Title: The Stability of the Solar System

Date: Jun 02, 2004 07:00 PM

URL: http://pirsa.org/04060000

Abstract: For over three hundred years, physicists and mathematicians have been trying to understand how stable the Earth really is. Could gravitational forces from other planets lead to drastic changes in EarthÂ's orbit? Will we collide with other planets or be ejected into interstellar space? <kw> stability, solar systems, Scott Tremaine, Copernicus, Copernican principle, Kepler, Newton, motion, gravity, N-Body, dynamical system, Laplace, round off error, gravitational microlensing, MOSTorbit, chaos </kw>

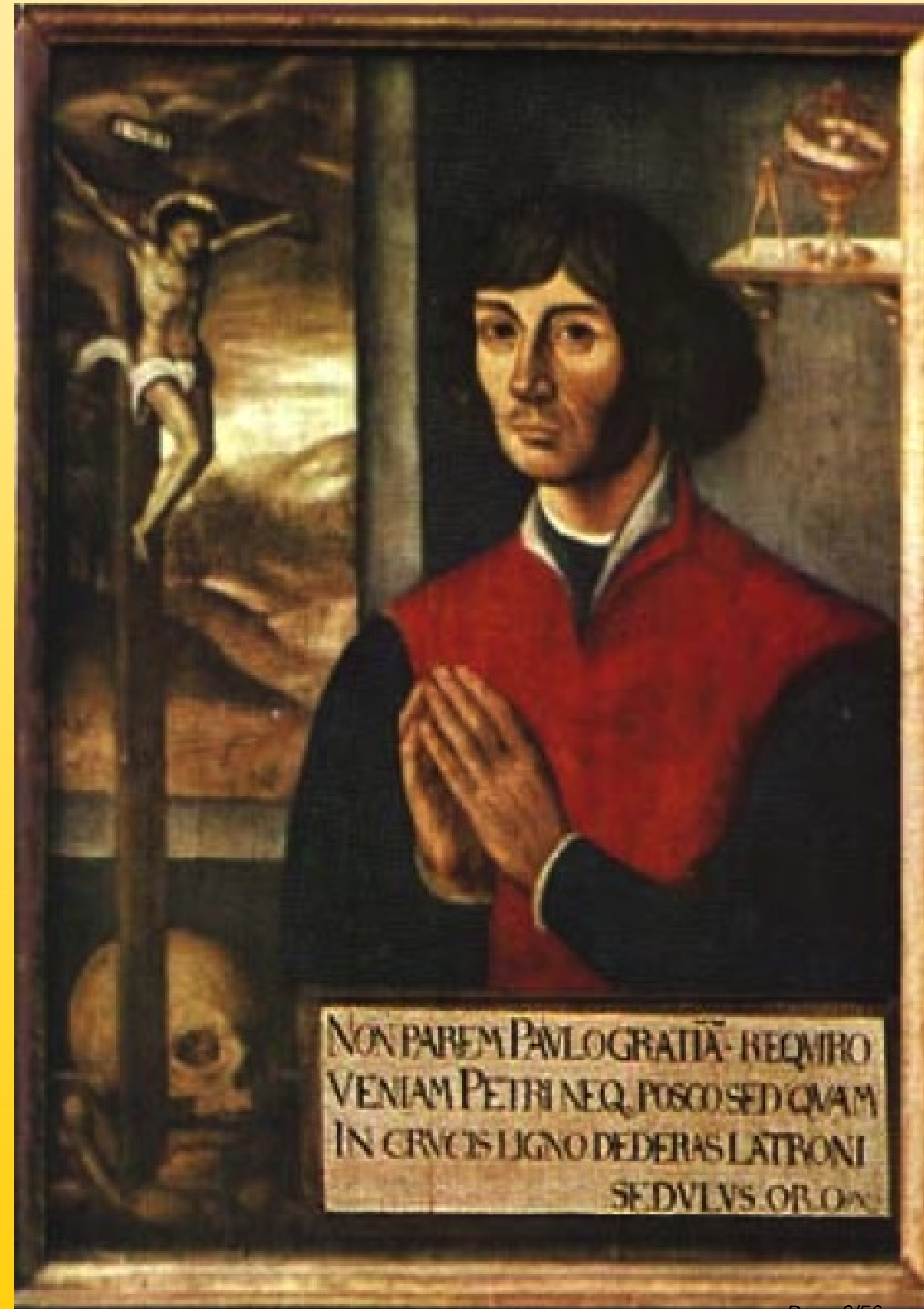
The stability of the solar system

Scott Tremaine
Princeton University

Nicolaus Copernicus

1473 - 1543

Copernicus showed that the Sun, not the Earth, was the centre of the solar system



Nicolaus Copernicus

1473 - 1543

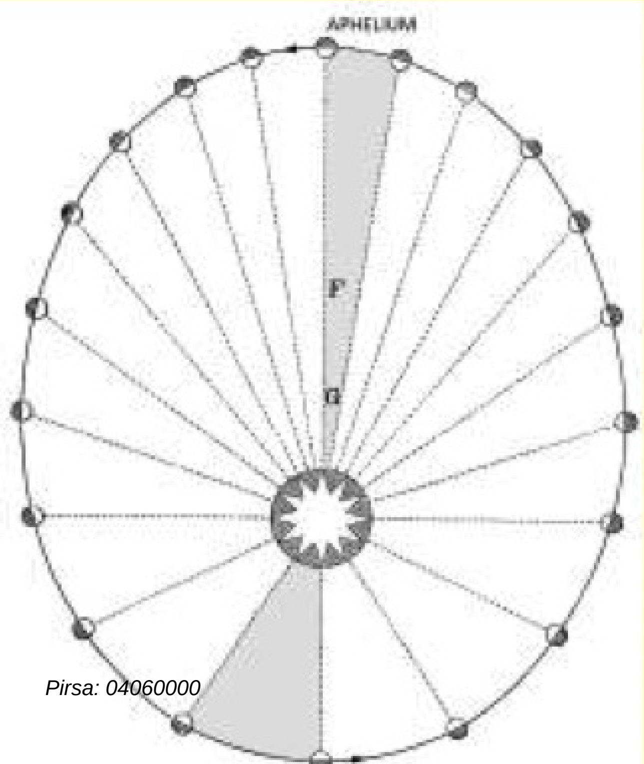
The Copernican Principle:

We are not located at a special place in the Universe, or at a special time in the history of the Universe

Johannes Kepler

1571-1630

Kepler showed that the motions of the planets obeyed three empirical laws



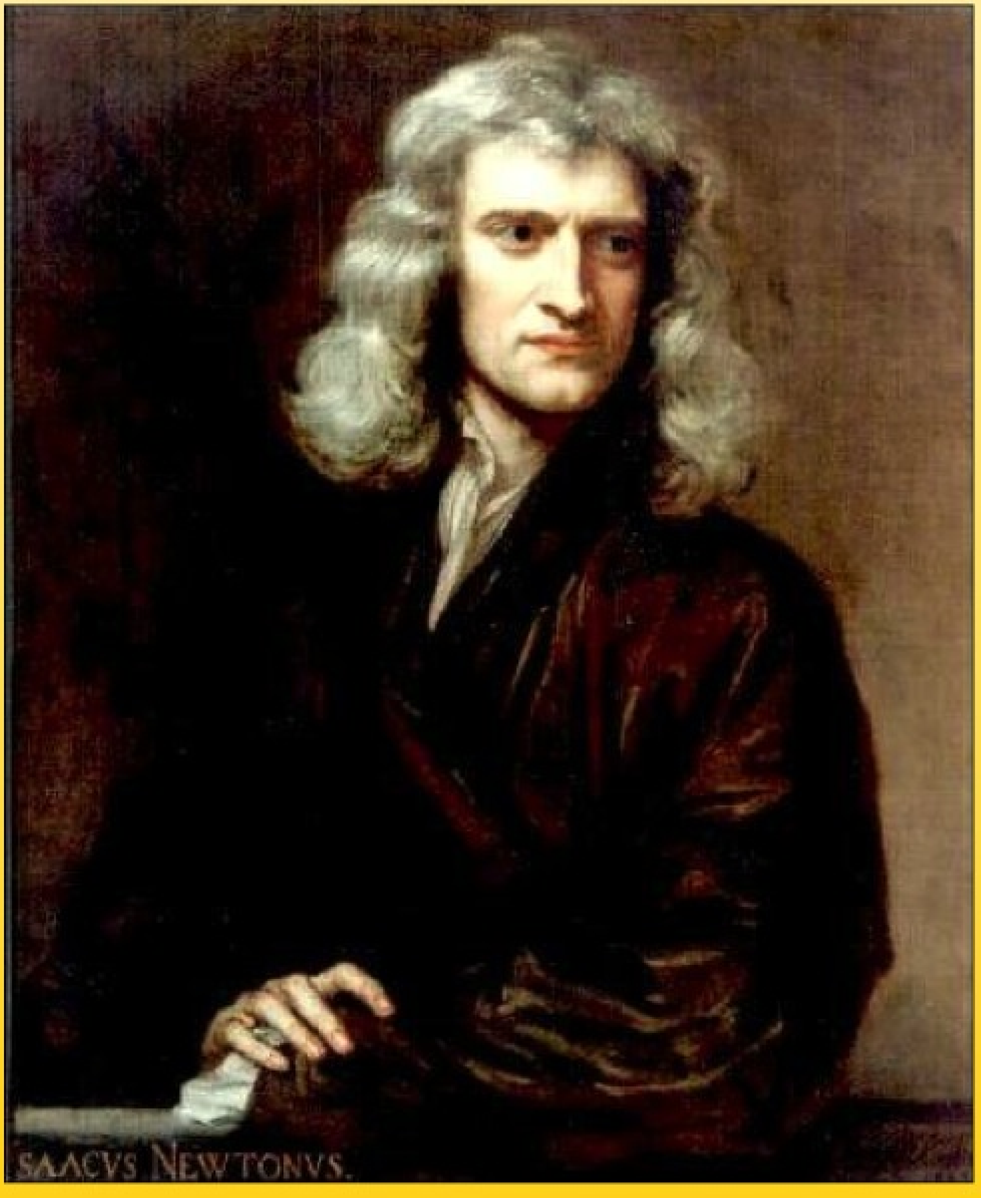
1. **The orbit of a planet is an ellipse with the Sun at one focus**
2. **A line joining the planet and the Sun sweeps out equal areas in equal time intervals**
3. **The square of the period of a planet is proportional to the cube of its semi-major axis**

Isaac Newton

1642-1747

Newton's law of motion:
force = mass × acceleration or F=ma

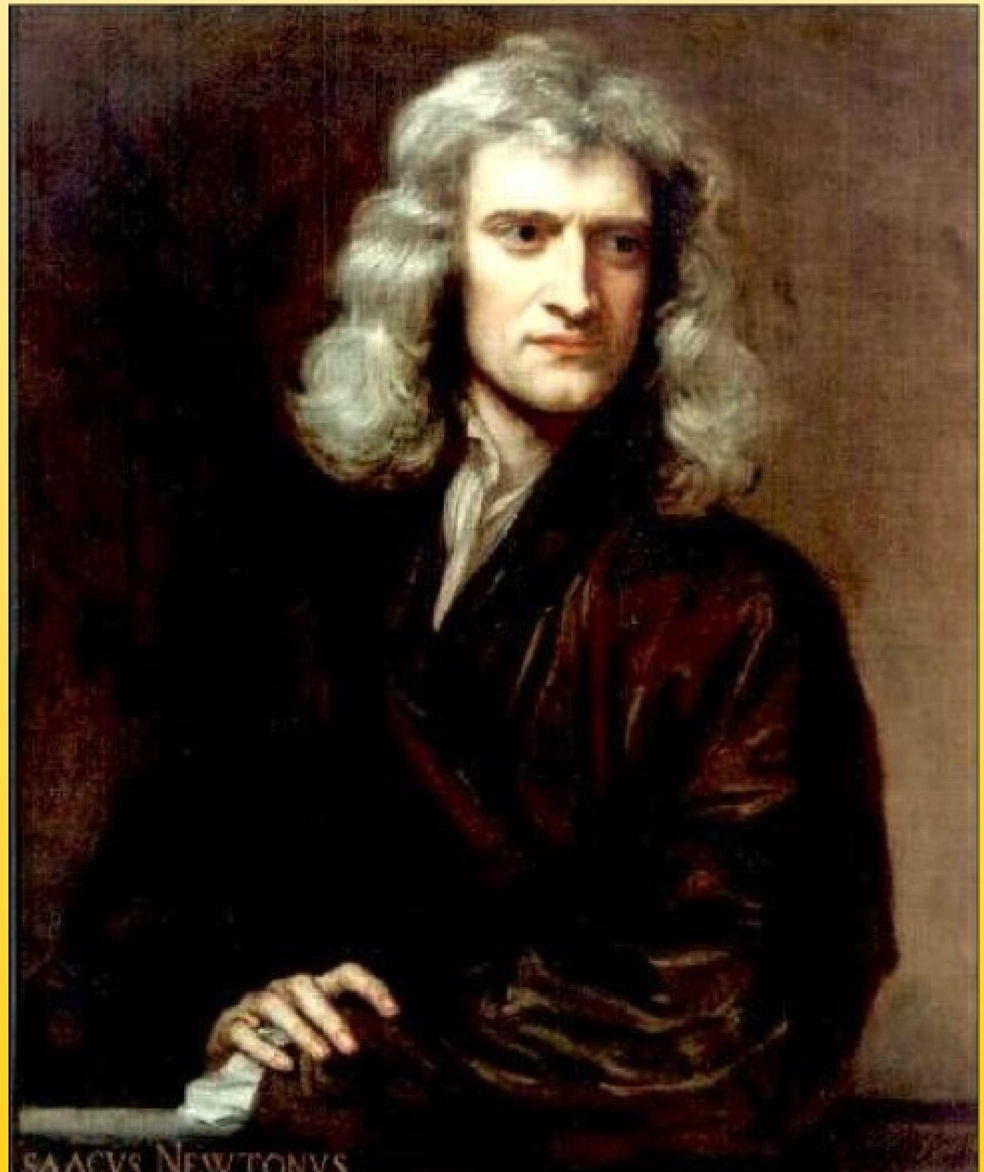
Newton's law of gravity:
force between two bodies of mass m and M at distance r is $F=GmM/r^2$



Isaac Newton

1642-1747

Newton solved the equations of motion for the Sun and one planet and showed that the motion of the planet satisfied Kepler’s laws



The gravitational N-body problem

The problem:

A point mass is surrounded by N much smaller masses on nearly circular orbits. Is the configuration stable over very long times?

For the solar system:

- **the point mass is the Sun**
- **N=9 (the nine planets)**
- **"very long times" = about 10 billion orbits (4.5 billion years since the formation of the solar system, 7 billion years until the Sun turns into a red giant)**

The gravitational N-body problem

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What have we left out?

- asteroids
- comets
- mass loss from Sun through radiation and solar wind
- drag of solar wind on planetary magnetospheres
- Galactic tidal forces
- satellites
- general relativity
- passing stars (closest passage about 500 AU)

All of these are either straightforward to calculate or small at the level of a part per billion or less

The gravitational N-body problem

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Why do we care?

- what is the fate of the Earth?
- why are there so few planets?
- why are the regions between the planets so clean?
- where do comets and meteors come from?
- was the present state of the solar system determined in the first 0.1% of its lifetime?
- what will other planetary systems look like?
- can we calibrate the geological timescale over the last 100 million years?
- how do dynamical systems behave over very long times?

Stability of the solar system

What Newton said:

"blind fate could never make all the Planets move one and the same way in Orbs concentric, some inconsiderable irregularities excepted, which could have arisen from the mutual Actions of Planets upon one another, and which will be apt to increase, until this System wants a reformation" – Opticks, Book III

"In the system of planets, Newton sees careful planning on the part of Providence that has ensured long-term stability; but the stability is not completely permanent…Providence must intervene to prevent gravitational collapse, and will thereby demonstrate a continuing concern for the welfare of mankind. In short, Providence has…a regular servicing contract with the solar system." – Hoskins (1985)

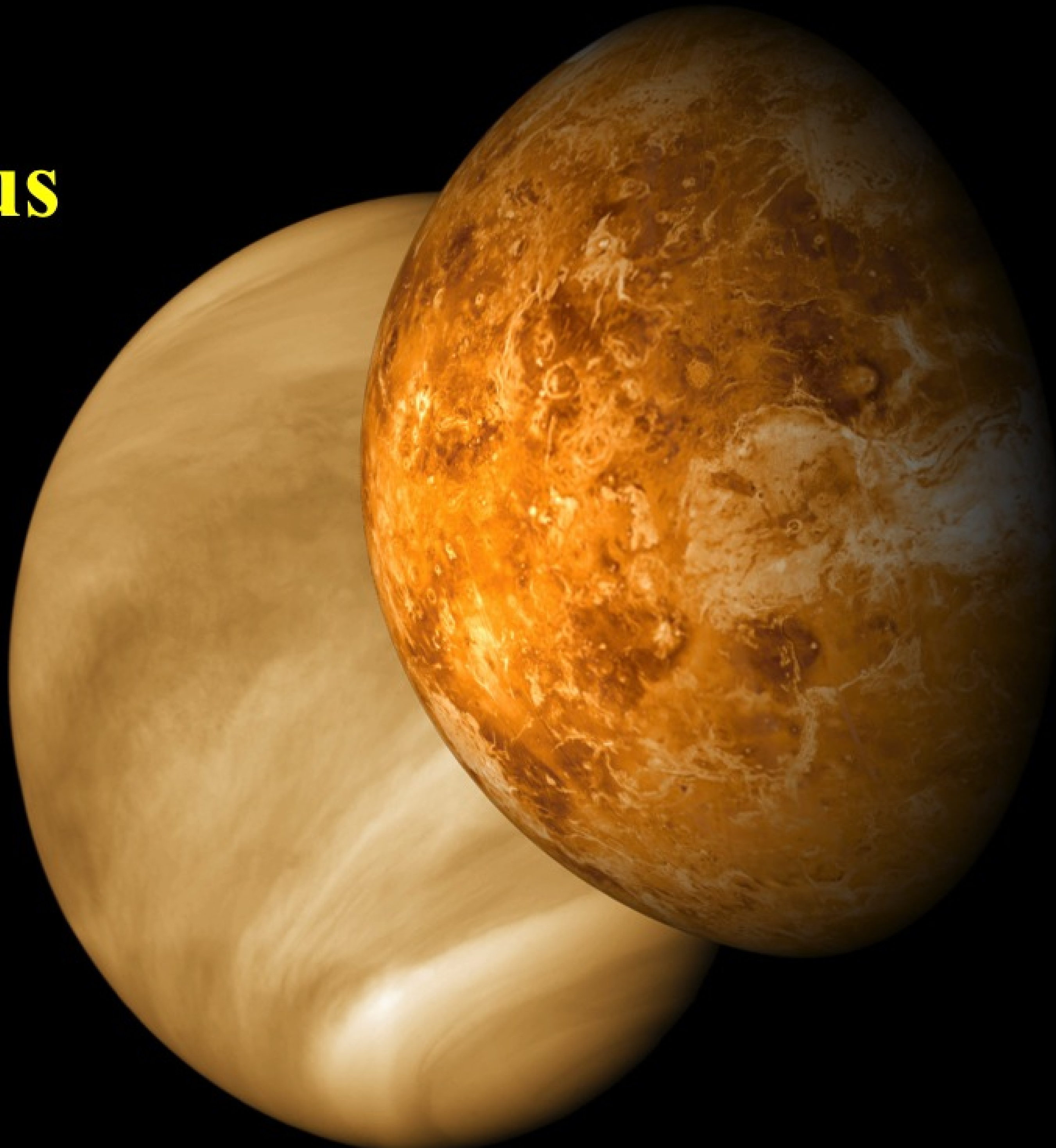
Stability of the solar system

What Laplace said:

"An intelligence knowing, at a given instant of time, all forces acting in nature, as well as the momentary positions of all things of which the universe consists, would be able to comprehend the motions of the largest bodies of the world and those of the smallest atoms in one single formula, provided it were sufficiently powerful to subject all data to analysis. To it, nothing would be uncertain; both future and past would be present before its eyes."

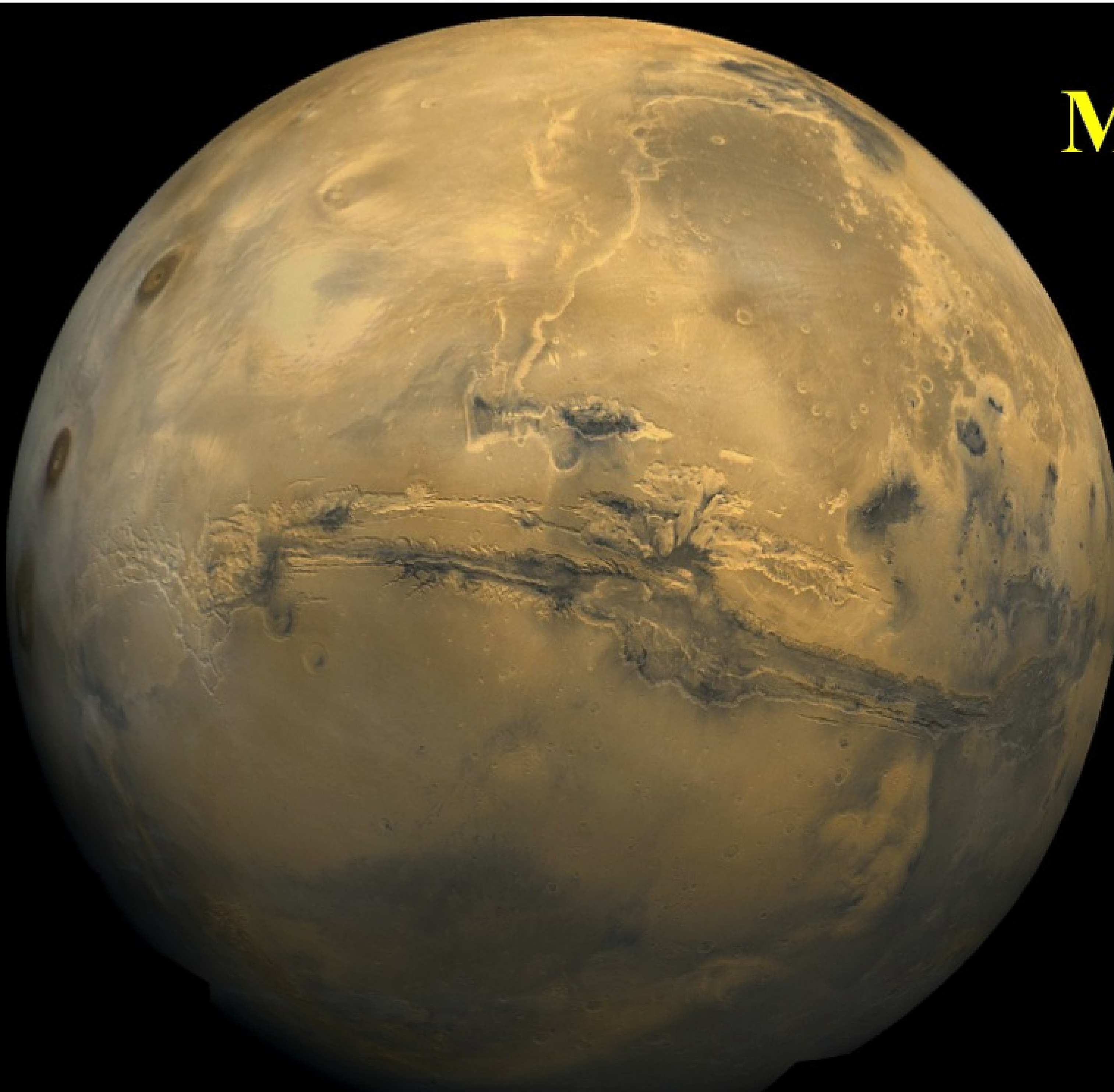
Mercury

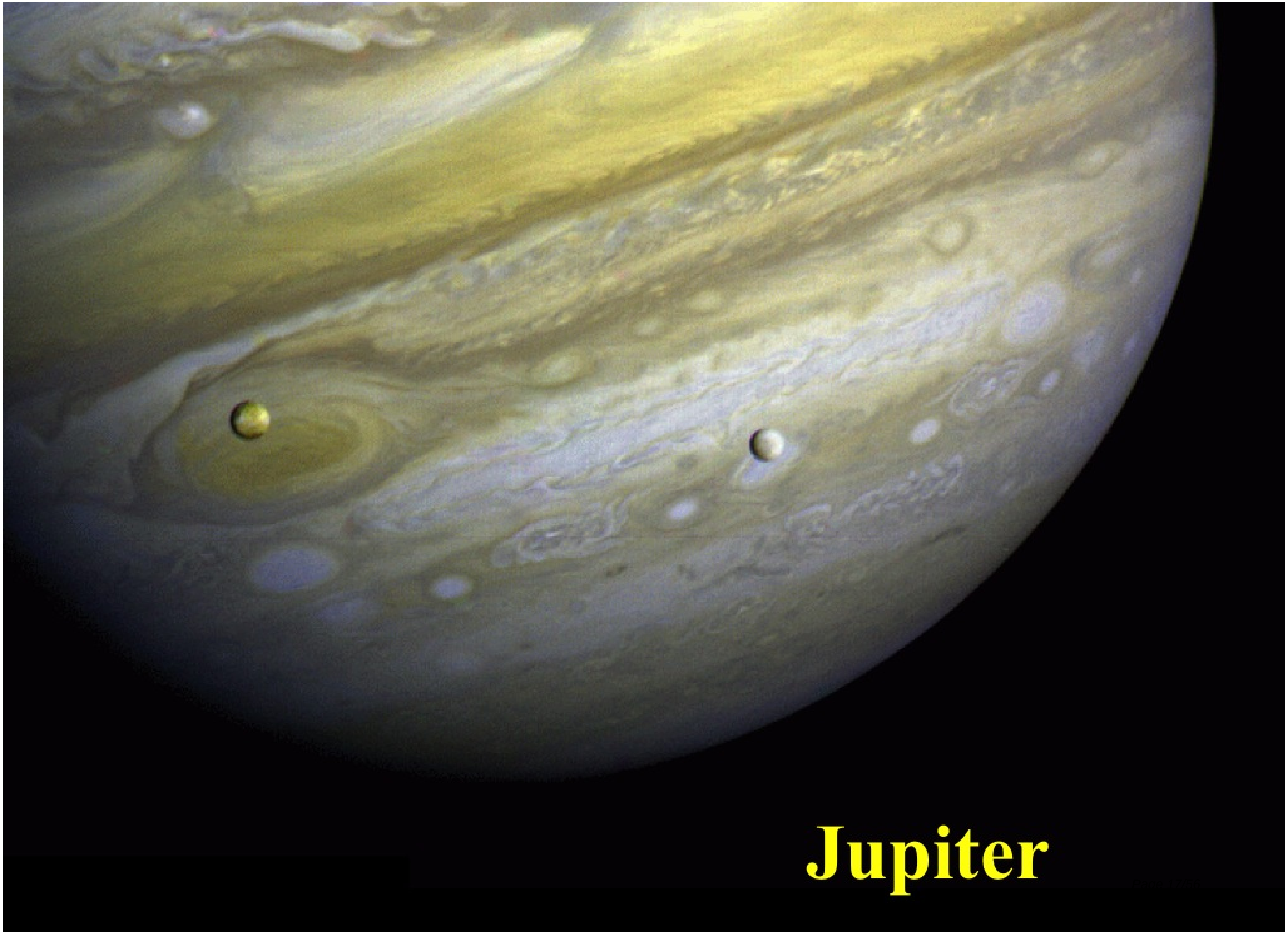
Venus



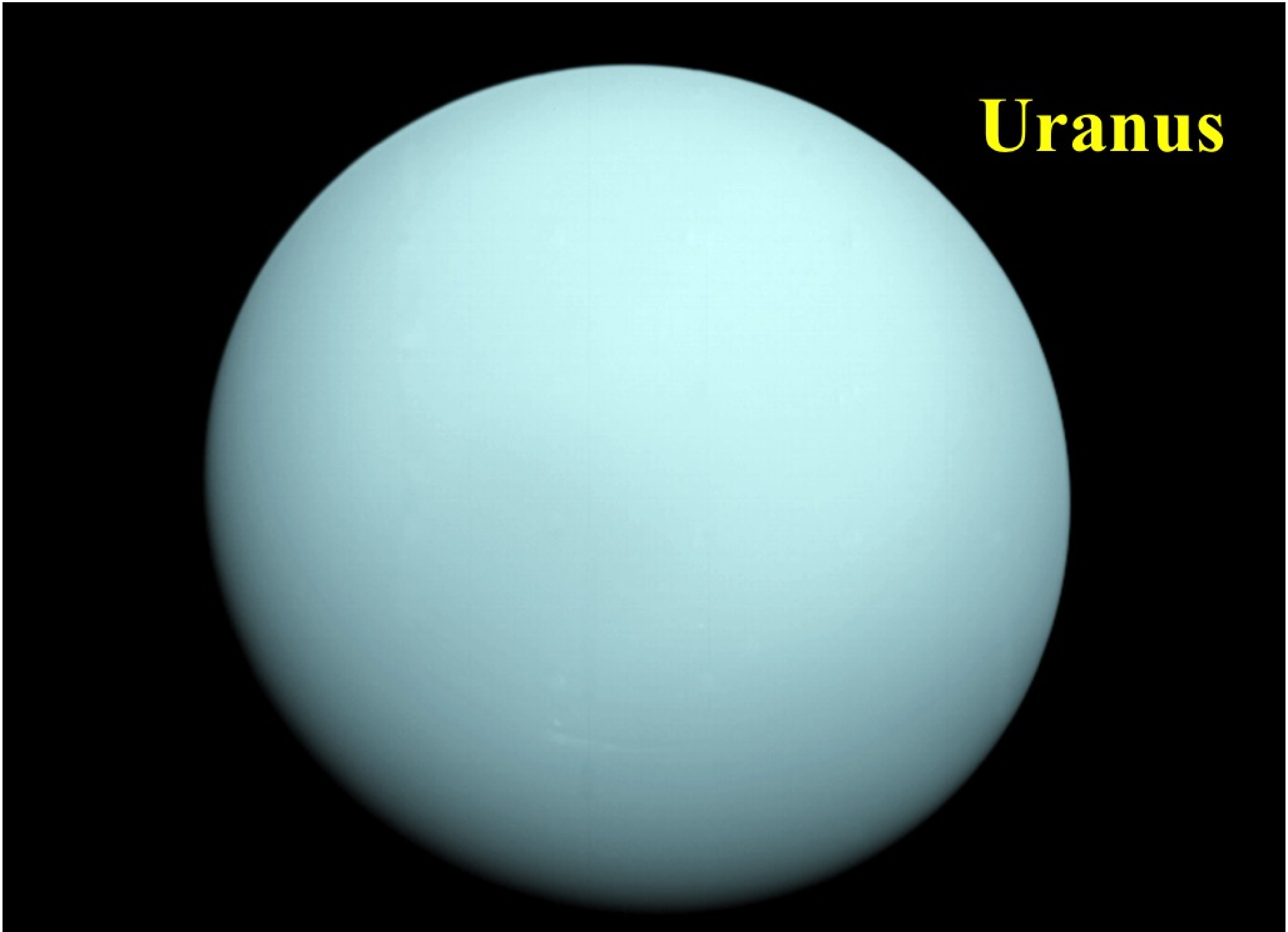

Earth
orbital radius = 1
astronomical unit =

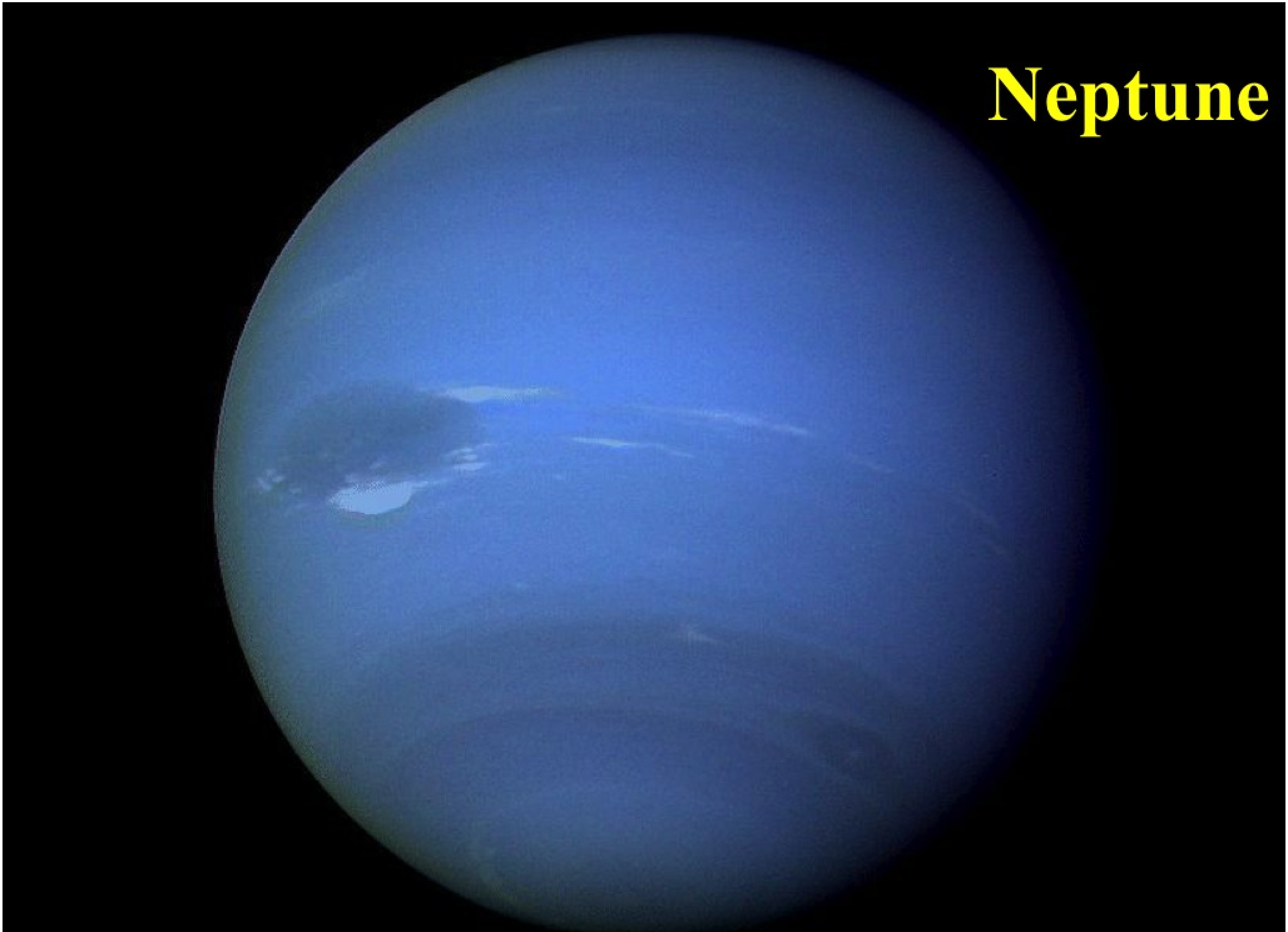
Mars




Jupiter

Saturn


Uranus


Neptune

Pluto and Charon


asteroids

comets

The gravitational N-body problem

The problem:

A point mass is surrounded by N much smaller masses on nearly circular orbits. Is the configuration stable over very long times?

How can we solve this?

- many famous mathematicians since Newton have attempted to find solutions, but only with limited success
- only feasible approach is numerical solution of equations of motion but:
  - needs sophisticated algorithms to avoid buildup of errors
  - difficult to parallelize
  - typical calculations last several months
  - roundoff error

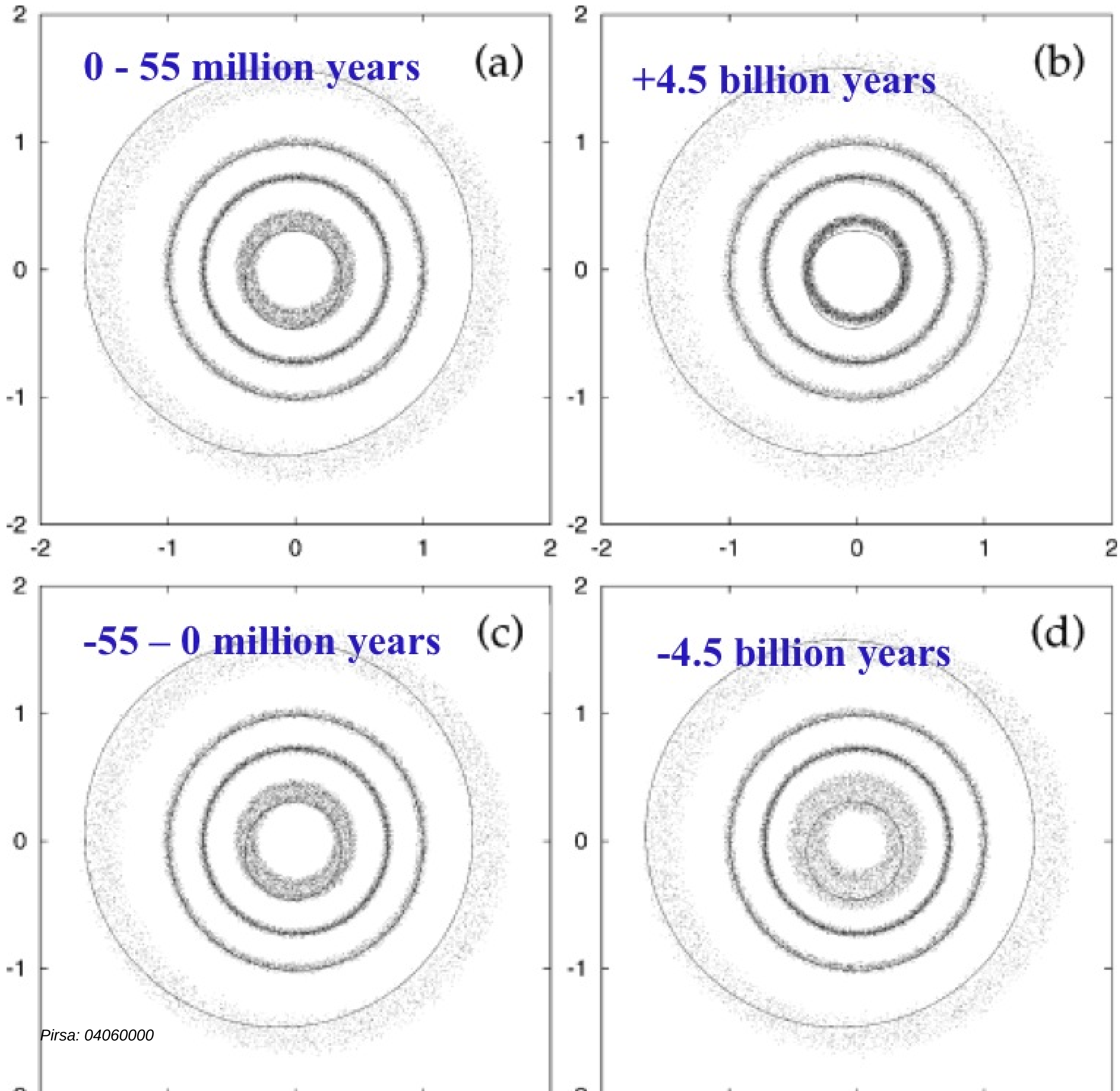
Roundoff error

Famous examples of problems due to roundoff error:

- new Vancouver stock exchange index was initialized in 1982 at 1000.0. After 22 months the index stood at 524.881 despite a rising market
- in 1991 Gulf War, Patriot missile defense system converted clock steps of 0.1 sec to decimal by multiplying by a 22-bit binary number; after 100 hours the accumulated error was 0.3 sec, which led to failure to intercept a Scud missile, resulting in 28 deaths

How to minimize roundoff error:

- always use highest possible precision (usually 53 bit)
- never use a fixed number that's not representable in the computer
- use optimal (IEEE-1985 compliant) arithmetic
- round 5's properly



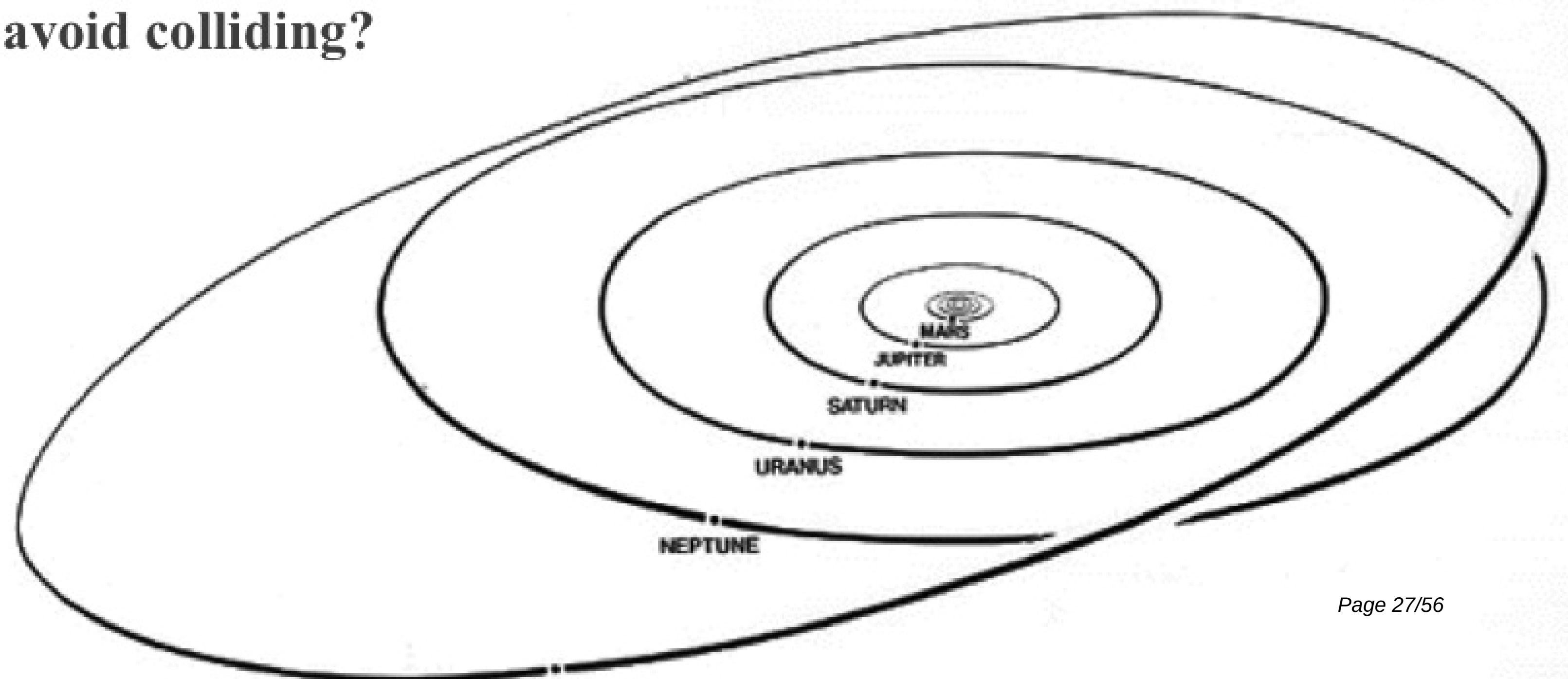
Ito & Tanikawa (2002)

Pluto's peculiar orbit

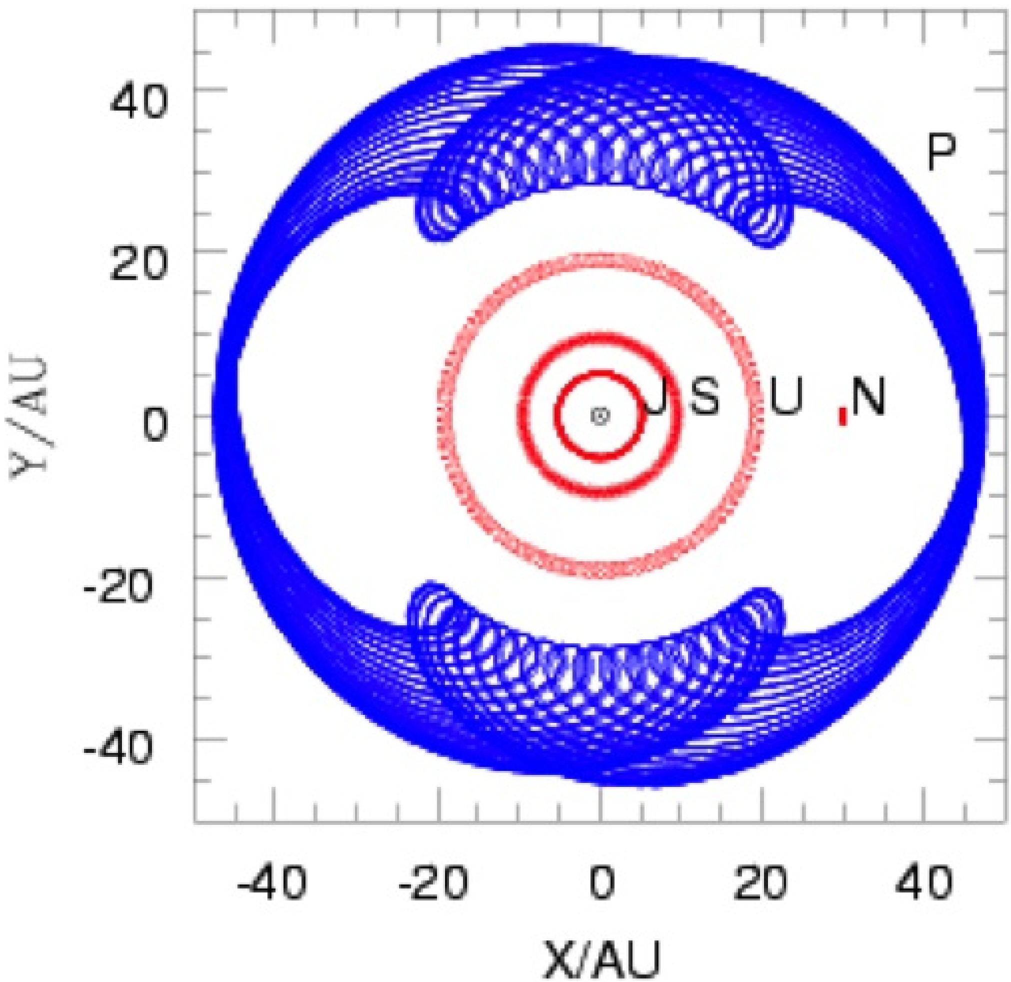
Pluto has:

- **the highest eccentricity of any planet (e = 0.250)**
- **the highest inclination of any planet (i = 17^{o})**
- **Pluto approaches closer to the Sun (29.6 AU) than Neptune's orbital radius (30.1 AU)**

How do they avoid colliding?



Pluto's peculiar orbit



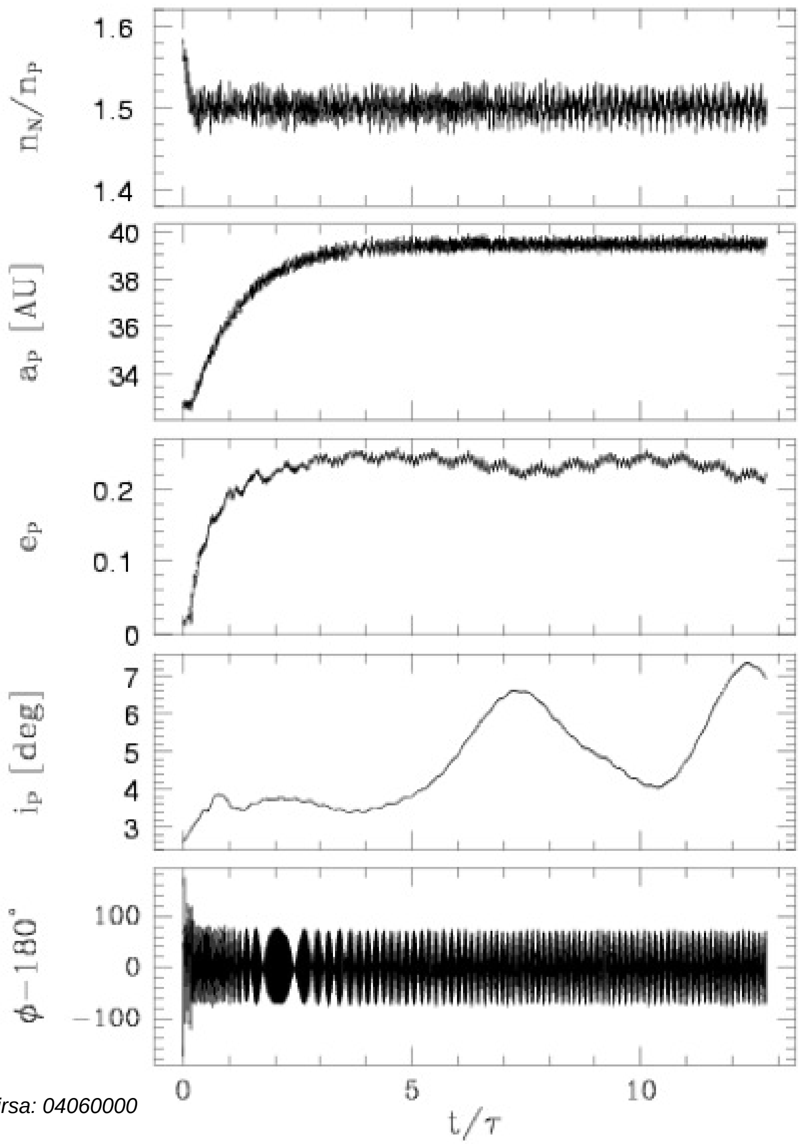
Orbital period of Pluto = 247.7 y

Orbital period of Neptune = 164.8 y

247.7/164.8 = 1.50 = 3/2

Resonance ensures that Pluto only crosses Neptune's orbit when it is approximately 90° away from Neptune

Pluto's peculiar orbit



•early in the history of the solar system there was debris (planetesimals) left over between the planets

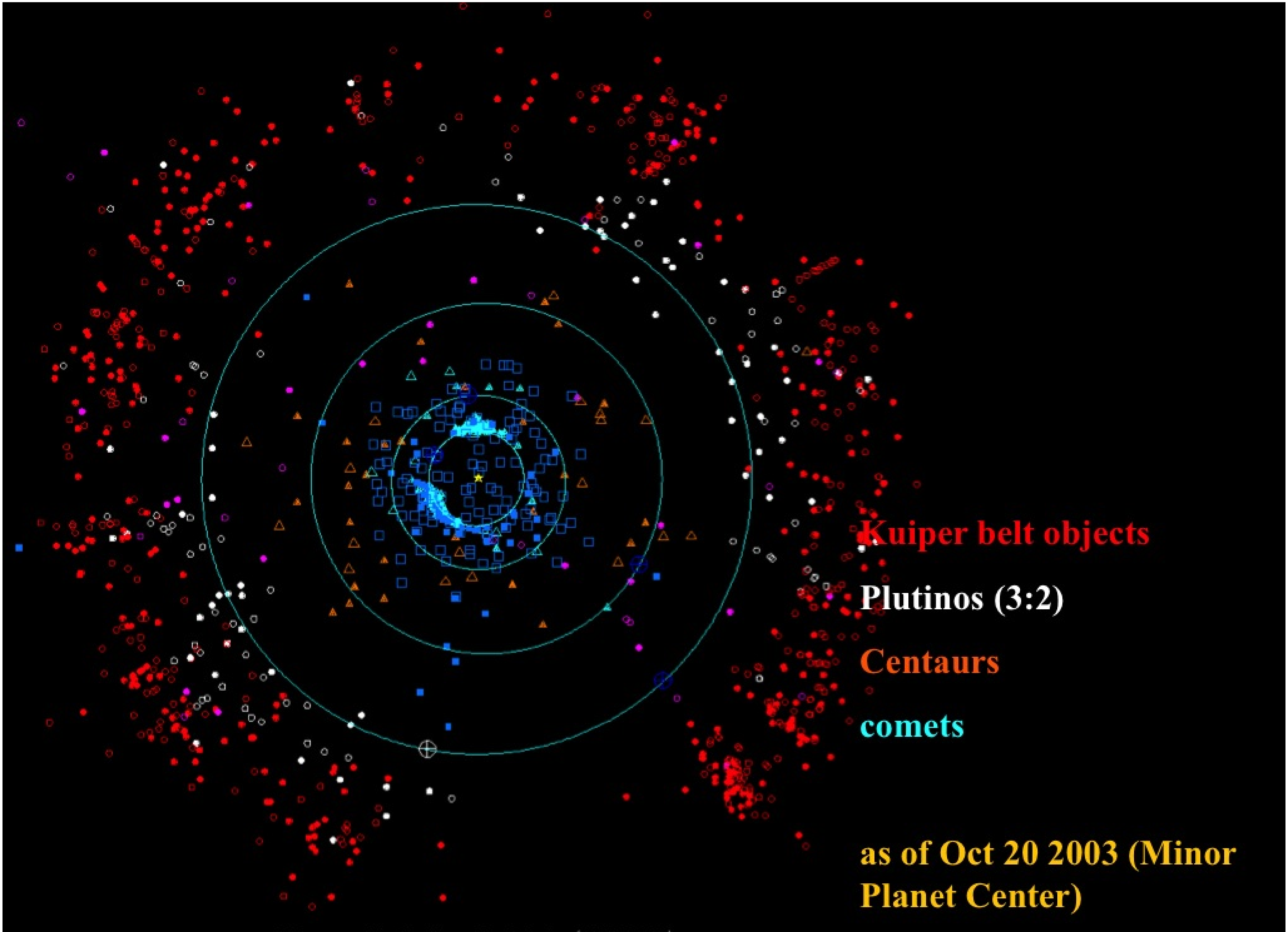
•ejection of this debris by Neptune caused its orbit to migrate outwards

•if Pluto were initially in a low-eccentricity, low-inclination orbit outside Neptune it is inevitably captured into 3:2 resonance with Neptune

•once Pluto is captured its eccentricity and inclination grow as Neptune continues to migrate outwards

•other objects may be captured in the resonance as well

Malhotra (1993)



Kuiper belt objects
Plutinos (3:2)
Centaurs
comets
as of Oct 20 2003 (Minor Planet Center)

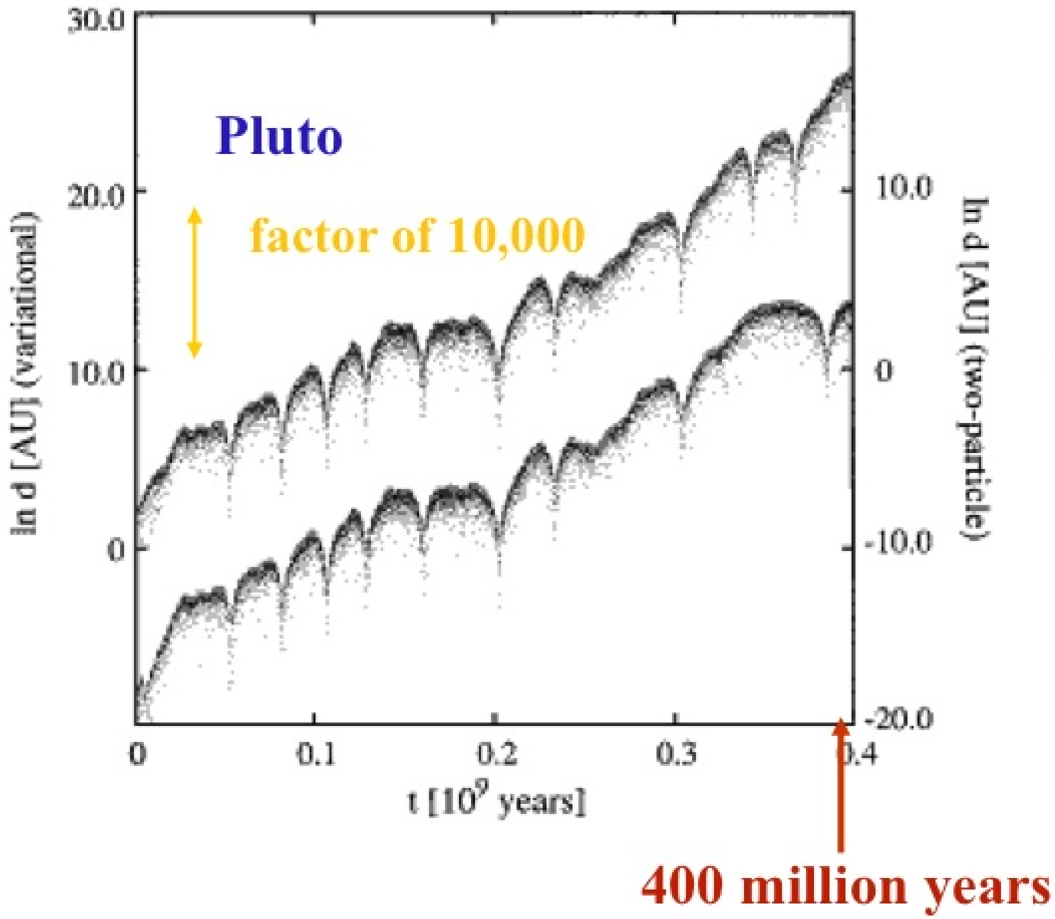
Two kinds of dynamical system

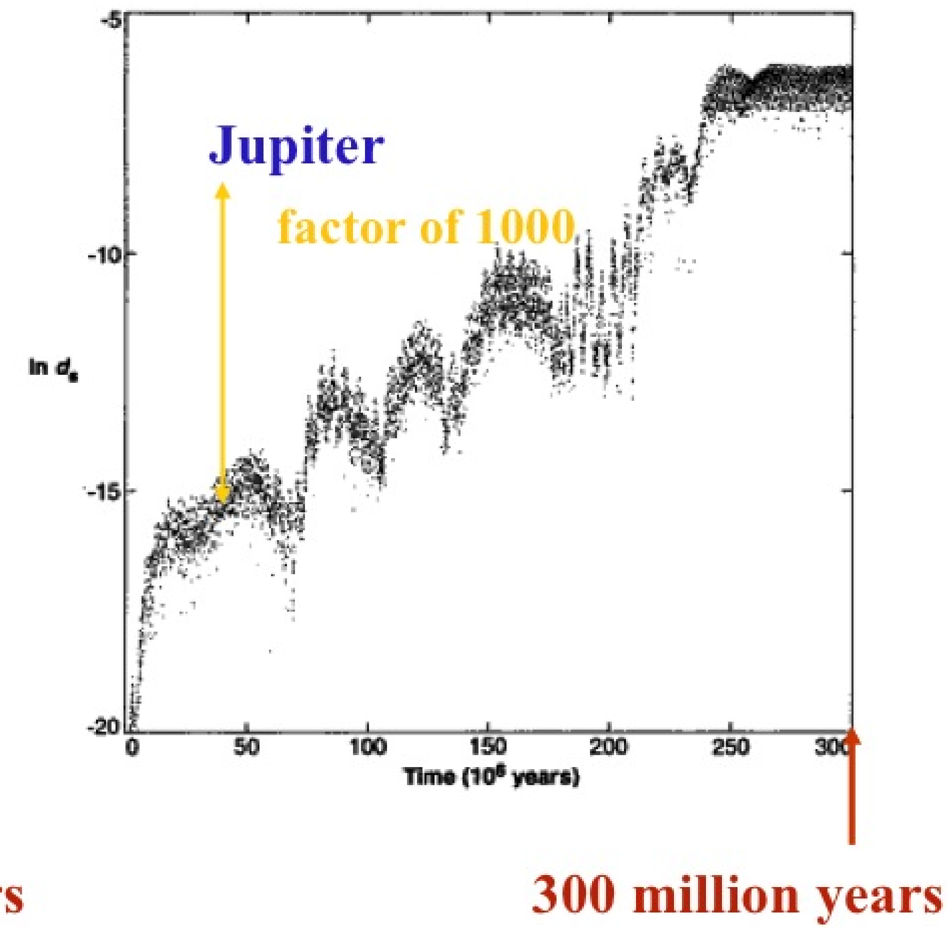
Regular

- highly predictable, “well-behaved”
- small differences grow linearly with time
- e.g. baseball, golf, pendulum, all problems in physics texts, planetary orbits on short timescales

Chaotic

- difficult to predict, “erratic”
- small differences grow exponentially at large times
- appears regular on short timescales
- e.g. roulette, dice, pinball, weather, billiards, double pendulum



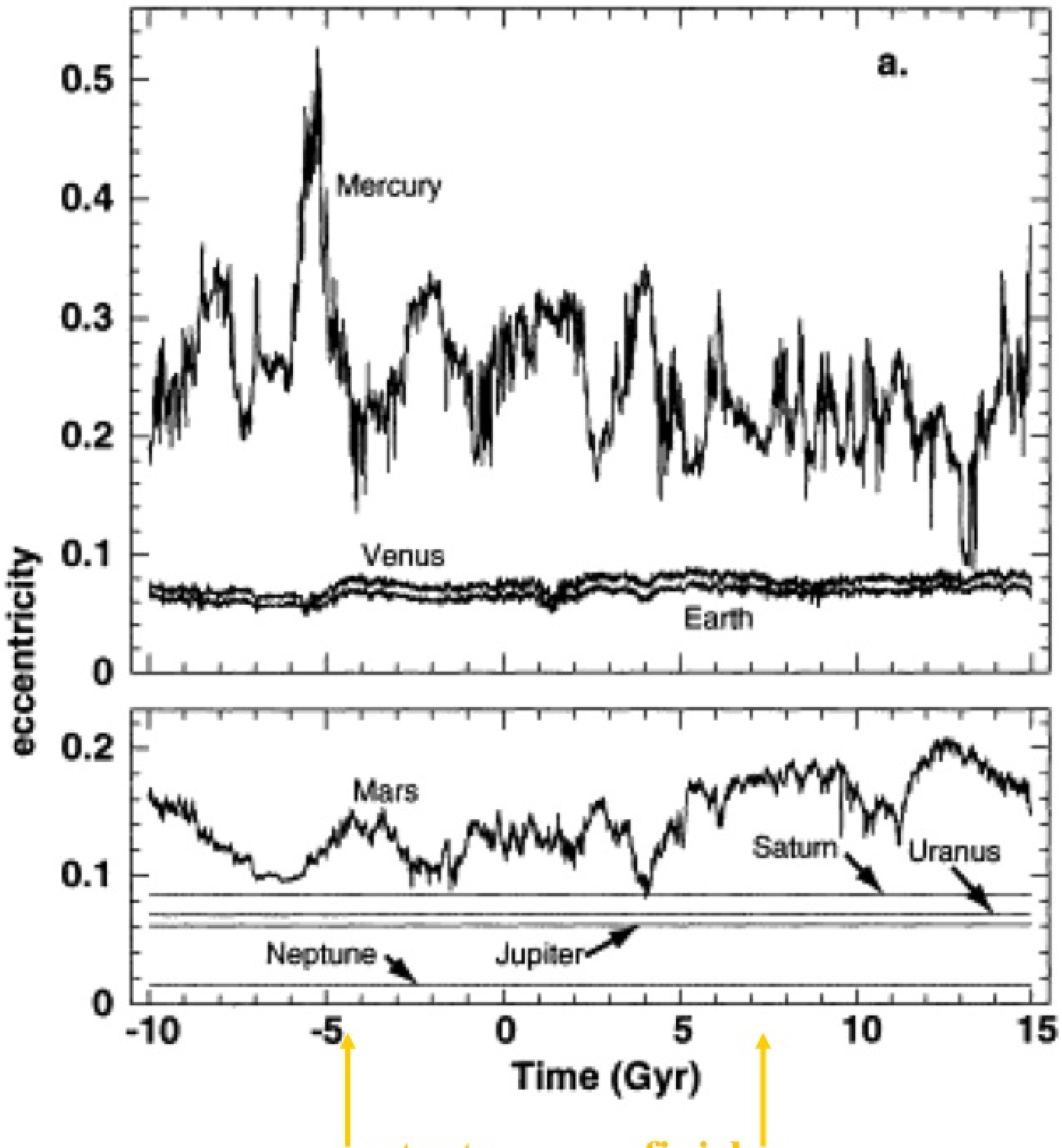


The orbit of every planet in the solar system is chaotic (Sussman & Wisdom 1988, 1992)

separation of adjacent orbits doubles on a timescale of 10 million years or increases by 10^{150} over lifetime of solar system

Consequences of chaos

- orbits of all of the planets are chaotic with doubling times for growth of small changes of ~10 million years, corresponding to increases by 10^{150} over lifetime of solar system
- positions of planets are not predictable on timescales longer than 100 million years (ejection of a single interplanetary spacecraft changes position of Earth completely in 200 Myr)
- shapes of orbits vary randomly on billion-year timescales

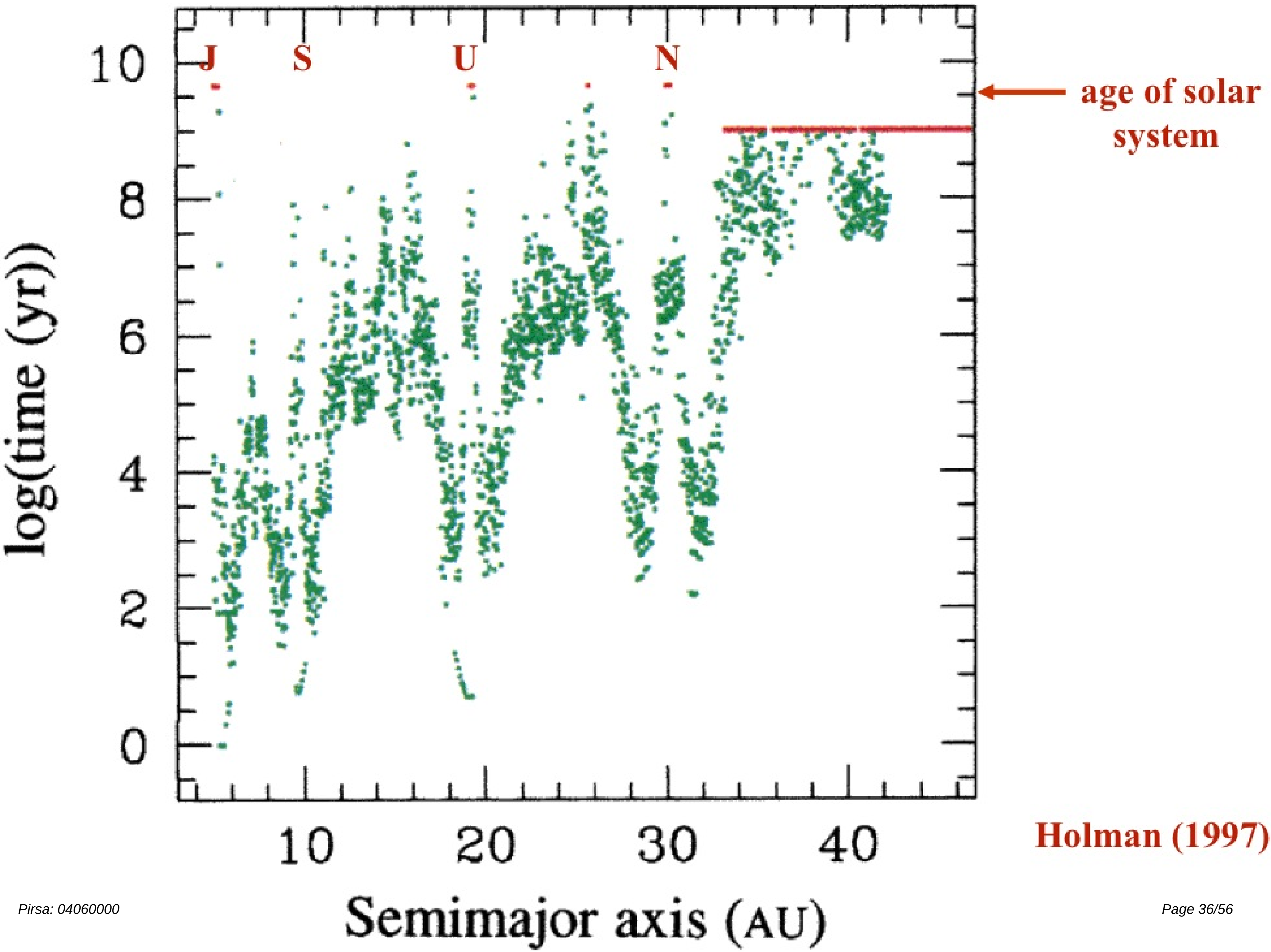


a.
eccentricity
Mercury
Venus
Earth
Mars
Saturn
Uranus
Neptune
Jupiter
0.5
0.4
0.3
0.2
0.1
0
0.2
0.1
0
-10
-5
0
5
10
15
Time (Gyr)

Laskar (1994)

Consequences of chaos

- orbits of all of the planets are chaotic with doubling times for growth of small changes of ~10 million years, corresponding to increases by 10^{150} over lifetime of solar system
- positions of planets are not predictable on timescales longer than 100 million years (ejection of a single interplanetary spacecraft changes position of Earth completely in 200 Myr)
- shapes of orbits vary randomly on billion-year timescales
- random changes in orbit shapes inevitably lead to ejection or collision of a low-mass planet. Thus solar system is unstable, although probably on *very* long timescales
- the solar system is a poor example of a clockwork universe
- most likely ejection has *already happened* one or more times


J
S
U
N
age of solar system
log(time (yr))
0
2
4
6
8
10
10
20
30
40
Semimajor axis (AU)

Holman (1997)

The gravitational N-body problem

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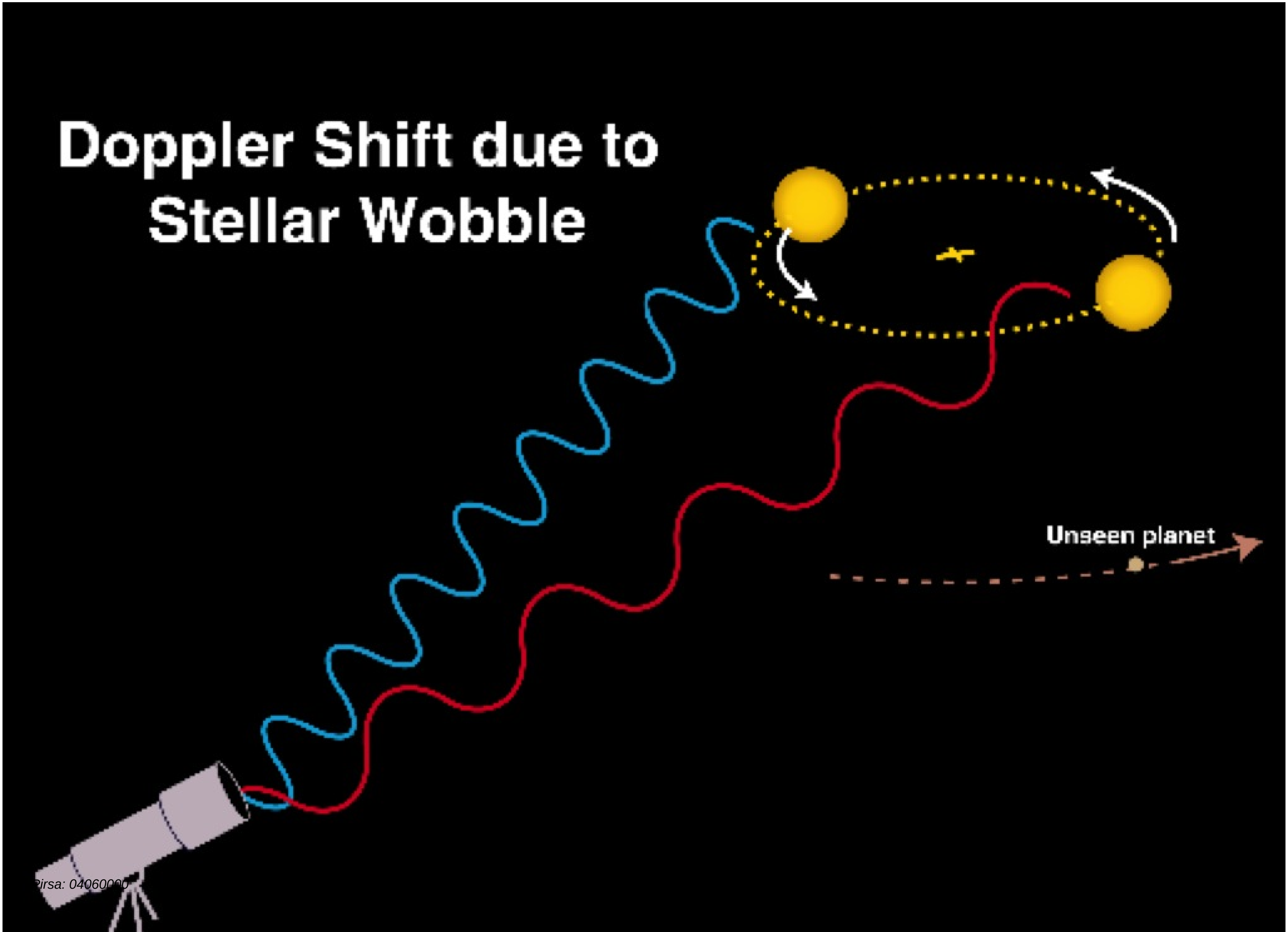
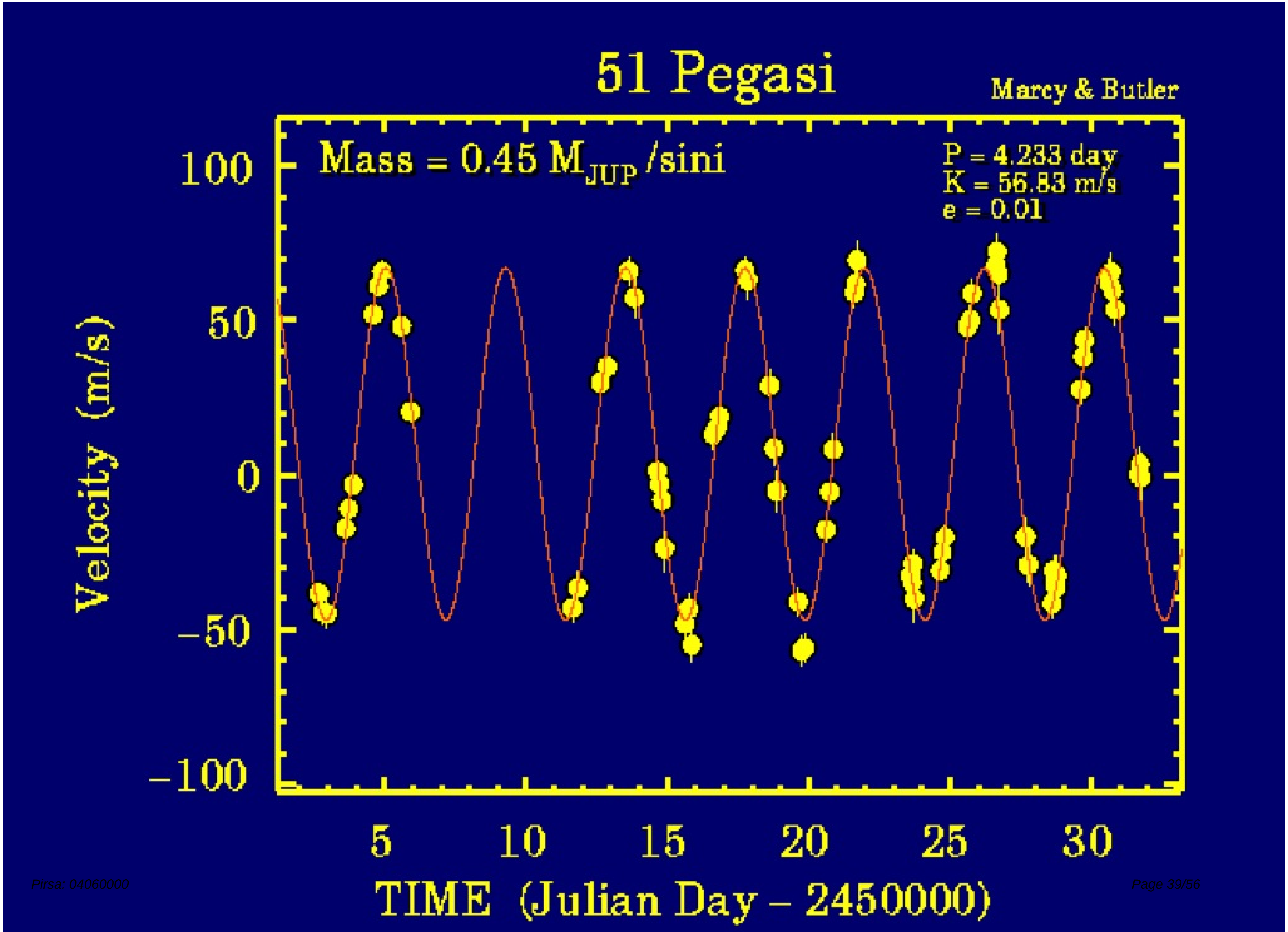
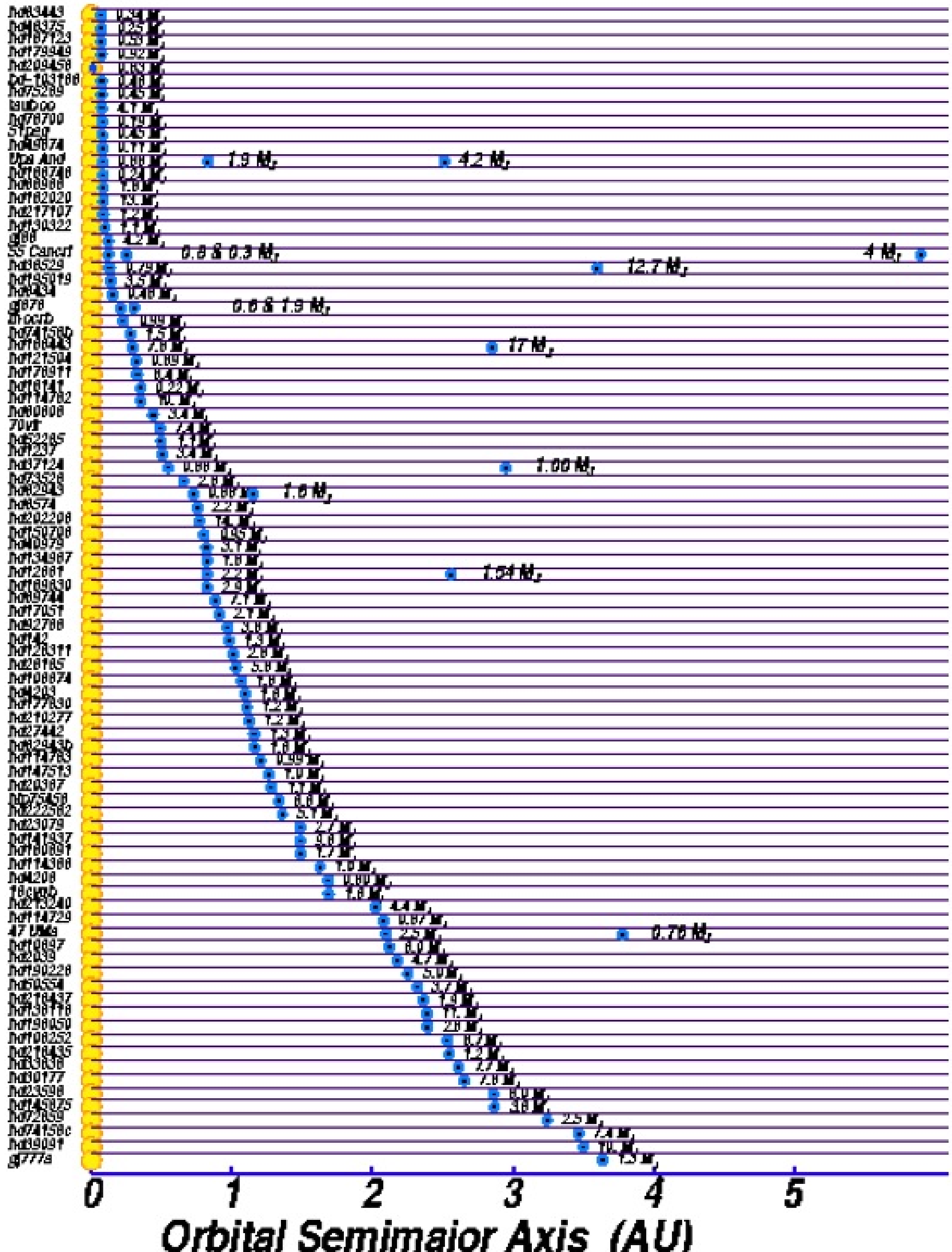
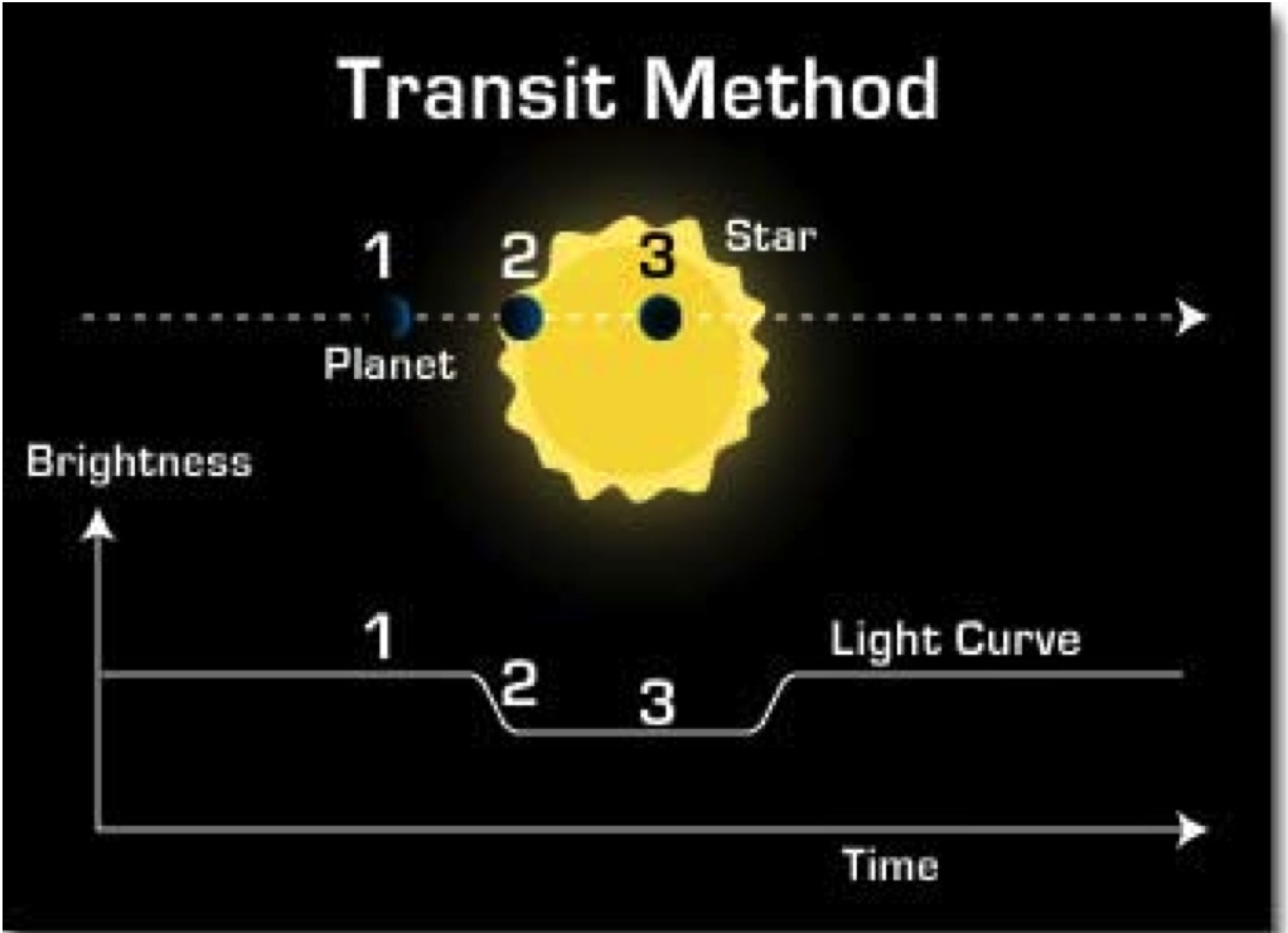

Doppler Shift due to
Stellar Wobble
Unseen planet


51 Pegasi
Marcy & Butler
Mass = 0.45 M JUP /sini
P = 4.233 day
K = 56.83 m/s
e = 0.01
Velocity (m/s)
100
50
0
−50
−100
5
10
15
20
25
30
TIME (Julian Day − 2450000)


0
1
2
3
4
5
Orbital Semimajor Axis (AU)

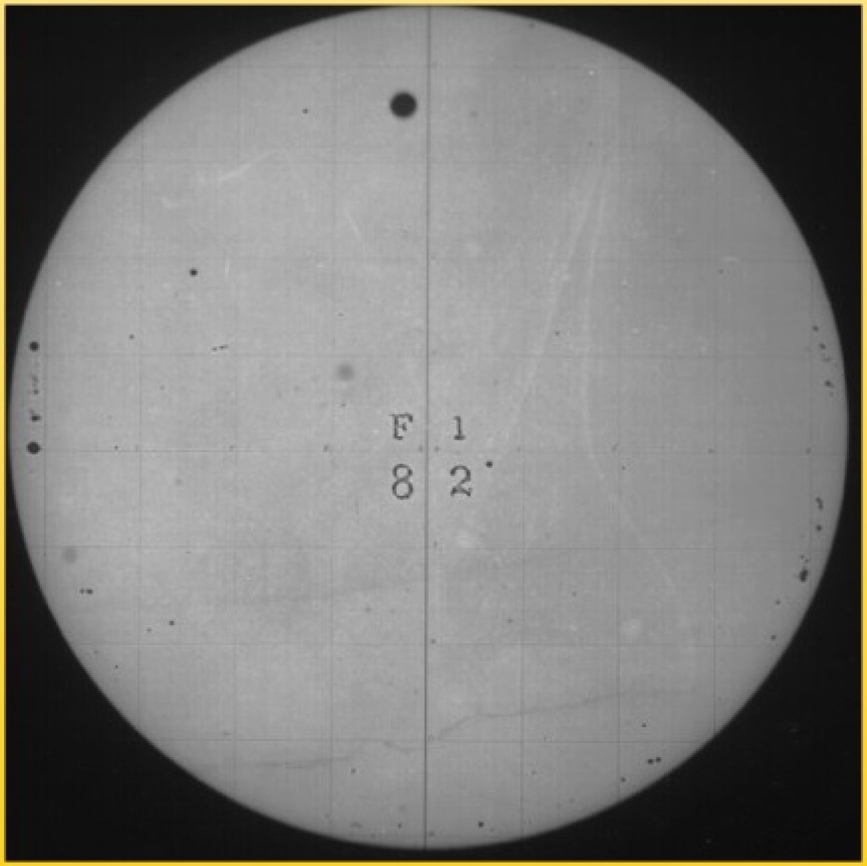


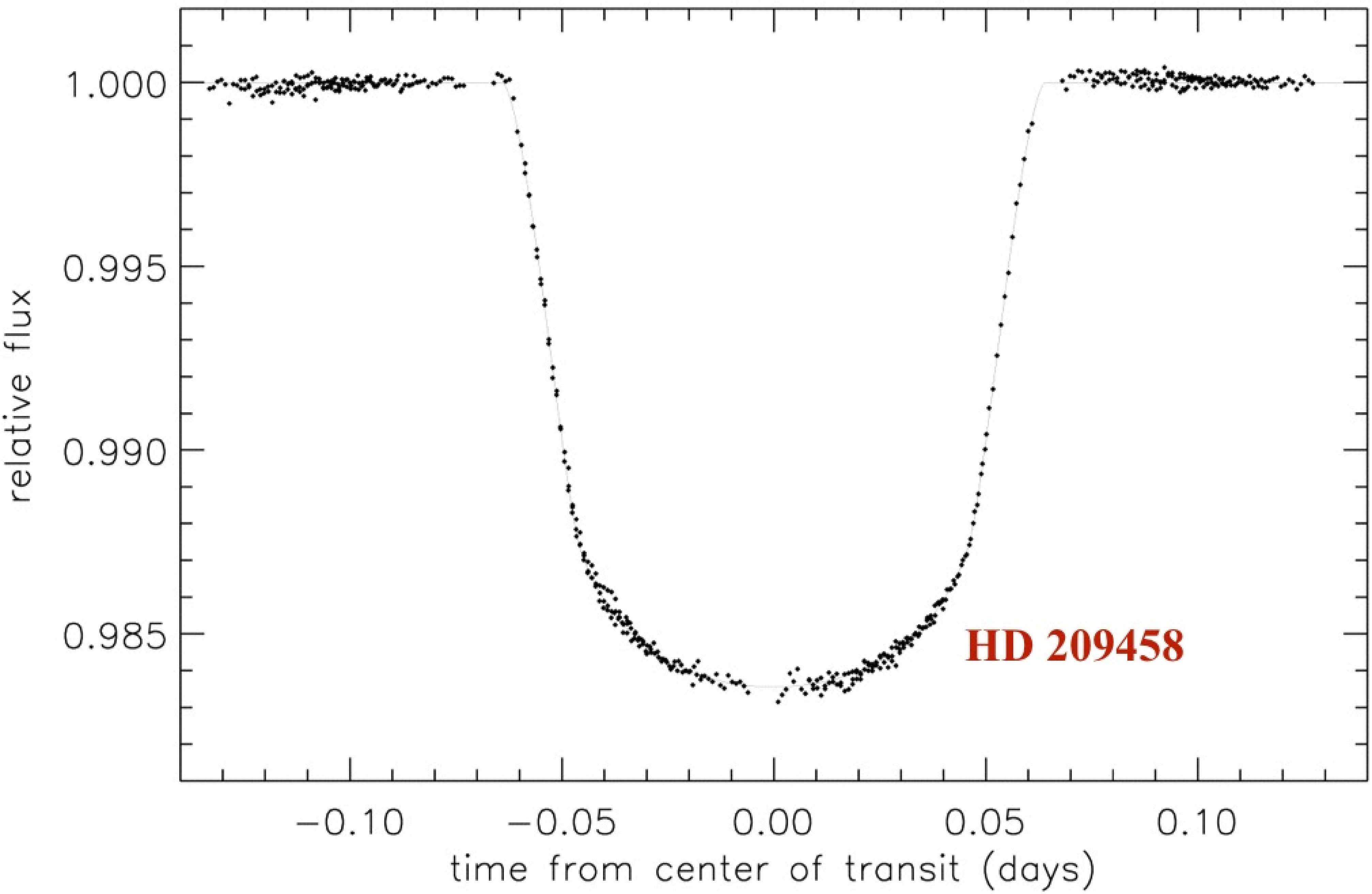
Transit Method
1
2
3
Star
Planet
Brightness
1
Light Curve
2
3
Time

Transit of Venus

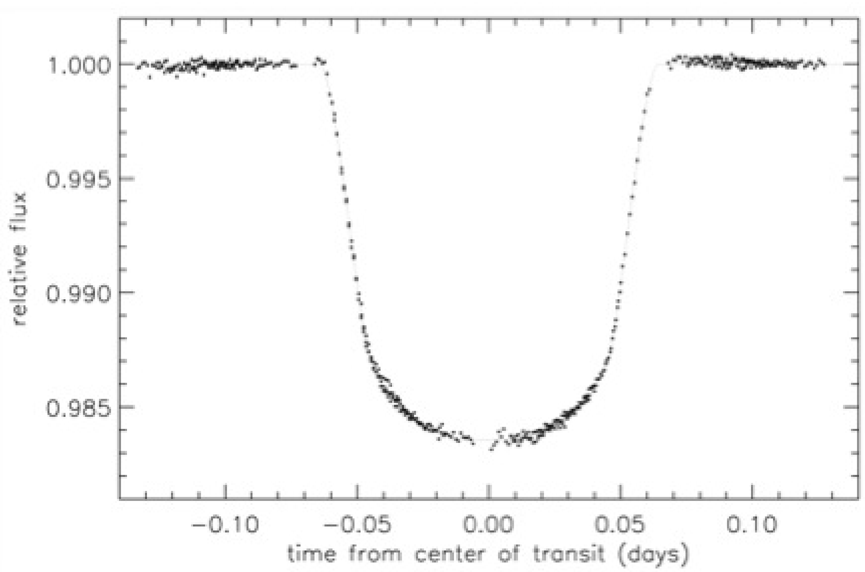
June 8, 2004

- last occurred in 1882, next in 2012, 2117
- entire transit visible from Europe but only the last stages can be seen here
- visible in Waterloo until 0725 local time (Sun < 17^{o} above the horizon)
- use indirect viewing **only**




1.000
0.995
0.990
0.985
relative flux
−0.10
−0.05
0.00
0.05
0.10
time from center of transit (days)
HD 209458

What do we know about the planet in HD 209458?



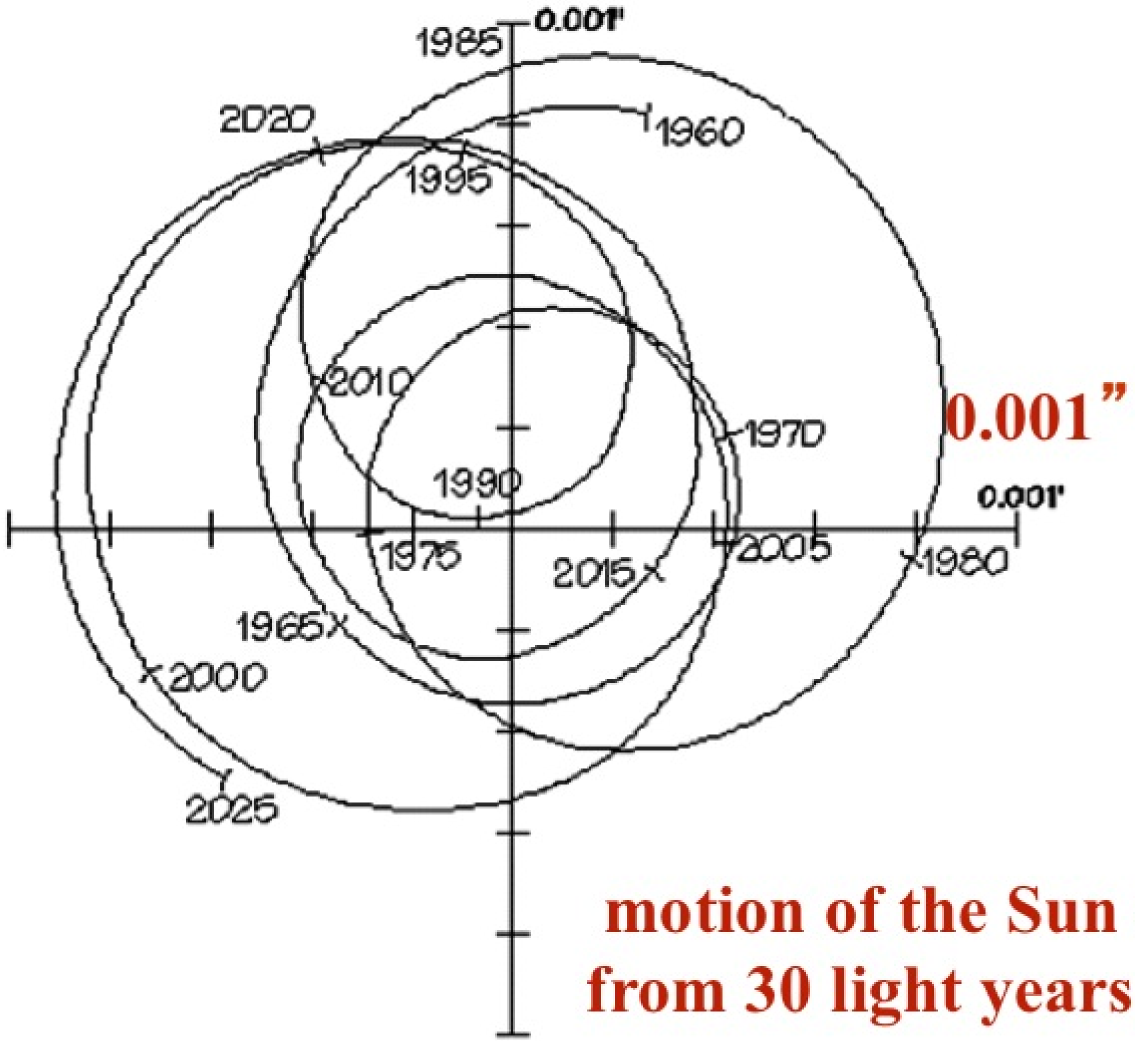
- mass is 0.69 Jupiter masses
- radius is 1.35 Jupiter radii, consistent with models of a heated Jupiter
- orbital period 3.52 days
- orbital radius 0.047 AU or 10 stellar radii
- sodium line detected from planetary atmosphere

MOST

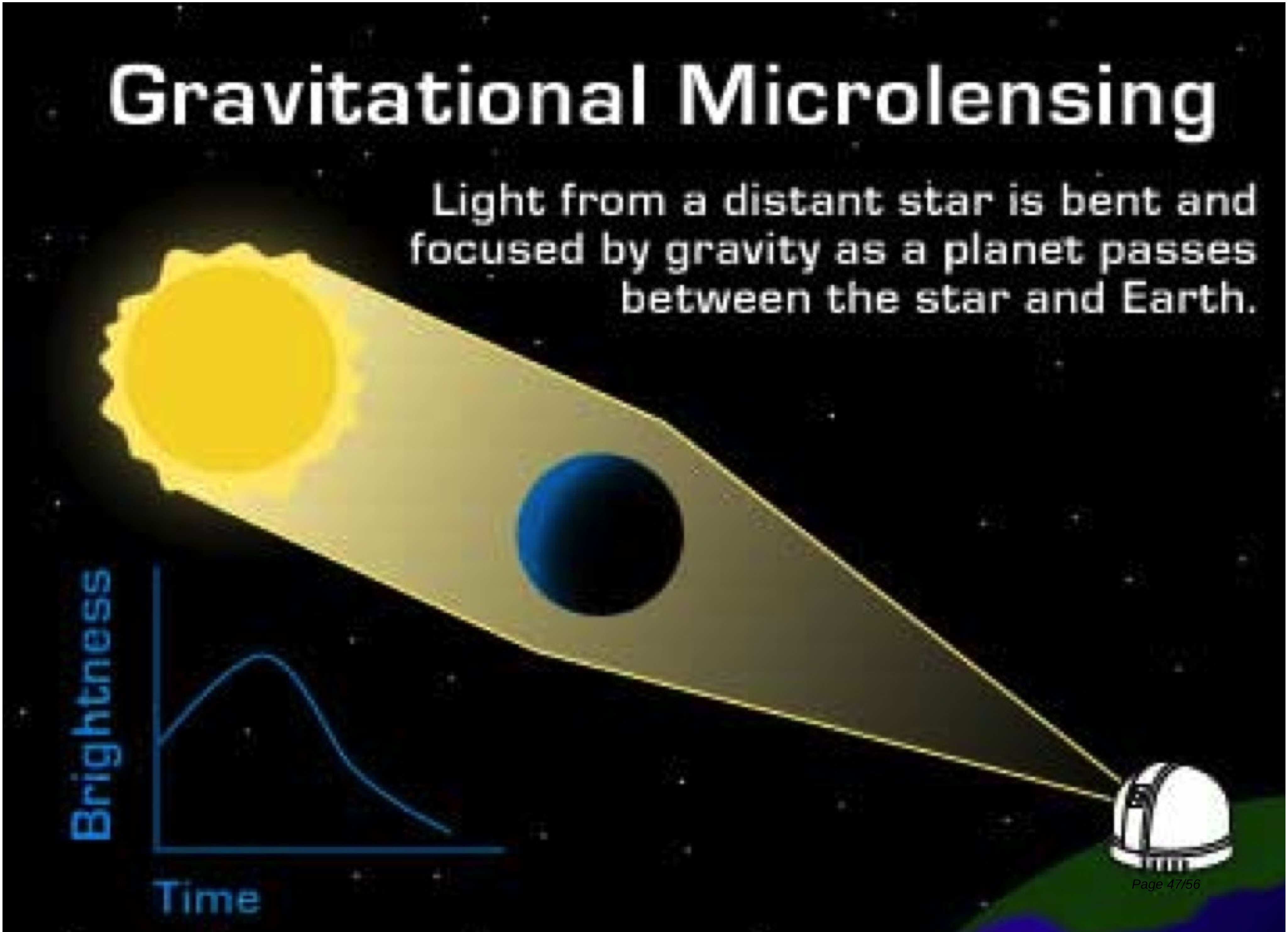
Microgravity and Oscillations of STars

launched June 30, 2003

astrometry

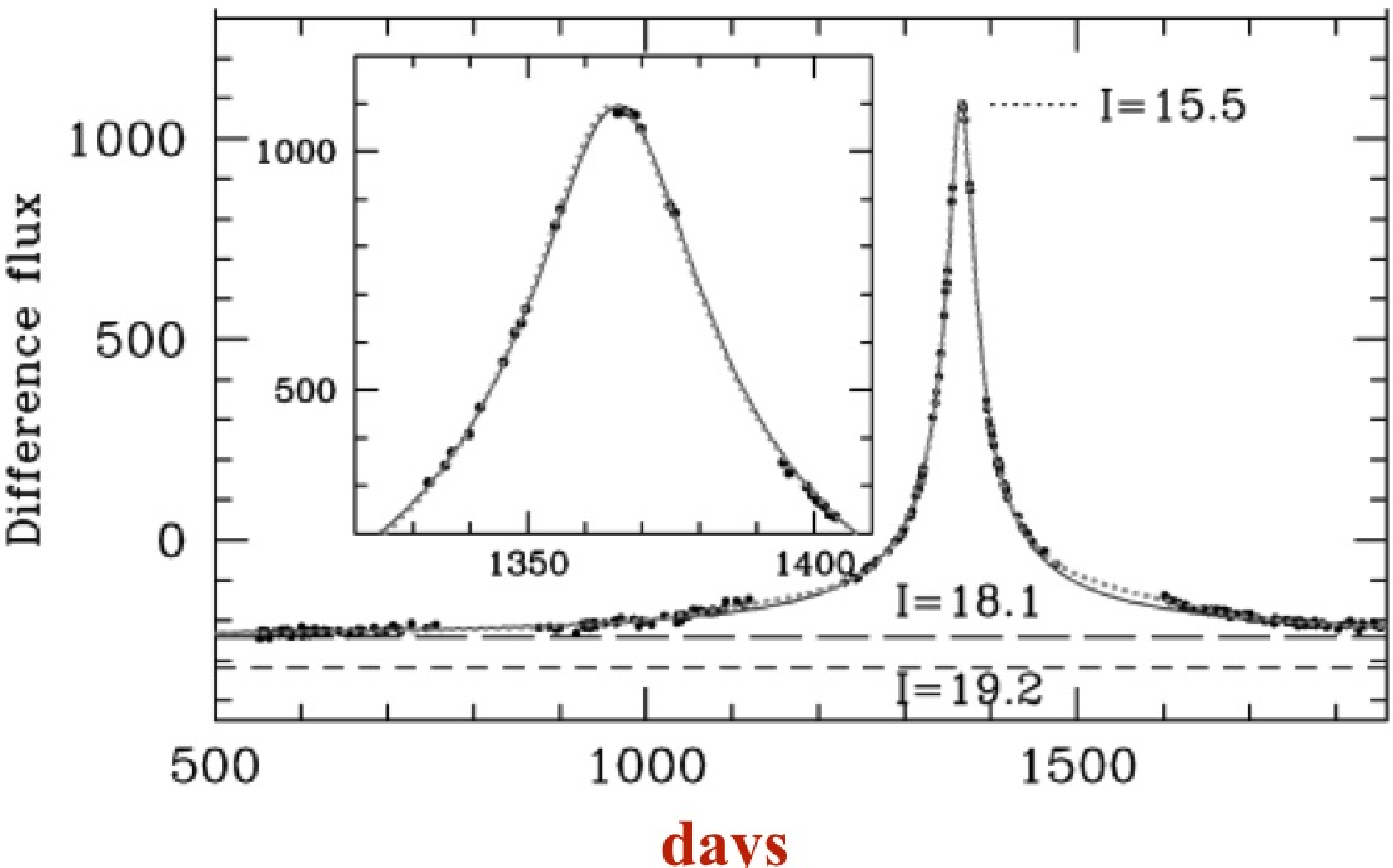


motion of the Sun as viewed from 30 light years away

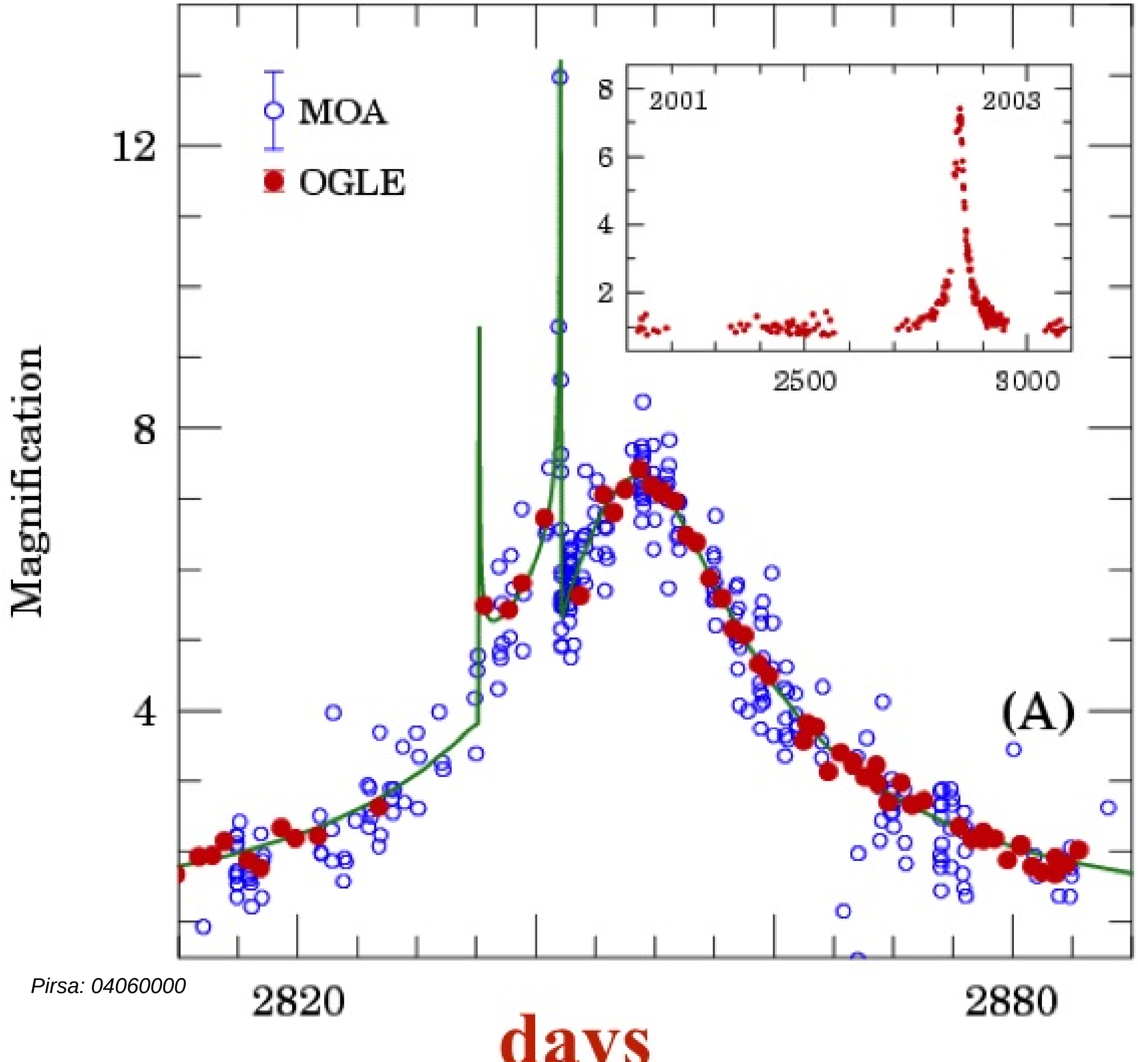


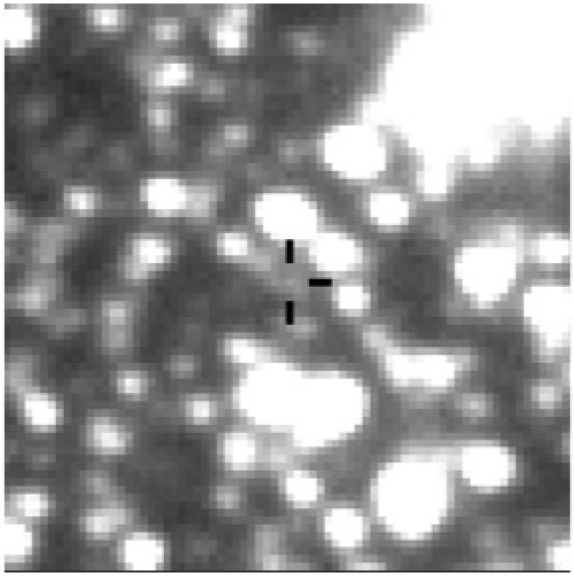
Gravitational Microlensing
Light from a distant star is bent and focused by gravity as a planet passes between the star and Earth.
Brightness
Time

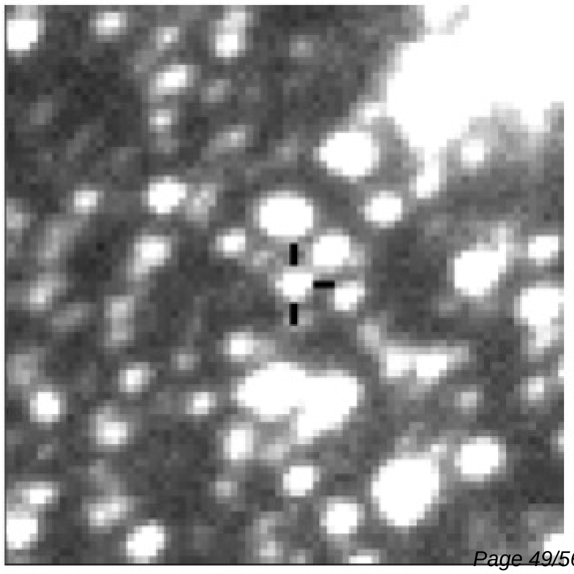
Gravitational microlensing

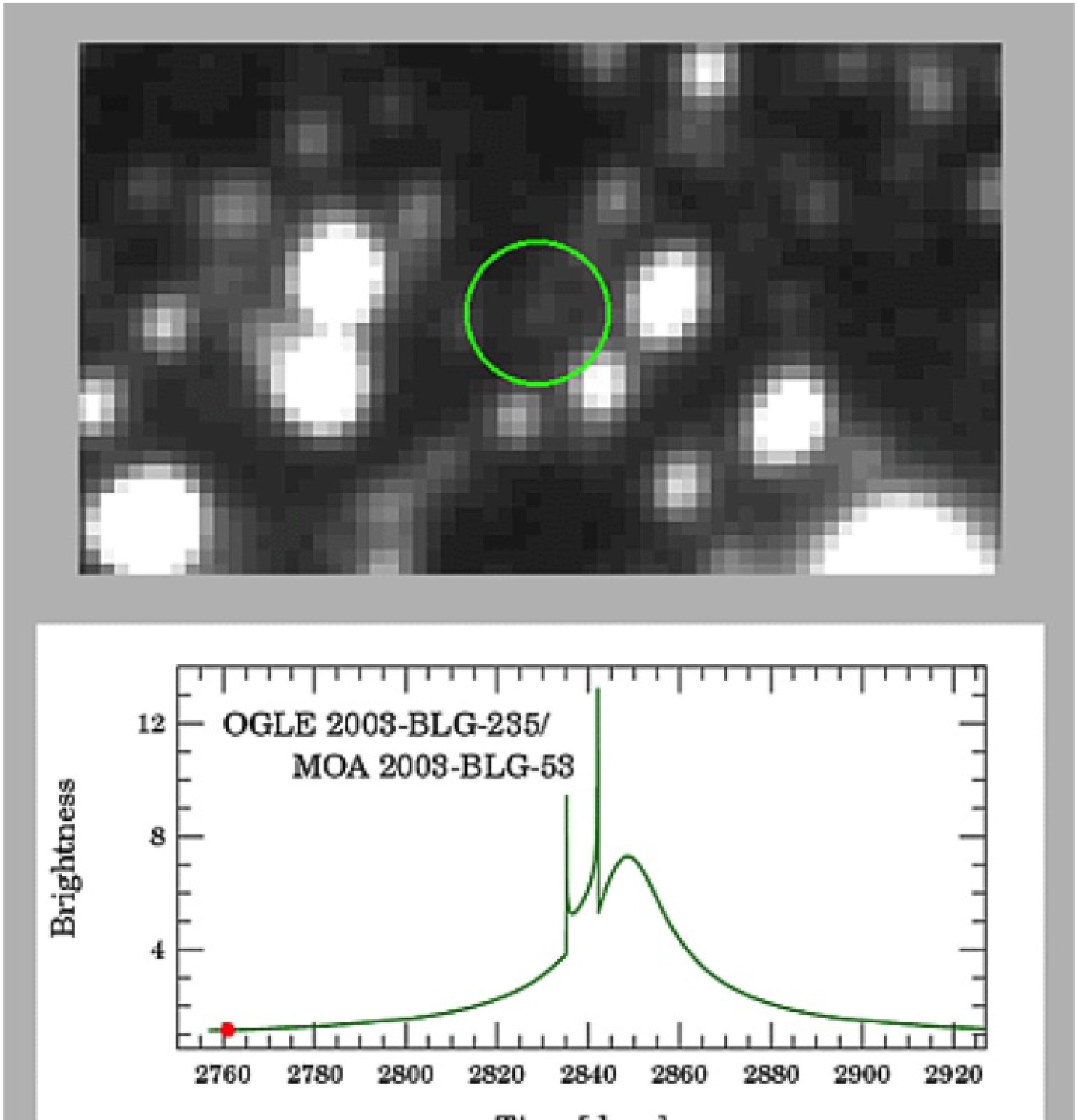


Gravitational microlensing








OGLE 2003-BLG-235/
MOA 2003-BLG-53
Brightness
12
8
4
2760
2780
2800
2820
2840
2860
2880
2900
2920

What we know

•planet formation is common: at least 10% of nearby stars host giant planets

•giant planets are found at distances 100 × closer to their host star than Jupiter is to the Sun

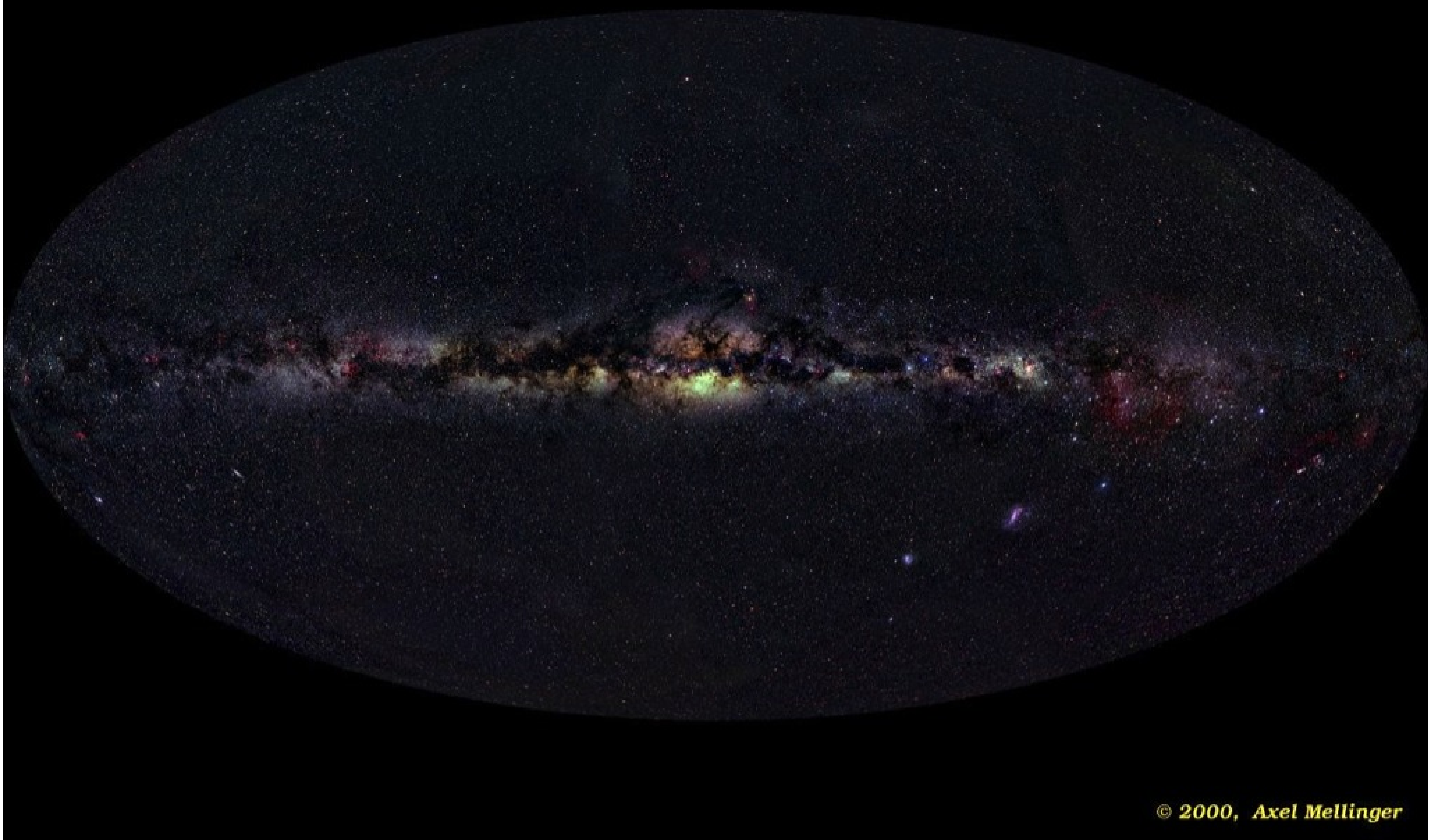
What we don’t know

•why are the planetary systems we have found so different from the solar system?

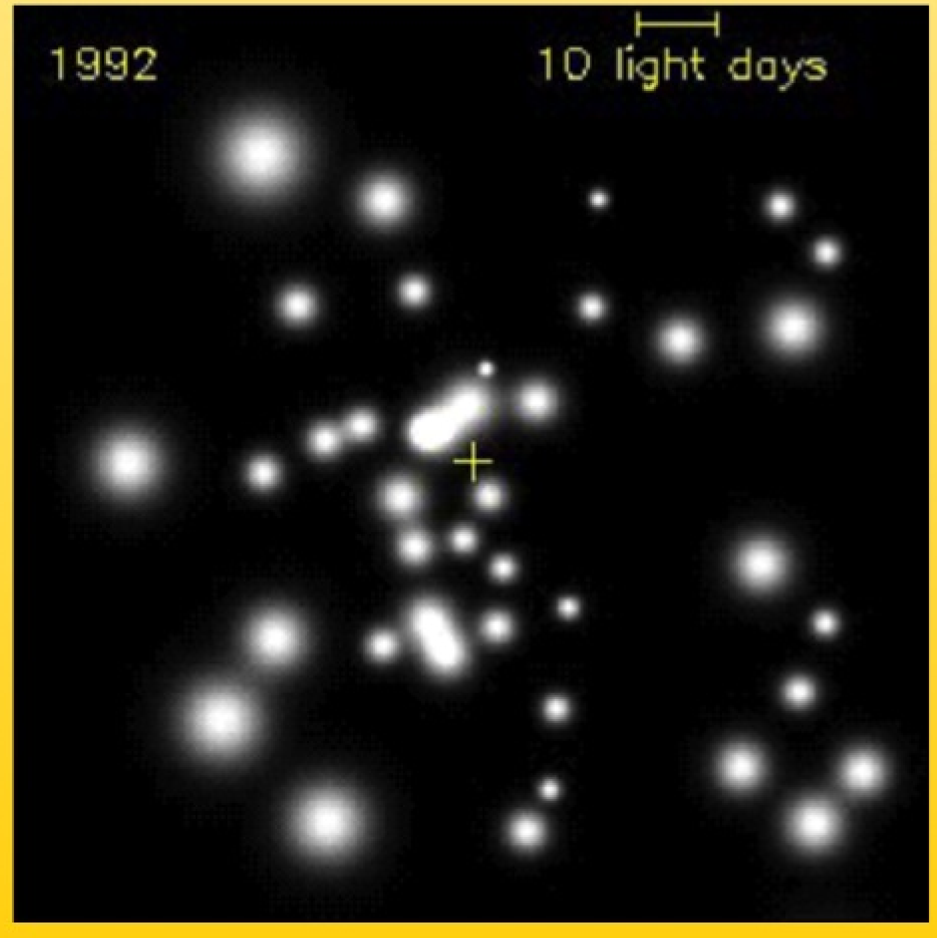
•how common are Earth-mass planets?

•how common are habitable planets?

The Milky Way galaxy



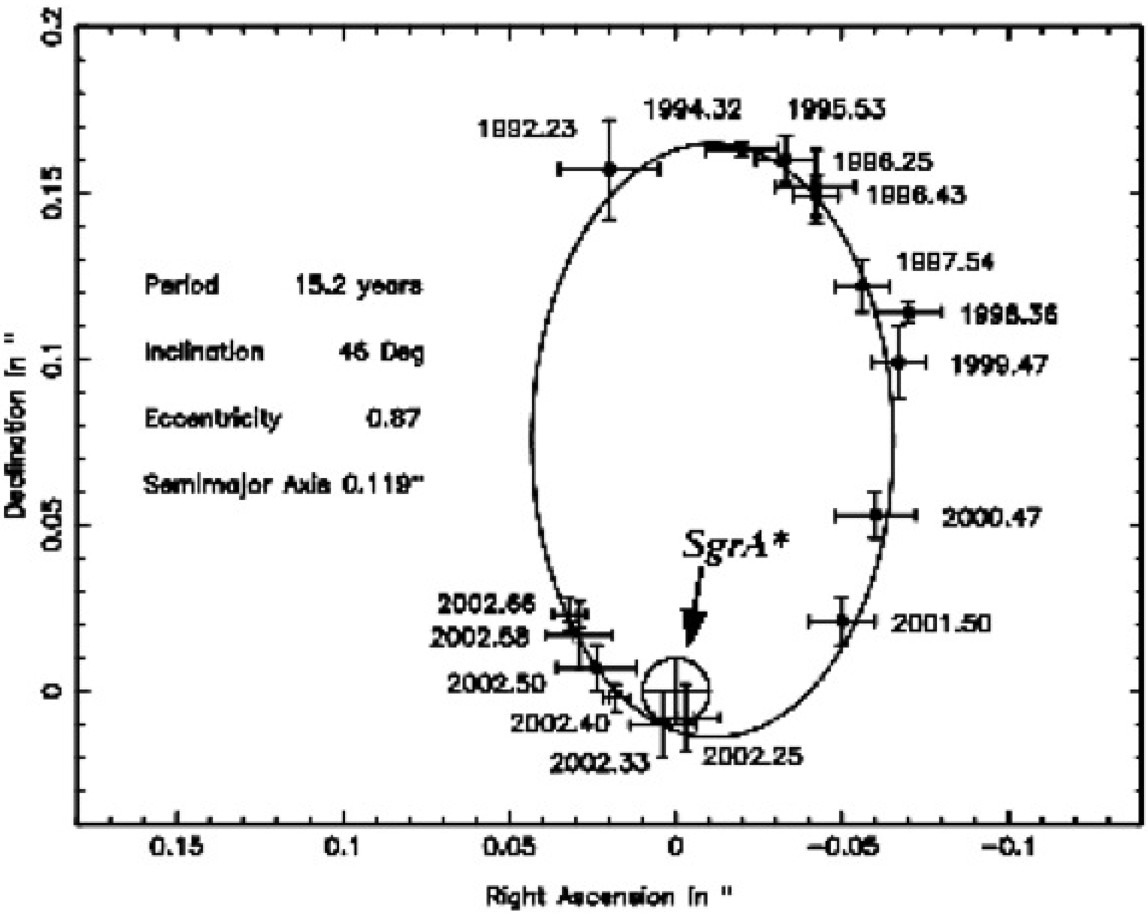
The Milky Way Galaxy



- center is believed to be marked by radio source Sagittarius A* (+)
- Sagittarius A* has long been suspected to be a massive black hole
- 10 light days ~ 1700 AU

Genzel et al. (2002)

The Milky Way Galaxy



- orbital period 15.8 years
- semi-major axis 1000 AU
- orbiting a central mass smaller than 100 AU with total mass 4.1 million solar masses
- no known astronomical object can be this massive and this compact other than a black hole
- but we are still far from the event horizon at 0.05 AU

Schodel et al. (2002)

The Black Hole Sessions

Yes, this is what all you closet Einsteins have been waiting for!
Black holes, quantum teleportation, time travel, extra dimensions, quantum computers and more. All the cool ideas theoretical physicists have come up with to try to make sense of the mysterious universe we live in. If you want to know more, *The Black Hole Sessions* are where you want to be.

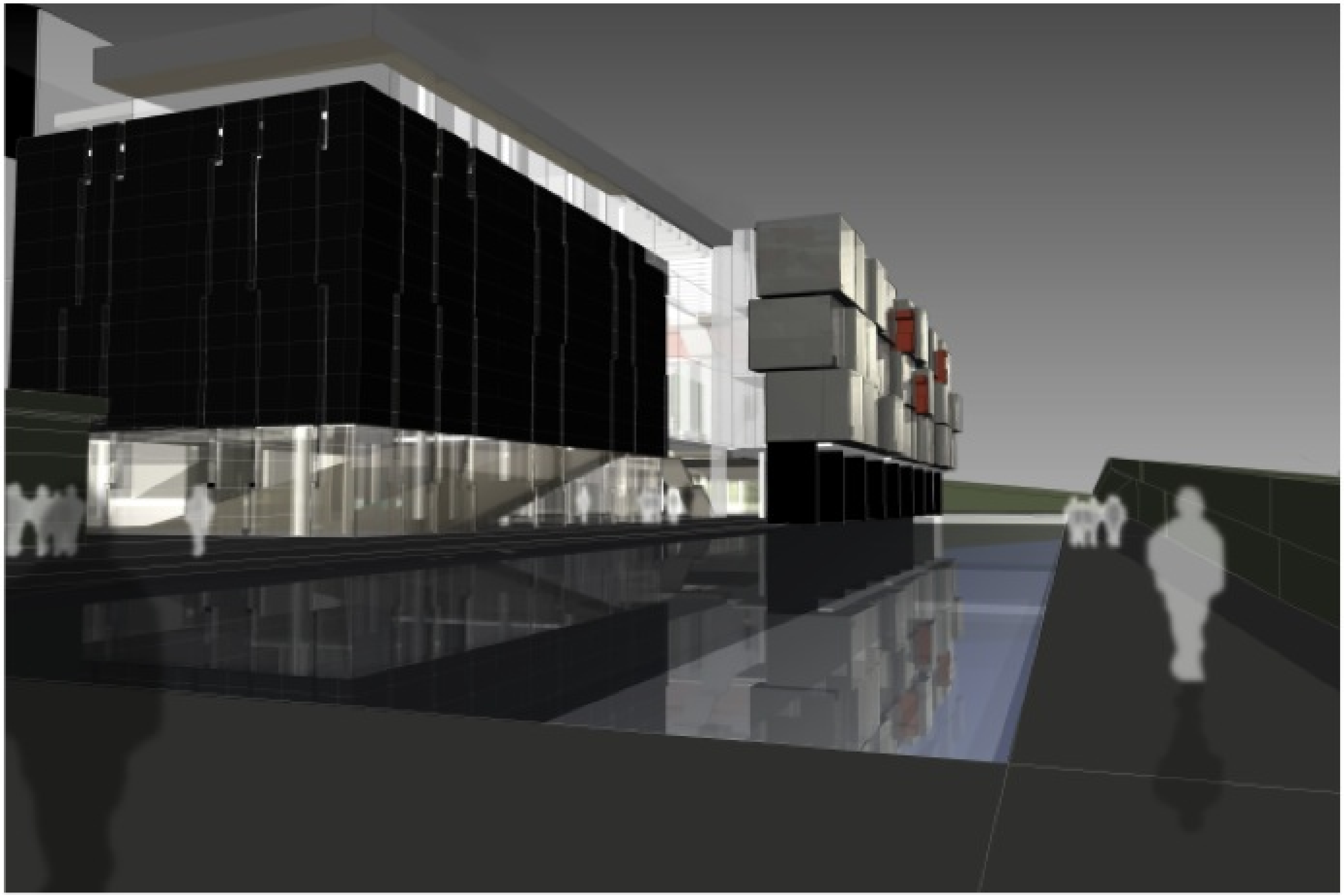
Where: Perimeter Institute for Theoretical Physics (where else?)
2nd Floor Conference Room
35 King Street North
(*Spacetime Square*—the clock tower building)
Uptown Waterloo

When: First Saturday of every month
10:00 a.m. till Noon

Why: You have to ask?

These sessions will be very informal, led by theoretical physicist Dr. Richard Epp, a.k.a. the Outreach Guy at Perimeter. Come prepared for some mind bending ideas and with any questions you want to discuss. Refreshments will be served.

PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS



One Space – One Time – One Event

PI/10/02/04