a linear combination of the projectors S and T onto the singlet and triplet subspaces:

$$P_e = s(e)\cdot S + t(e)\cdot T$$

Optimize s(e) and t(e): $\Rightarrow$ $s(e) = \delta(e-e_s)$

$t(e) = \delta(e-e_t)$

One finds: $e_s = 1/3$

$e_t = 5/9$

$$\Delta_{min} = \frac{2}{27} \qquad \left(\Delta_{a\,priori} = \frac{1}{12}\right)$$

General case N, M

Rotational invariance: $[P_e, U^{\otimes(N+M)}]=0$ for all unitary U.

$$\frac{N}{2}\otimes\frac{M}{2}=\frac{|N-M|}{2}\oplus\cdots\oplus\frac{|N+M|}{2}$$

$$\Downarrow$$

$$I_N\otimes I_M=I_{|N-M|}\oplus\cdots\oplus I_{|N+M|}$$

Hence, each POVM element P_e is a linear combination of the projectors I_j

$$P_e=\sum_{j=\frac{1}{2}|N-M|}^{\frac{1}{2}|N+M|} q_j(e)\,I_j$$

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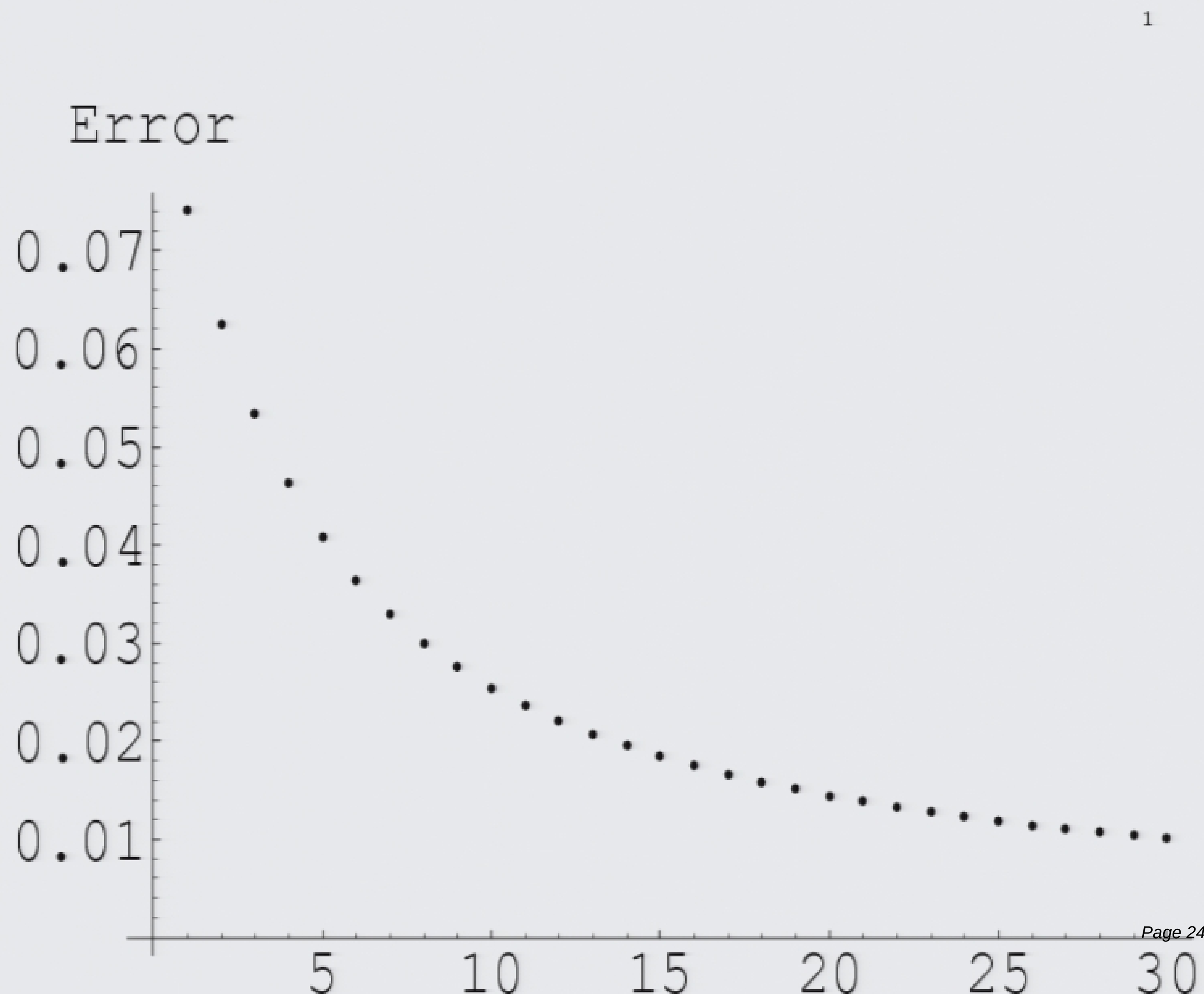
Case N=M



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Case N=1, M

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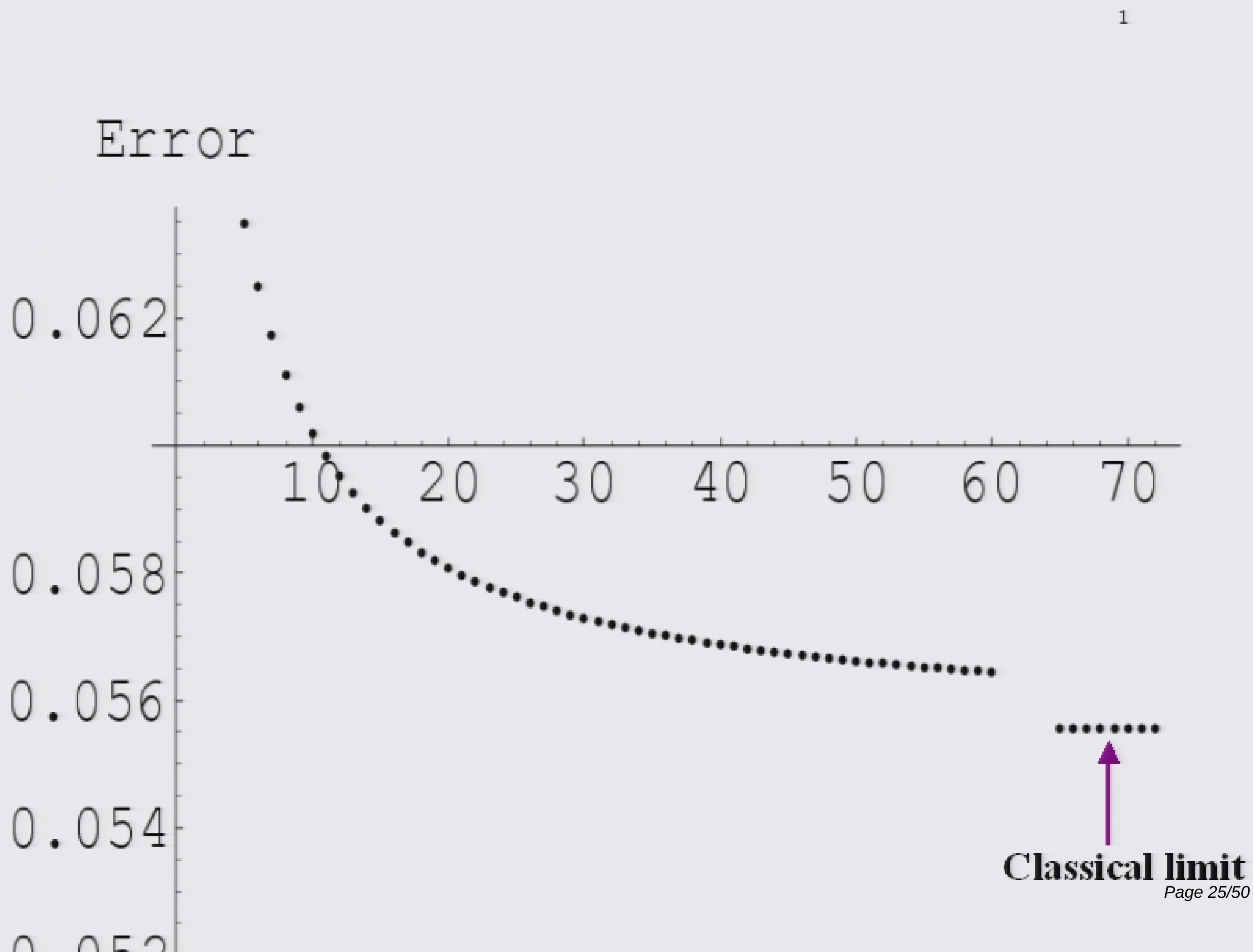
1
Error
0.062
0.058
0.056
0.054
10
20
30
40
50
60
70
Classical limit

Anti-parallel spins ?

Given $\psi_1, \psi_1^{\perp}, \psi_2$ we like to estimate $\left|\left\langle \psi_1 \middle| \psi_2 \right\rangle\right|^2$

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Anti-parallel spins ?

Given $\psi_1, \psi_1^{\perp}, \psi_2$ we like to estimate $\left|\langle \psi_1 | \psi_2 \rangle\right|^2$

$$\Delta^{anti-\|}(1,2) = \Delta^{\|}(1,2) = \frac{5}{72}$$

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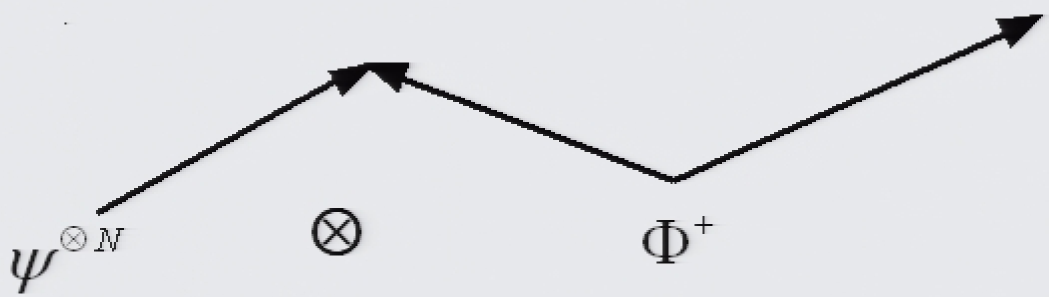
Conclusion

- All Q measurements are estimations of relative states.
- An important example are the Bell-analyzers used in Q teleportation.



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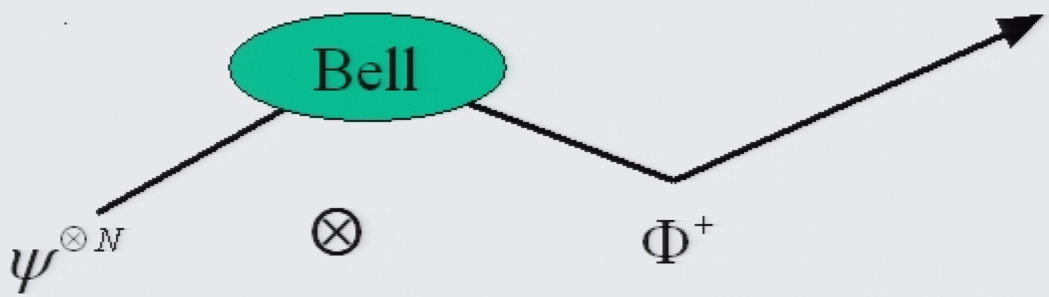

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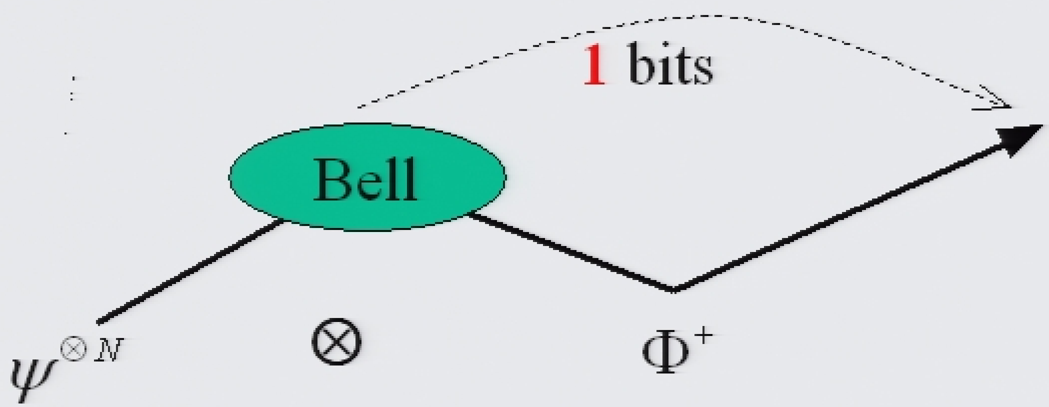

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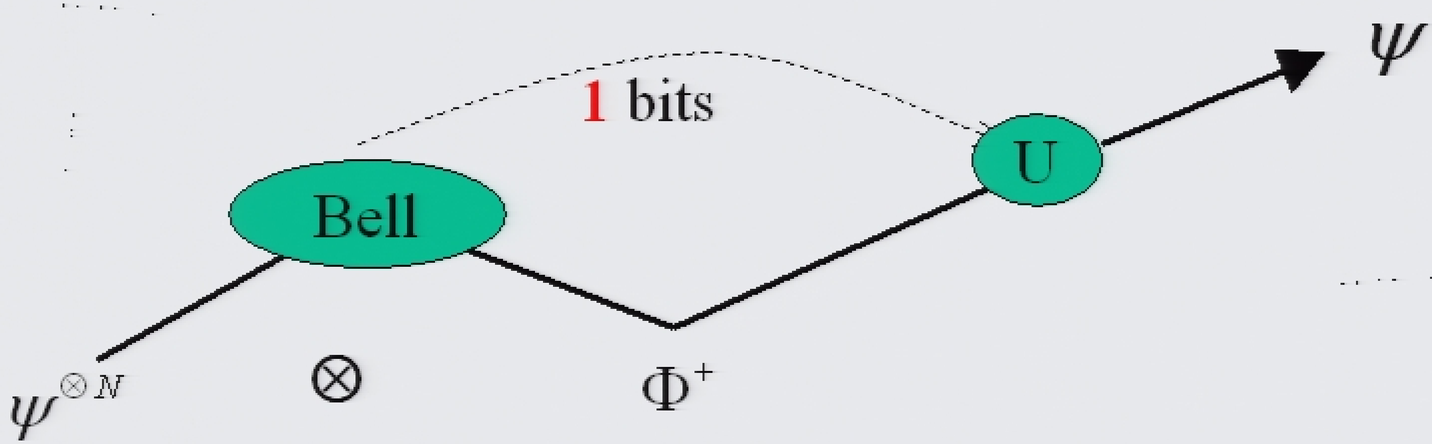

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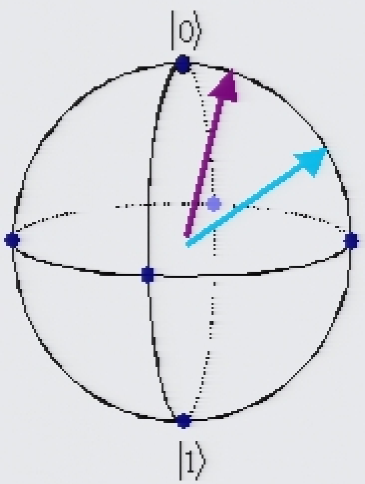

To estimate relative states one should not get information about the individual states. In some sense the estimation of $\vec{m}_1 - \vec{m}_2$ **is incompatible with that of** $\vec{m}_1 + \vec{m}_2$

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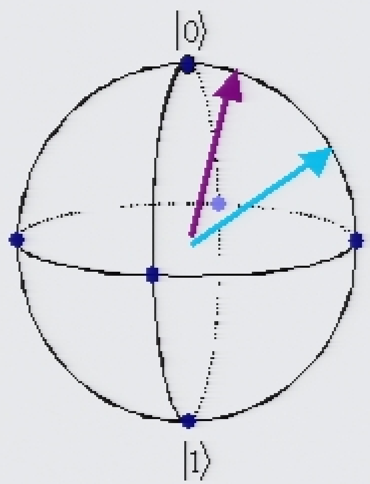

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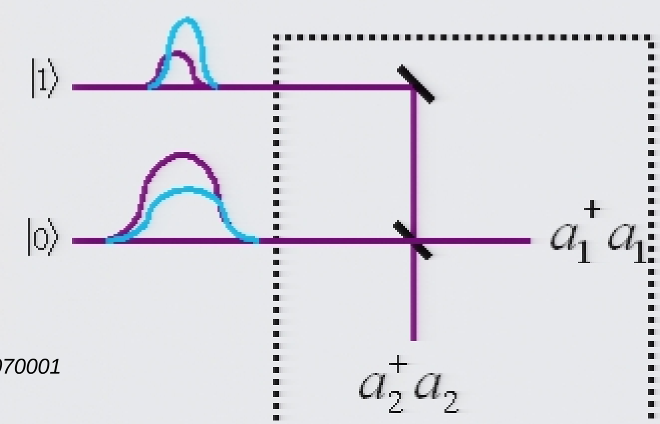

Is the measurement of $a_1^+ a_1 - a_2^+ a_2$
- equivalent to the measurements of $a_1^+ a_1$ and $a_2^+ a_2$ with substraction of the classical results?
- a coherent measurement?
- optimal for relative state estimation?

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To Q-teleport or not to Q-teleport, is this the question ?

Experimental setup

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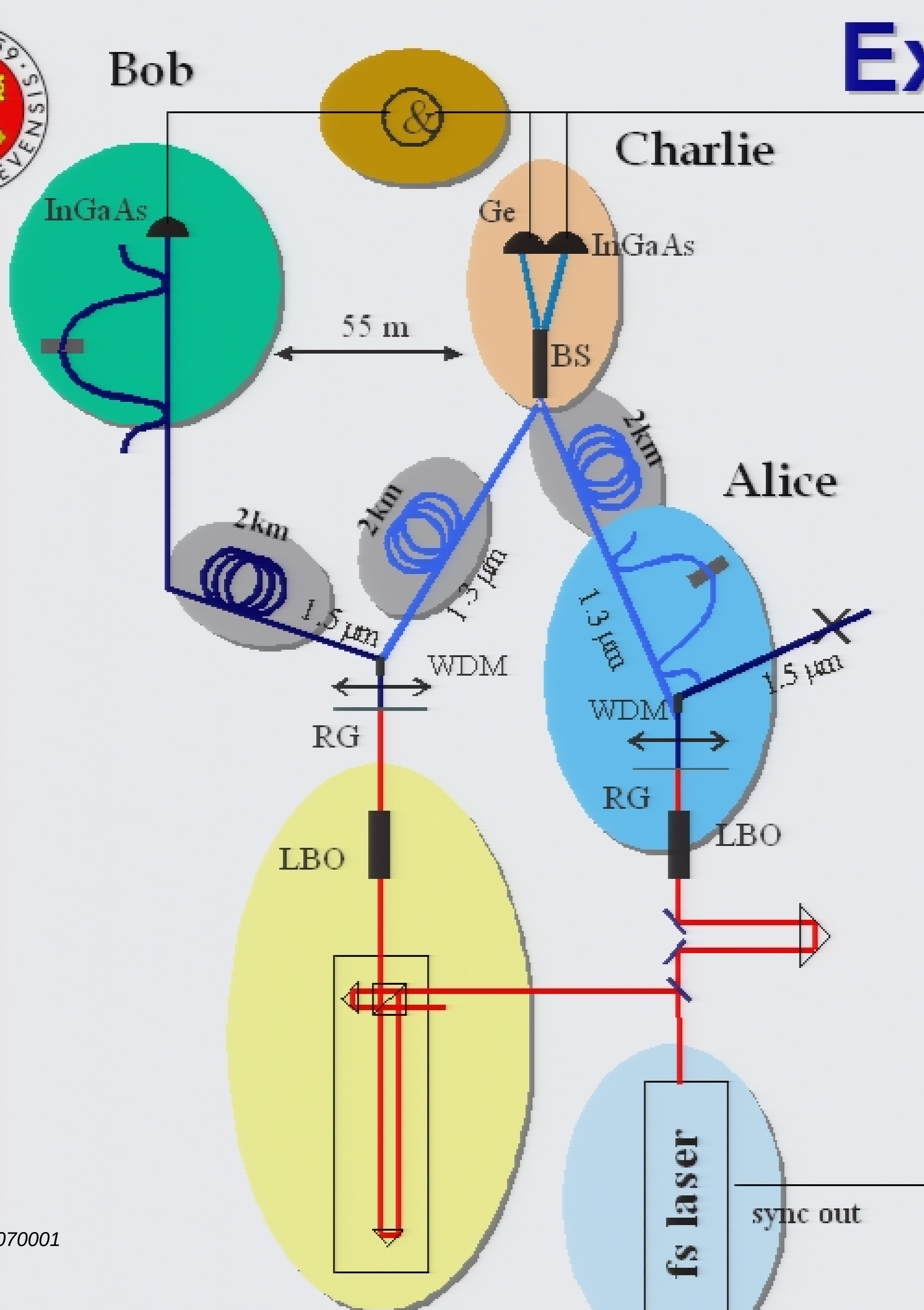


fs laser @ 710 nm

Alice:creation of qubits to be teleported

creation of entangled qubits

Charlie:the Bell measurement

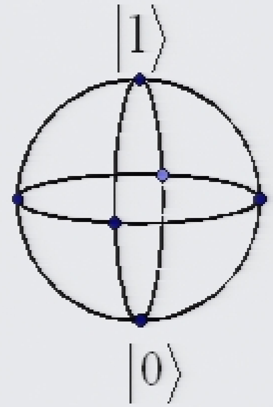
Bob:analysis of the teleported qubit, 55 m from Charlie

2 km of optical fiber

coincidence electronics

H. de Riedmaten et al.,

results



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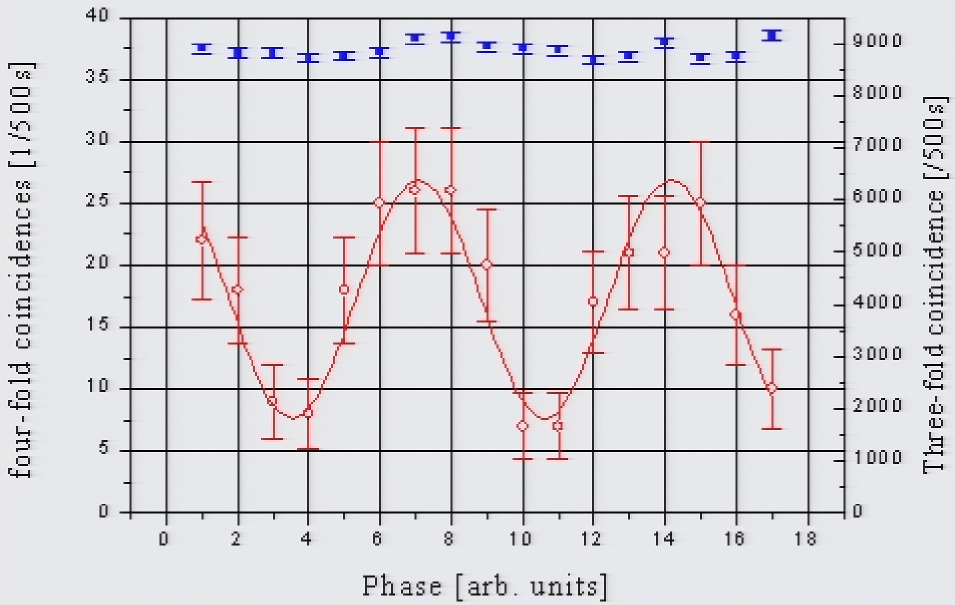
Equatorial states



Raw visibility : $V_{raw} = 55 \pm 5\ \%$

$$F_{eq} = \frac{1+V_{raw}}{2} = 77.5 \pm 2.5\ \%$$

results

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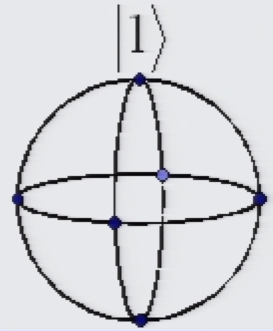
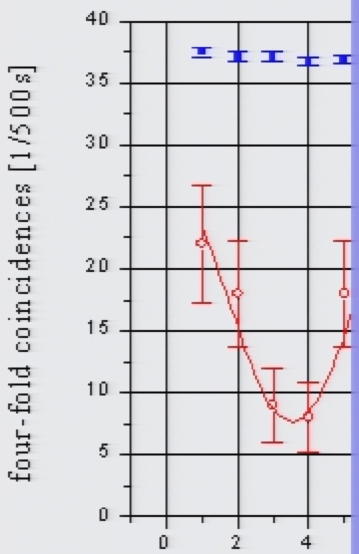


Equatorial states



$F = \frac{C_{correct}}{\ldots}$ wrong

± 3%

± 3%

77.5 ± 3 %

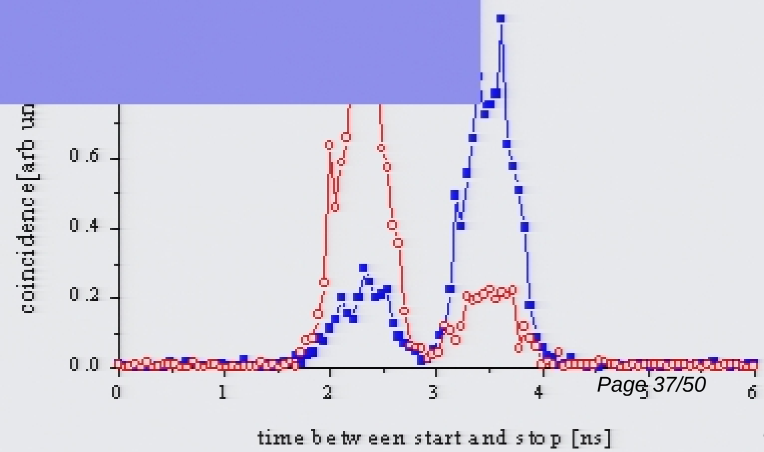
th poles

Mean Fidelity

$$F_{mean} = \frac{2}{3}F_{eq} + \frac{1}{3}F_p$$

$$= 77.5 \pm 2.5\ \%$$

» 67 % (no entanglement)



Raw visibility : $V_{raw} = 55 \pm 5\ \%$

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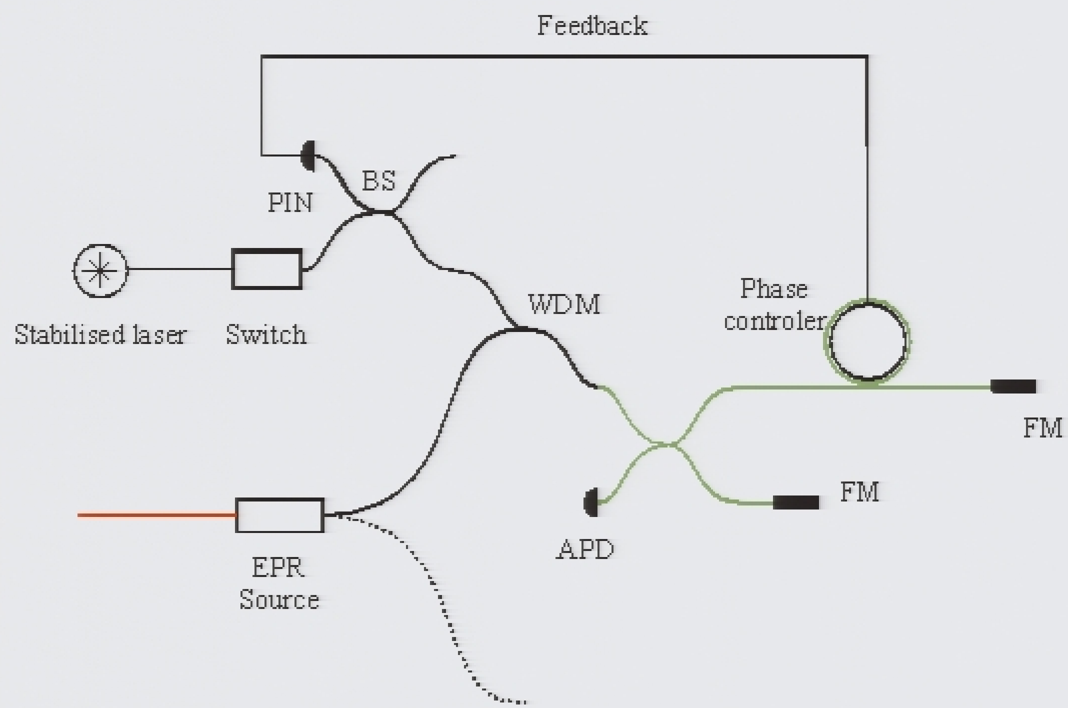
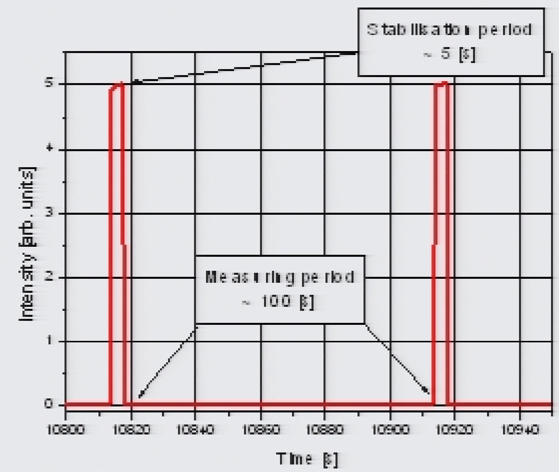
Stabilisation of the interferometers

- Idea: verify from time to time the phase





Every 100 s the phase is brought back to a given value

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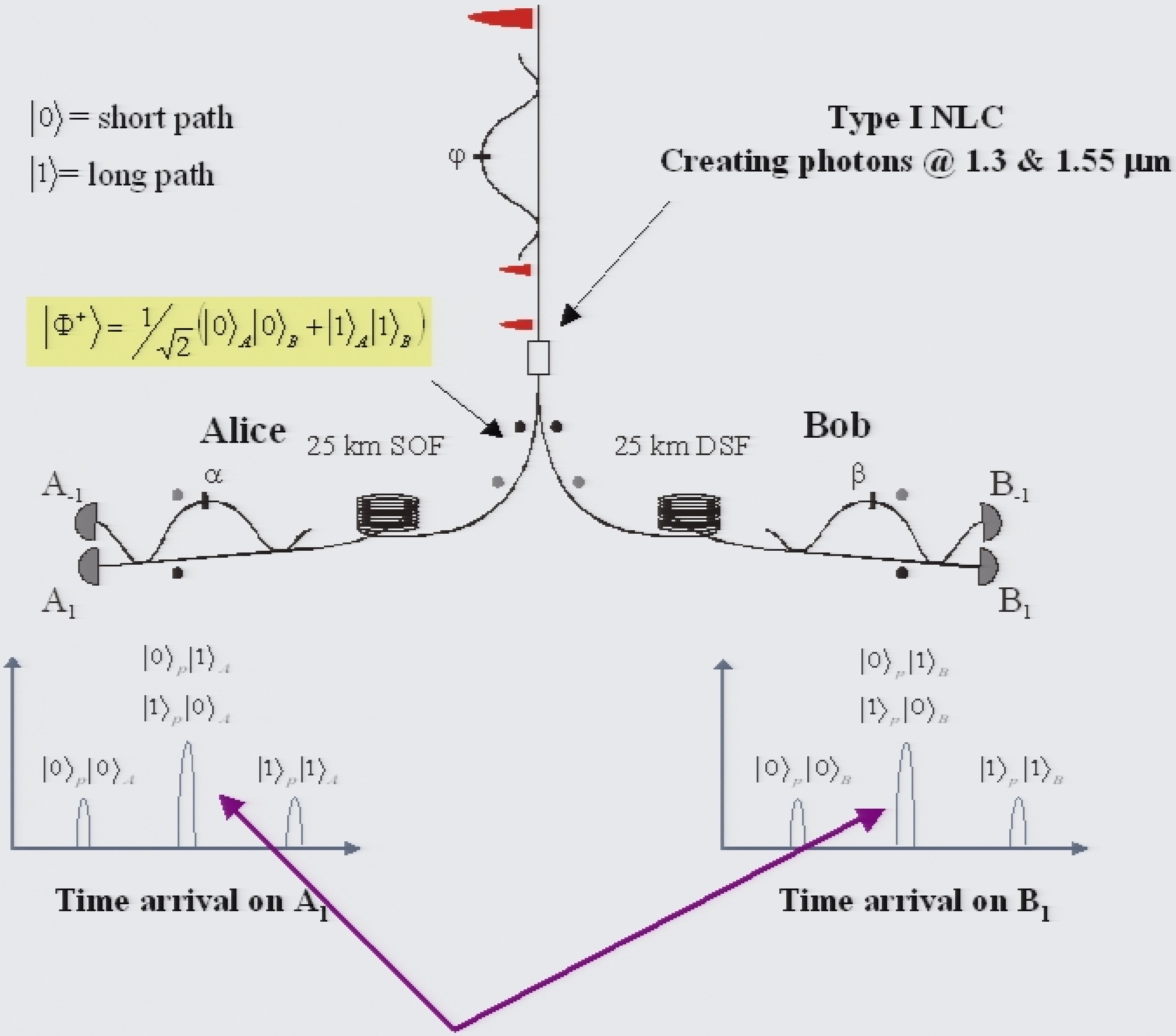
Bell test over 50 km



$$P_{ij}(\alpha,\beta) \propto 1 + ijV\cos(\alpha+\beta)$$

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Bell test over 50 km

- With phase control we can choose four different settings $\alpha = 0^\circ$ or 90° and $\beta = -45^\circ$ or 45°
- Violation of Bell inequalities:

$$S = E(\alpha = 0^\circ, \beta = -45^\circ) + E(\alpha = 90^\circ, \beta = -45^\circ) + E(\alpha = 0^\circ, \beta = 45^\circ) - E(\alpha = 90^\circ, \beta = 45^\circ)$$

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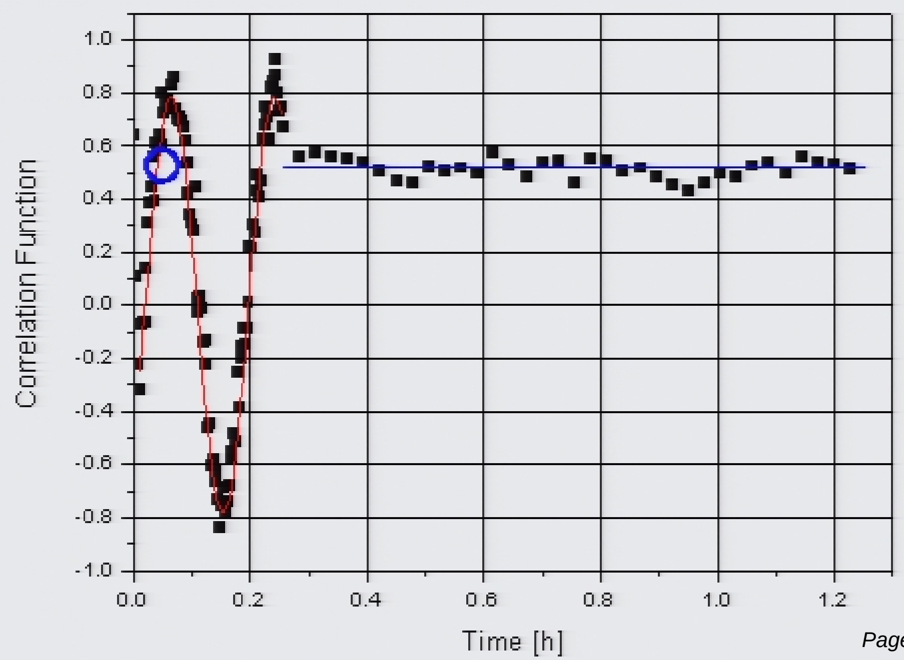
$$\boxed{E(\alpha=0^\circ, \beta=-45^\circ) = 0.518 \pm 0.006}$$



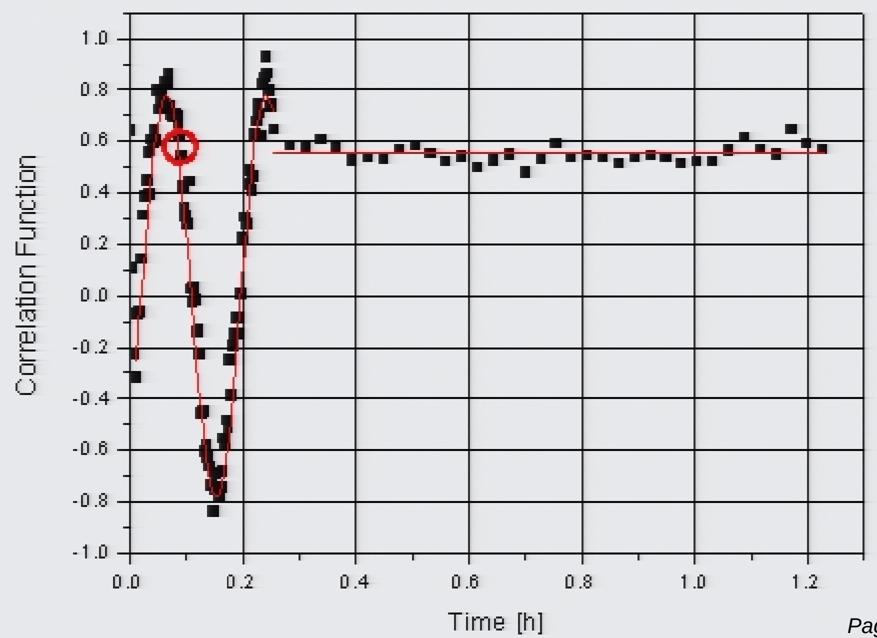
Bell test over 50 km

- With phase control we can choose four different settings $\alpha = 0^\circ$ or 90° and $\beta = -45^\circ$ or 45°
- Violation of Bell inequalities:

$$S = \boxed{E(\alpha = 0^\circ, \beta = -45^\circ)} + \boxed{E(\alpha = 90^\circ, \beta = -45^\circ)} + E(\alpha = 0^\circ, \beta = 45^\circ) - E(\alpha = 90^\circ, \beta = 45^\circ)$$

$$\boxed{E(\alpha = 0^\circ, \beta = -45^\circ) = 0.518 \pm 0.006}$$

$$\boxed{E(\alpha = 90^\circ, \beta = -45^\circ) = 0.554 \pm 0.005}$$



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Bell test over 50 km

- With phase control we can choose four different settings $\alpha = 0^\circ$ or 90° and $\beta = -45^\circ$ or 45°
- Violation of Bell inequalities:

$$S = \boxed{E(\alpha=0^\circ,\beta=-45^\circ)} + \boxed{E(\alpha=90^\circ,\beta=-45^\circ)} + \boxed{E(\alpha=0^\circ,\beta=45^\circ)} - \boxed{E(\alpha=90^\circ,\beta=45^\circ)}$$

$$E(\alpha=0^\circ,\beta=-45^\circ) = 0.518 \pm 0.006$$

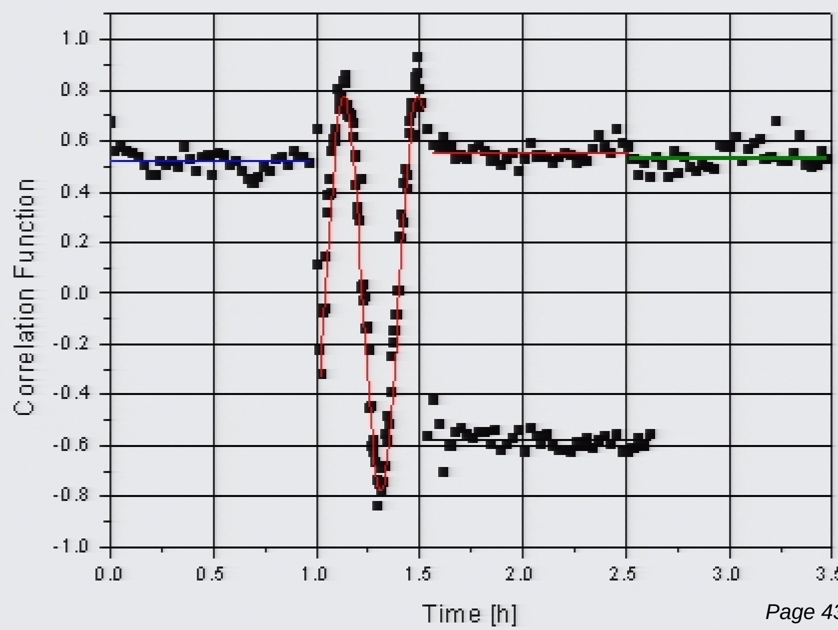
$$E(\alpha=90^\circ,\beta=-45^\circ) = 0.554 \pm 0.005$$

$$E(\alpha=0^\circ,\beta=45^\circ) = 0.533 \pm 0.006$$

$$E(\alpha=90^\circ,\beta=45^\circ) = 0.581 \pm 0.007$$

$$S = 2.185 \pm 0.012$$

Violation of Bell inequalities by more than 15 σ



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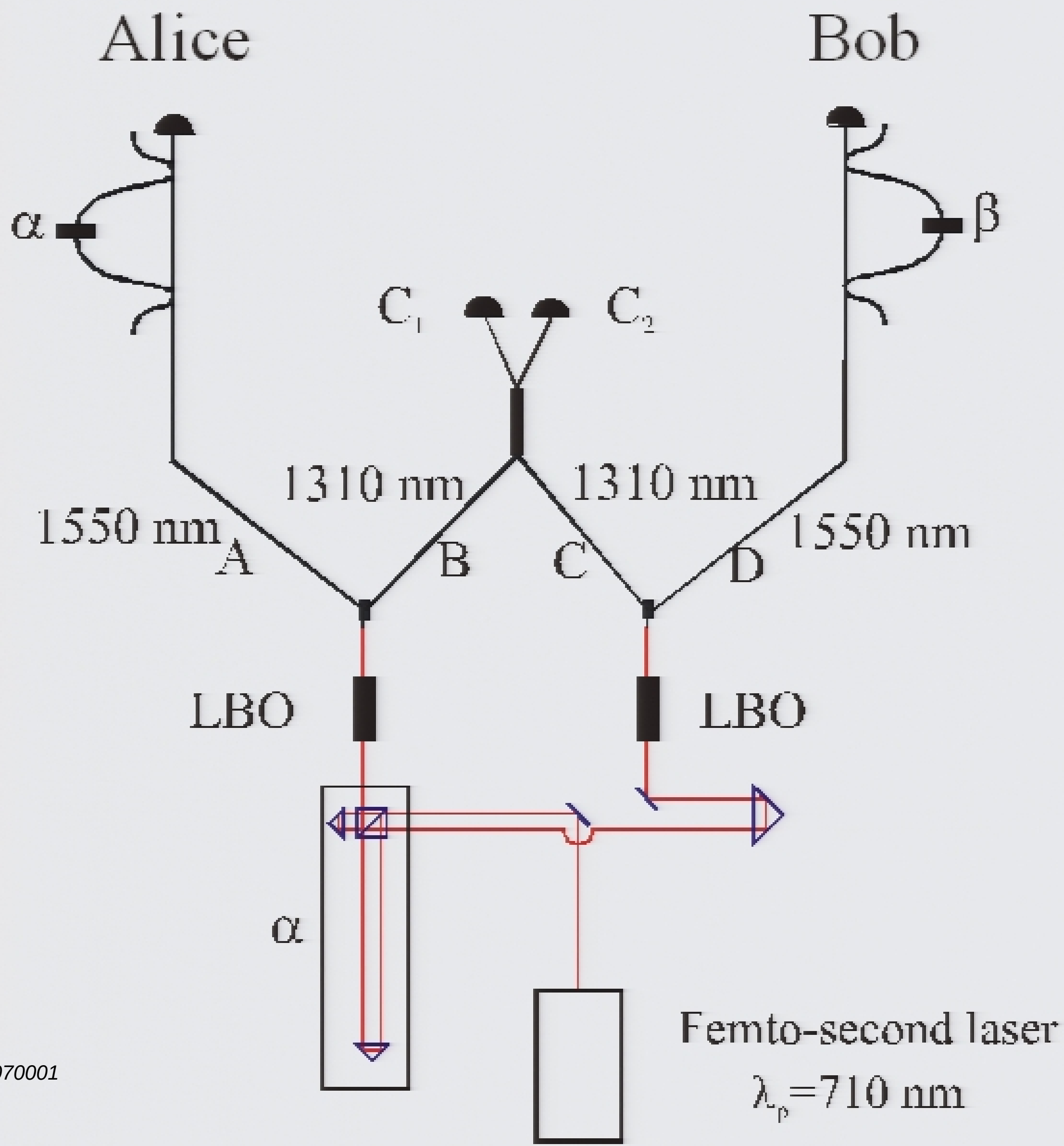
The experiment



The experiment

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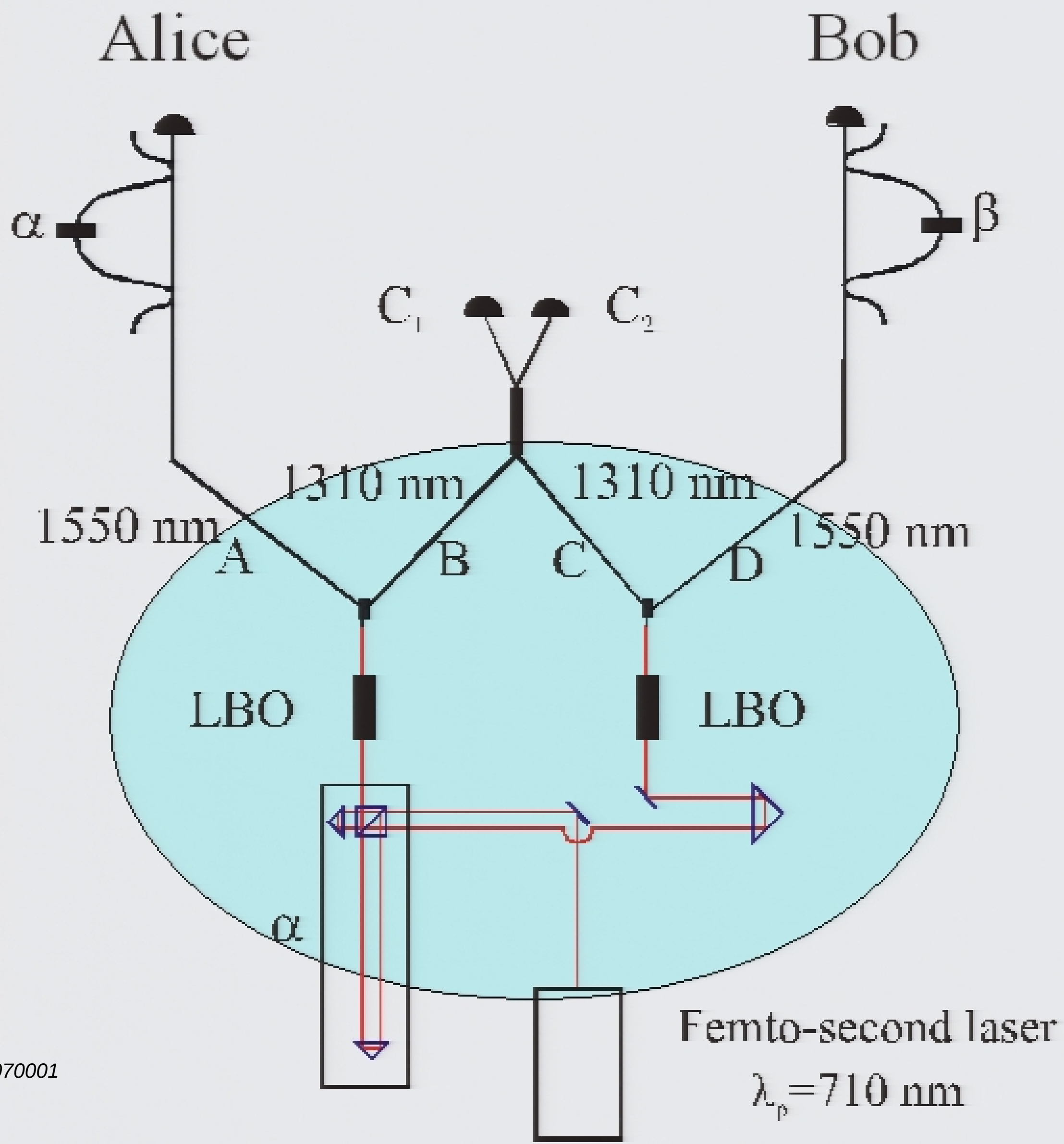

Sources of time-bin entangled photons

$$\left|\Phi^{(+)}\right\rangle_{AB} = \frac{1}{\sqrt{2}}\left(\left|0_A,0_B\right\rangle + e^{i\delta}\left|1_A,1_B\right\rangle\right)$$

$$\left|\Phi^{(-)}\right\rangle_{CD} = \frac{1}{\sqrt{2}}\left(\left|0_A,0_B\right\rangle - e^{i\delta}\left|1_A,1_B\right\rangle\right)$$

The experiment

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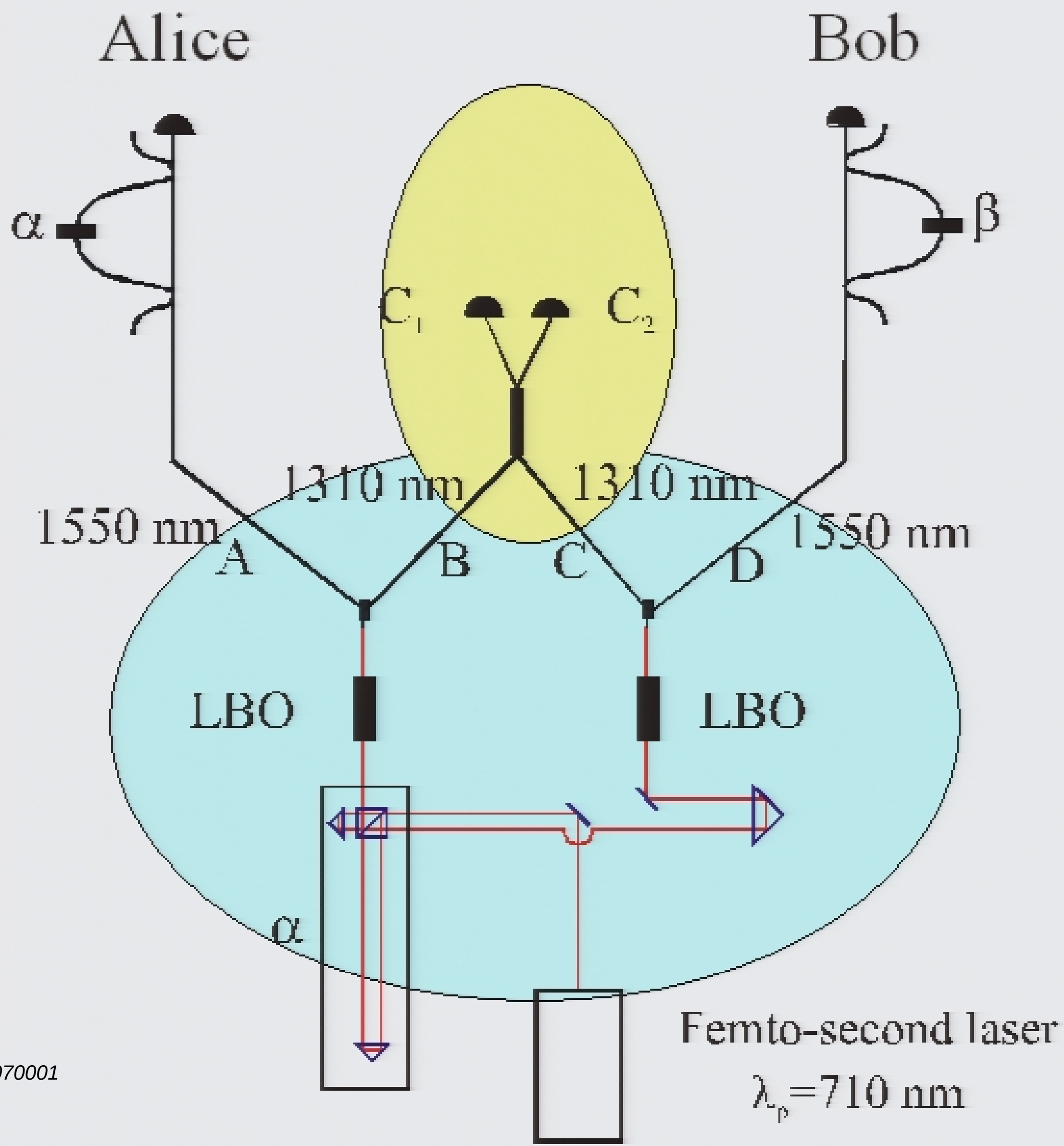

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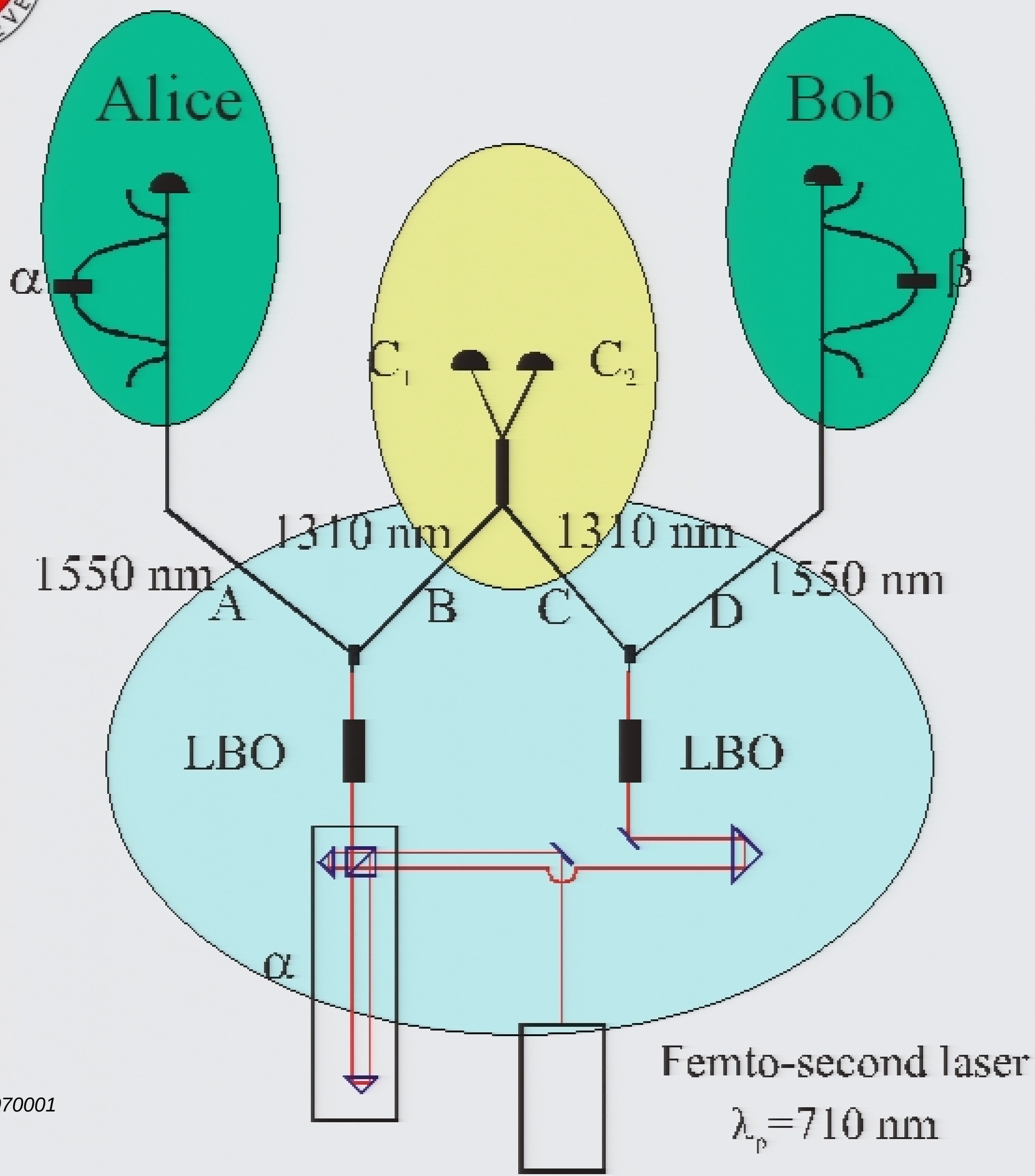
Bell state measurement

$$\left|\Psi^{(-)}\right\rangle_{BC} = \frac{1}{\sqrt{2}}\left(\left|0_B,1_C\right\rangle - \left|1_B,0_C\right\rangle\right)$$

$$\left|\Psi^{(+)}\right\rangle_{AD} = \frac{1}{\sqrt{2}}\left(\left|0_B,1_C\right\rangle + \left|1_B,0_C\right\rangle\right)$$

The experiment



Sources of time-bin entangled photons

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Bell state measurement

$$|\Psi^{(-)}\rangle_{BC} = \frac{1}{\sqrt{2}}\left(|0_B,1_C\rangle - |1_B,0_C\rangle\right)$$

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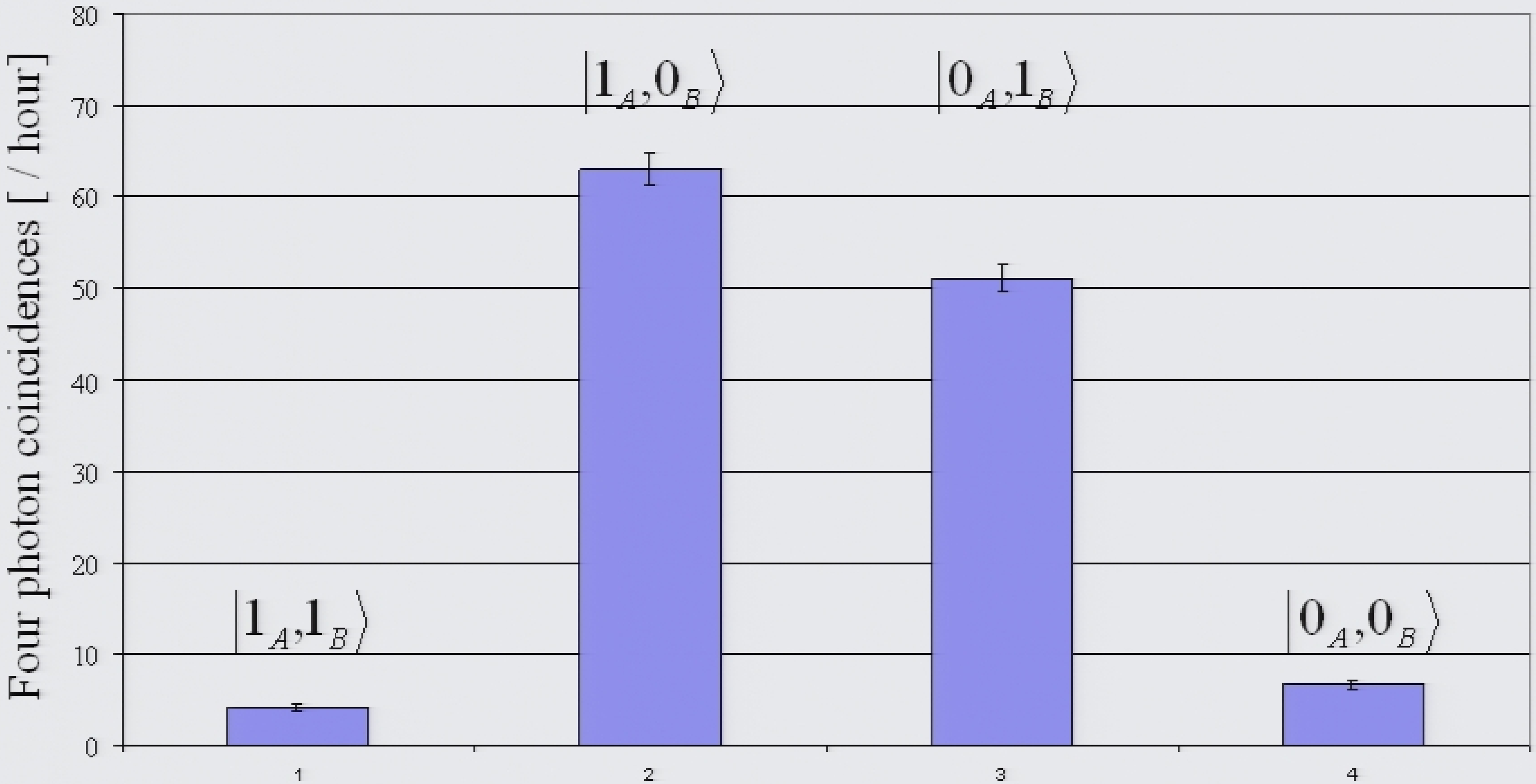
Entanglement analysis

Results: computational basis



Fidelity = (91,3 ± 2,5) %

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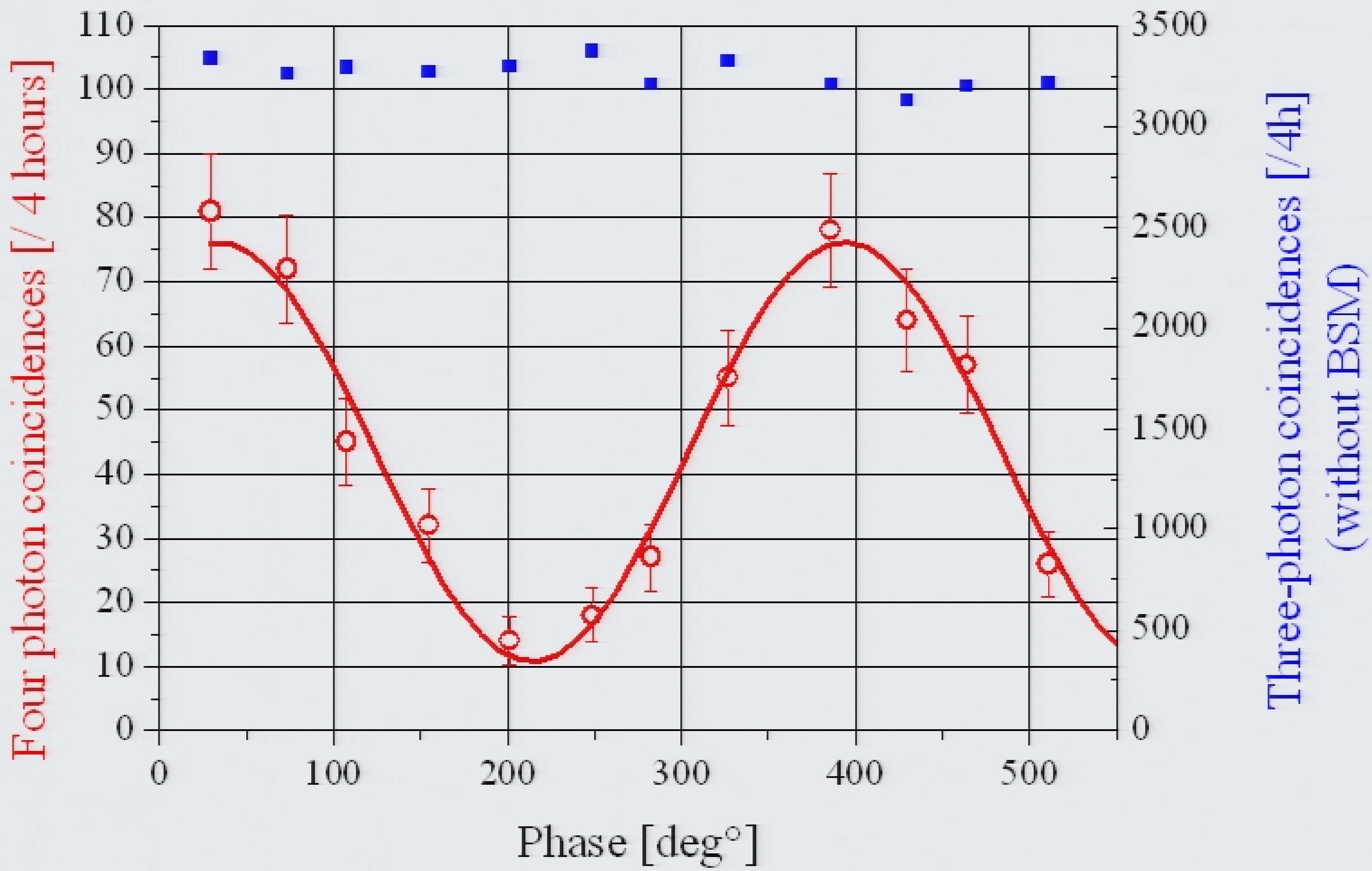
Superposition basis: first results



$V = (75 \pm 5.5)\ \%$

$$F = \frac{1+V}{2} = (87.5 \pm 2.6)\%$$

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Thank you !