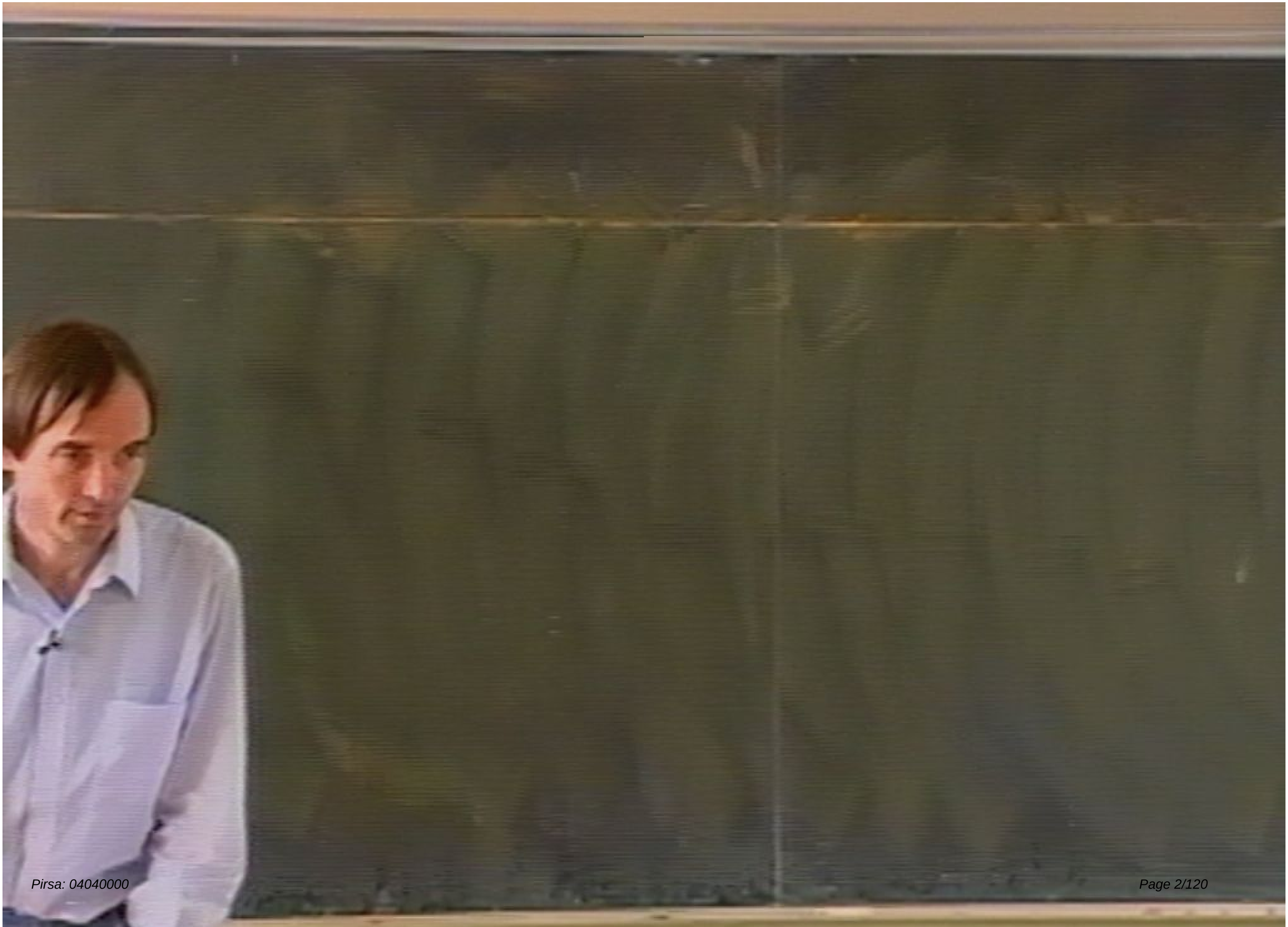
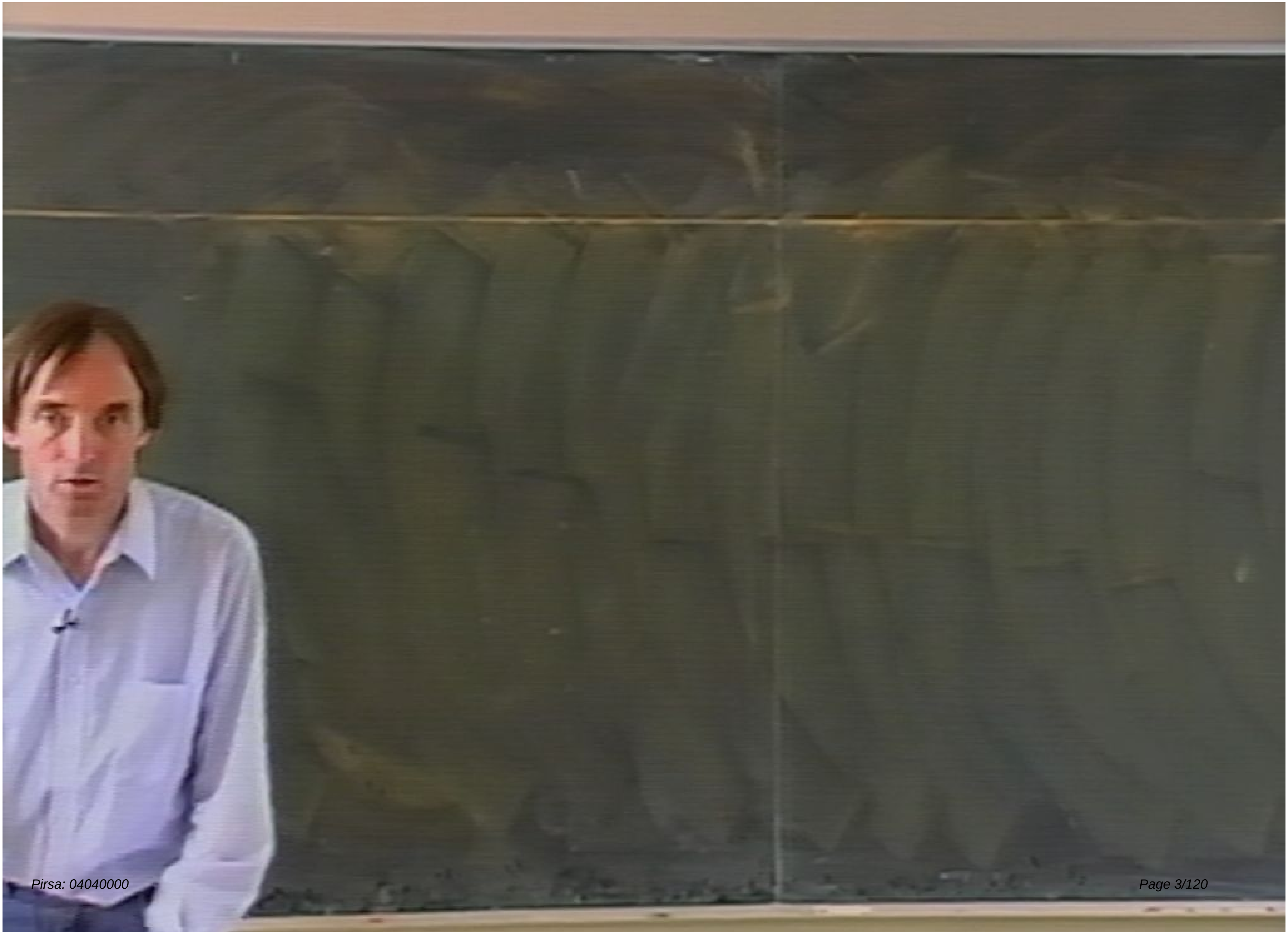
Title: Theory of Cosmological Perterbations - Lecture 3, Part 1

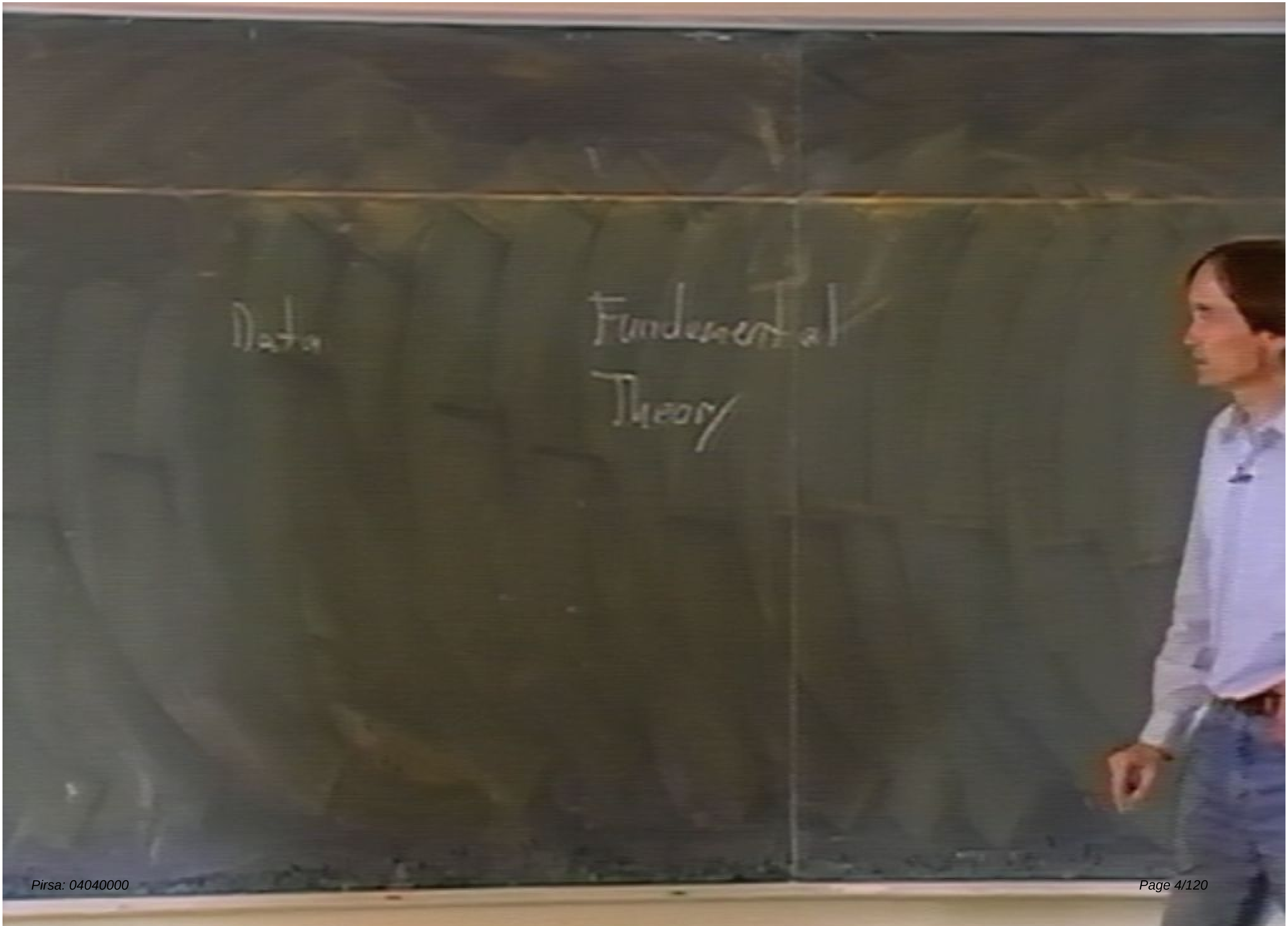
Date: Apr 20, 2004 11:00 AM

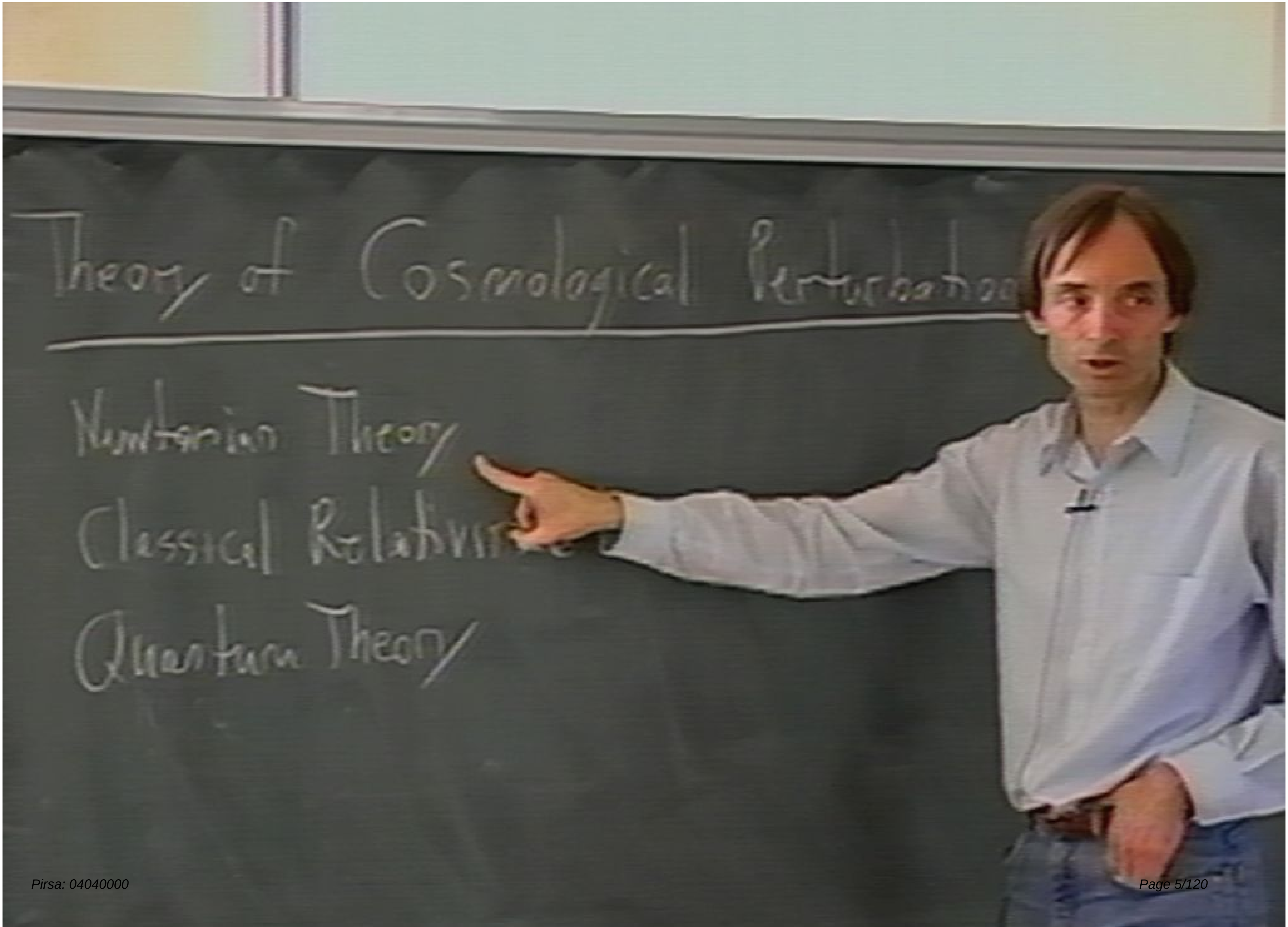
URL: http://pirsa.org/04040000

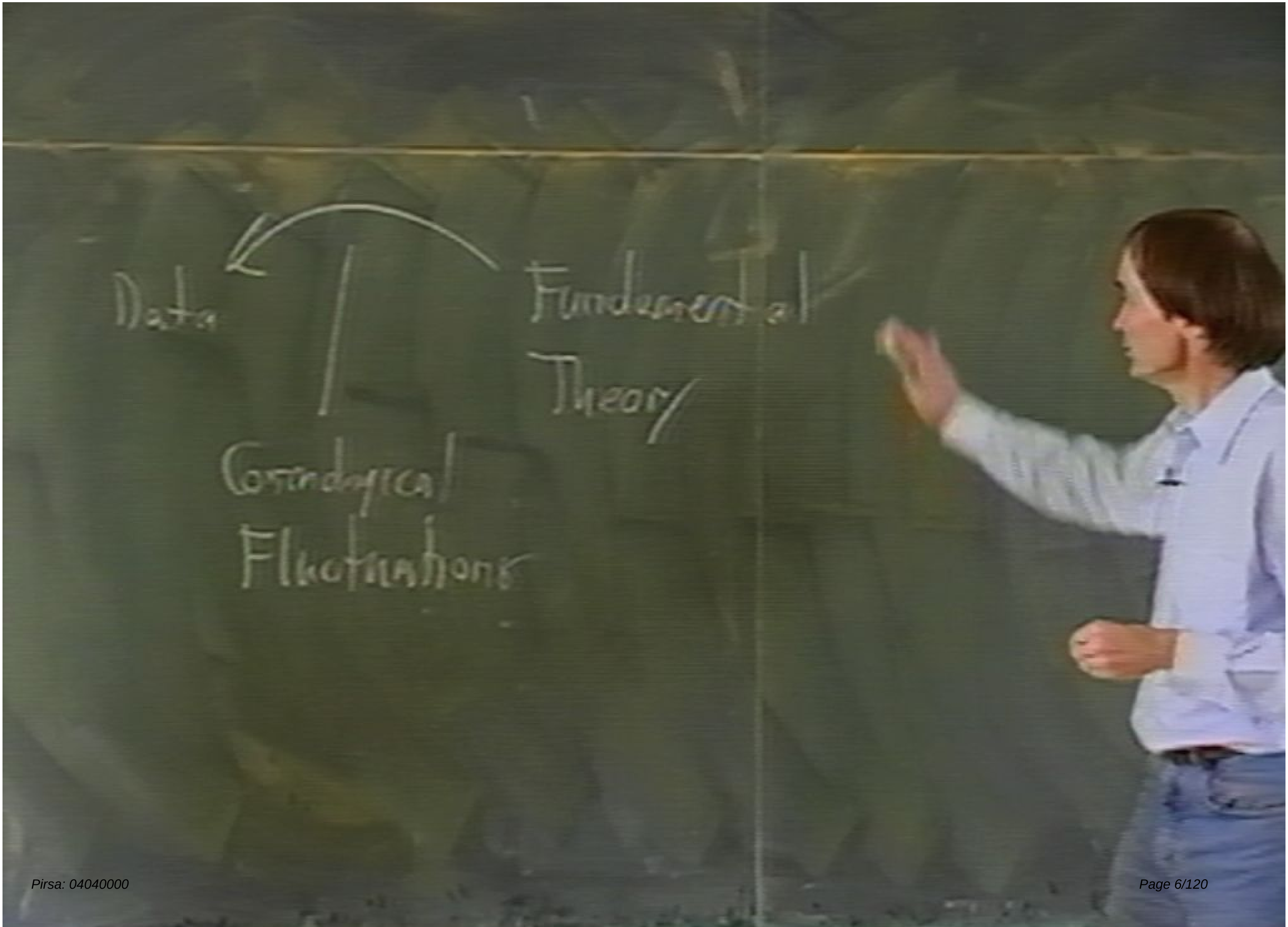
Abstract:

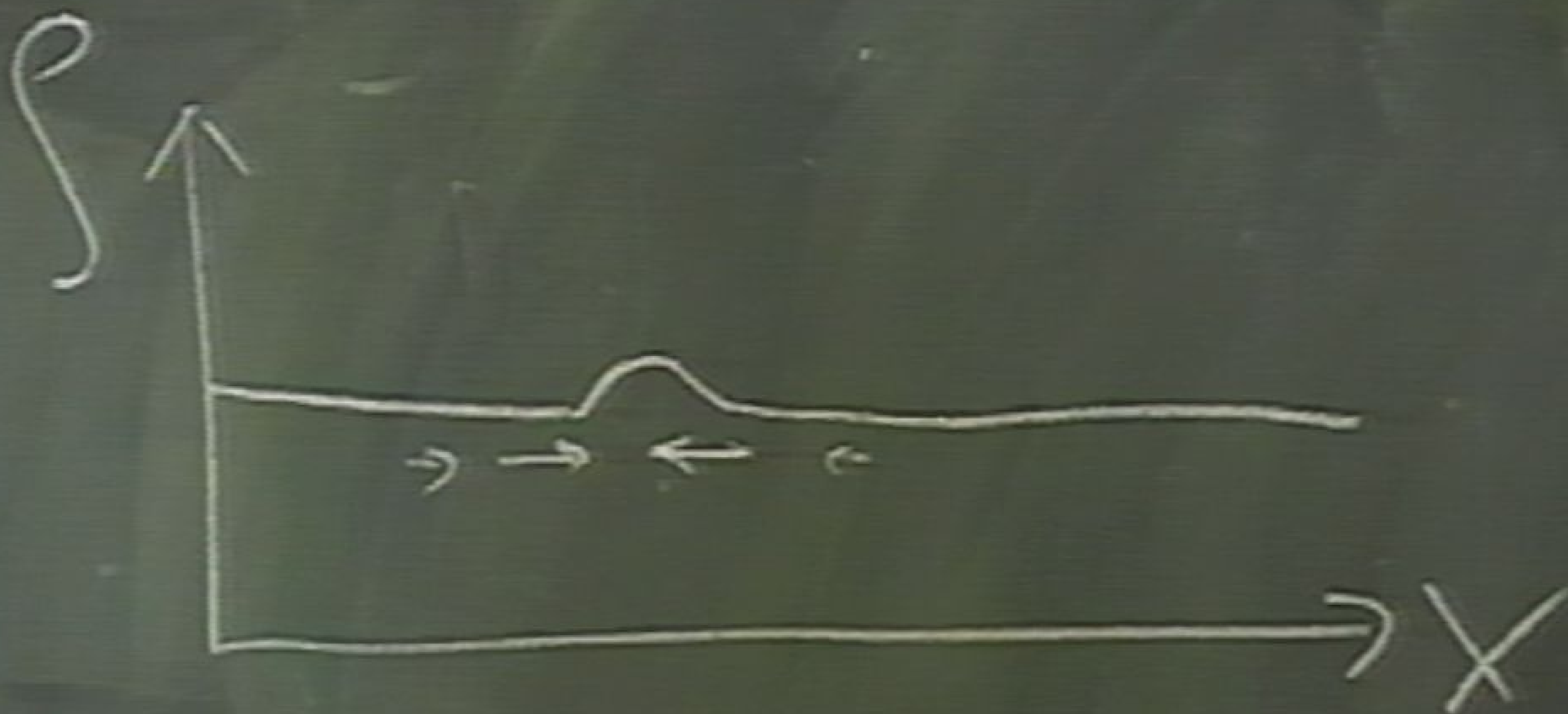


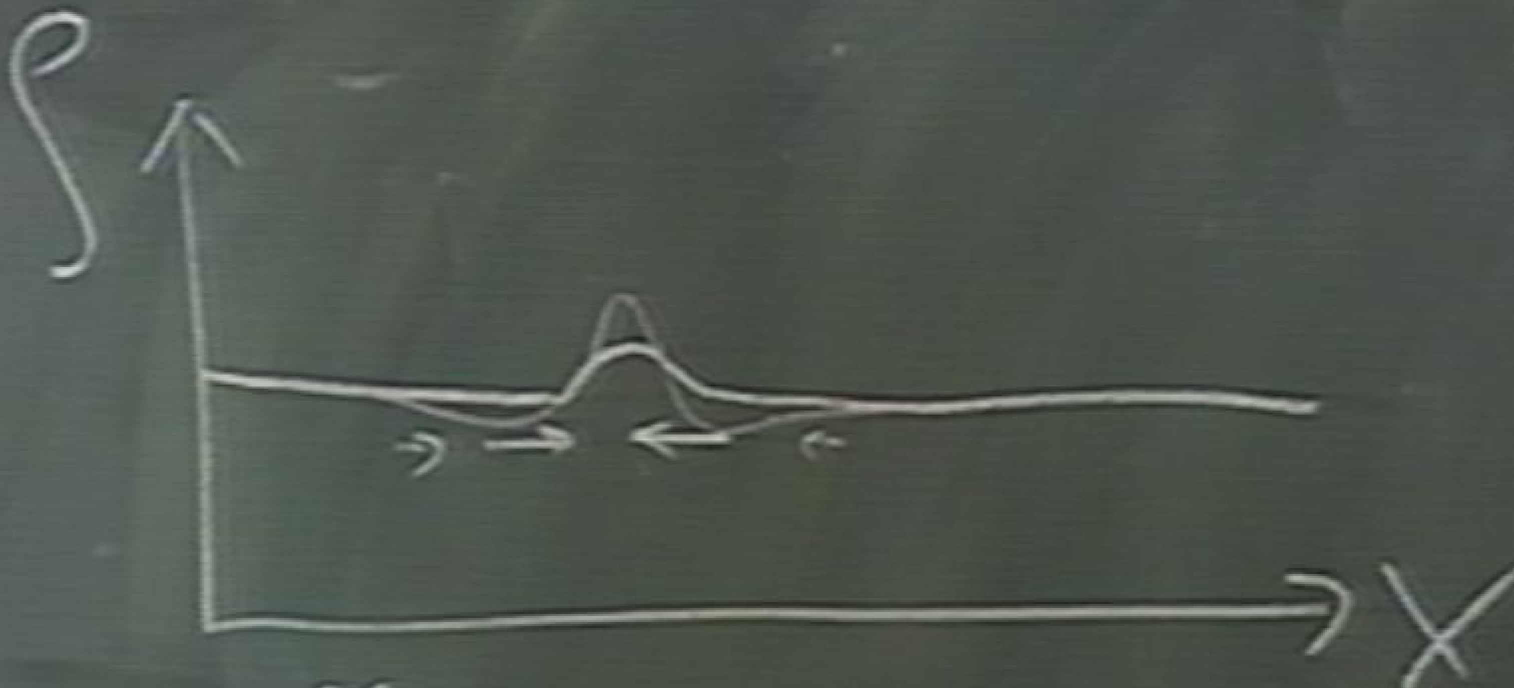



Data
Fundamental
Theory


Theory of Cosmological
Newtonian Theory
Classical
Quantum Theory

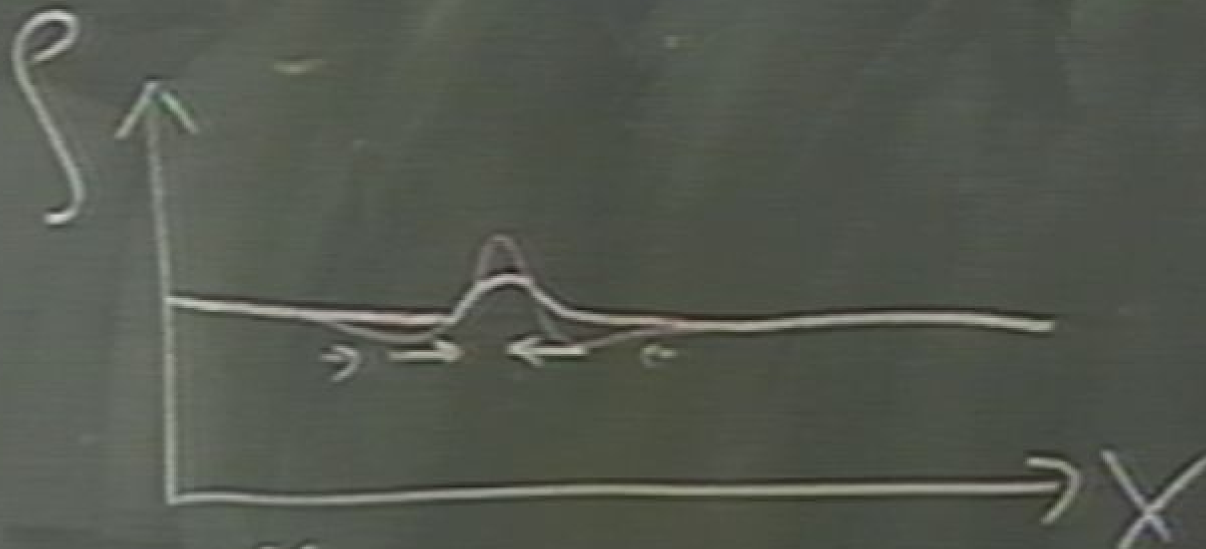

Data
Fundamental
Theory
Cosmological
Fluctuations


ρ
X

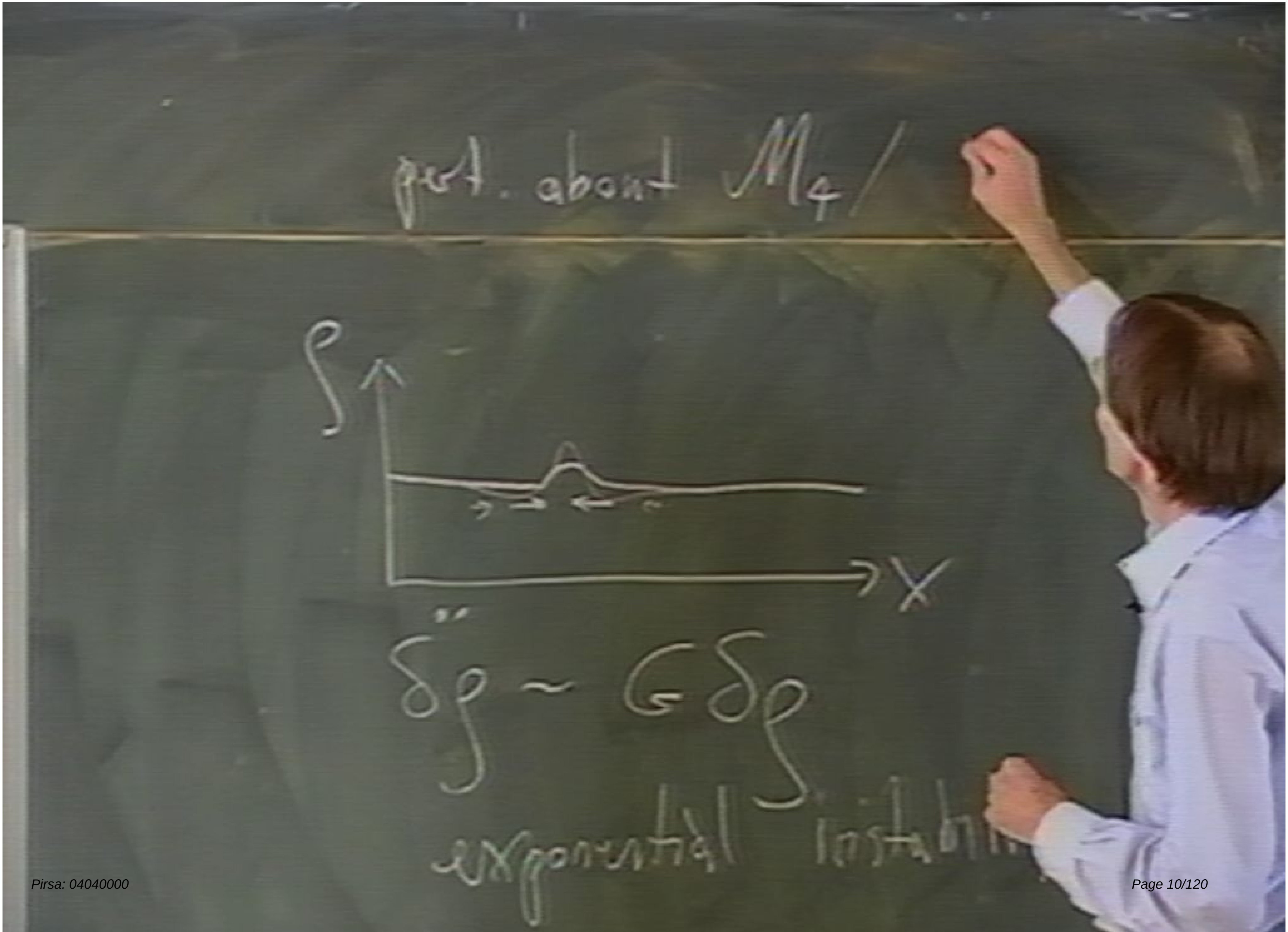


$$\delta\rho \sim G$$

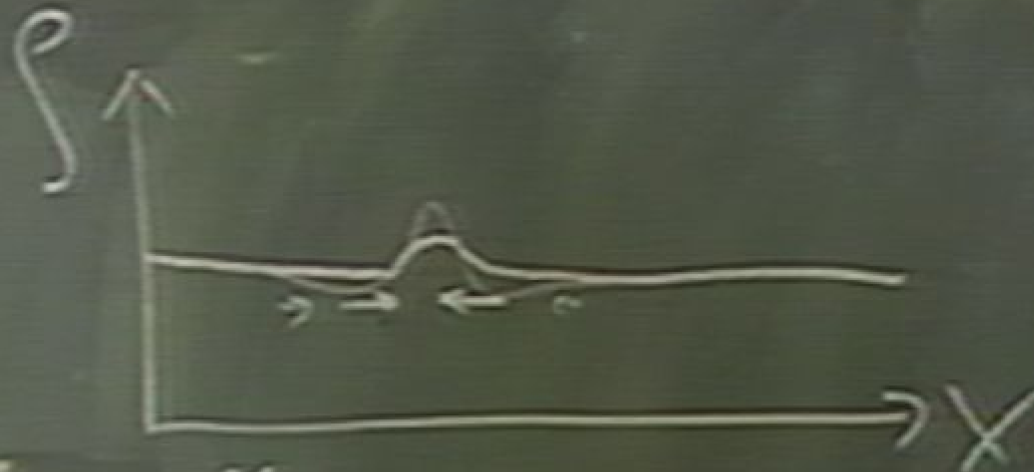
pert. about M_4



$$\ddot{\delta\rho} \sim G\,\delta\rho$$


pert. about M_4 /
ρ
x
$\ddot{\delta\rho} \sim G\,\delta\rho$
exponential

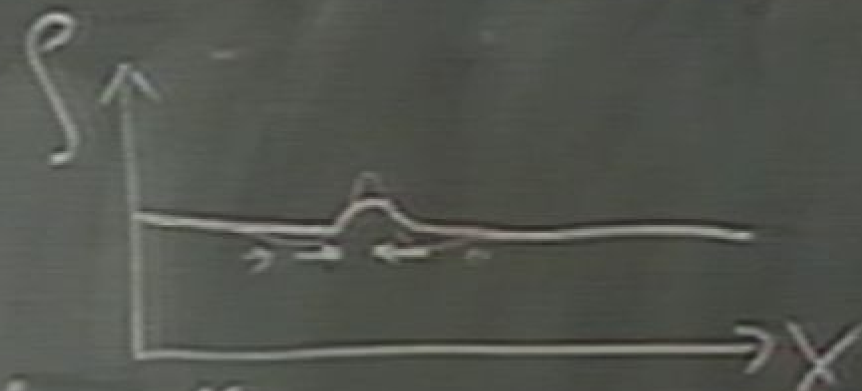
pert. about $\mathcal{M}_4$ / FRW



$$H\dot{\delta\rho} + \ddot{\delta\rho} \sim G\,\delta\rho$$

exponential instability

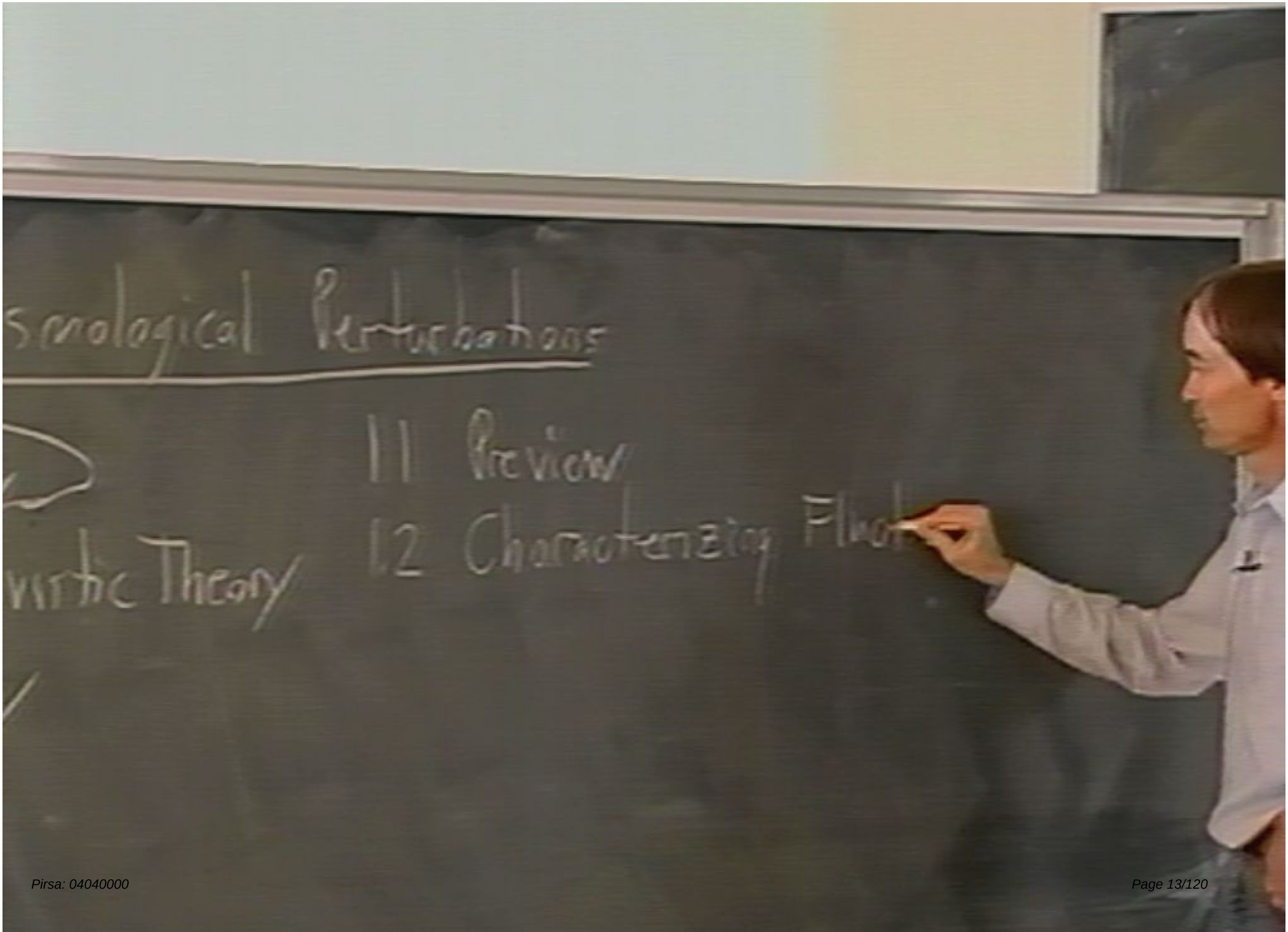
pert. about M_4 / FRW

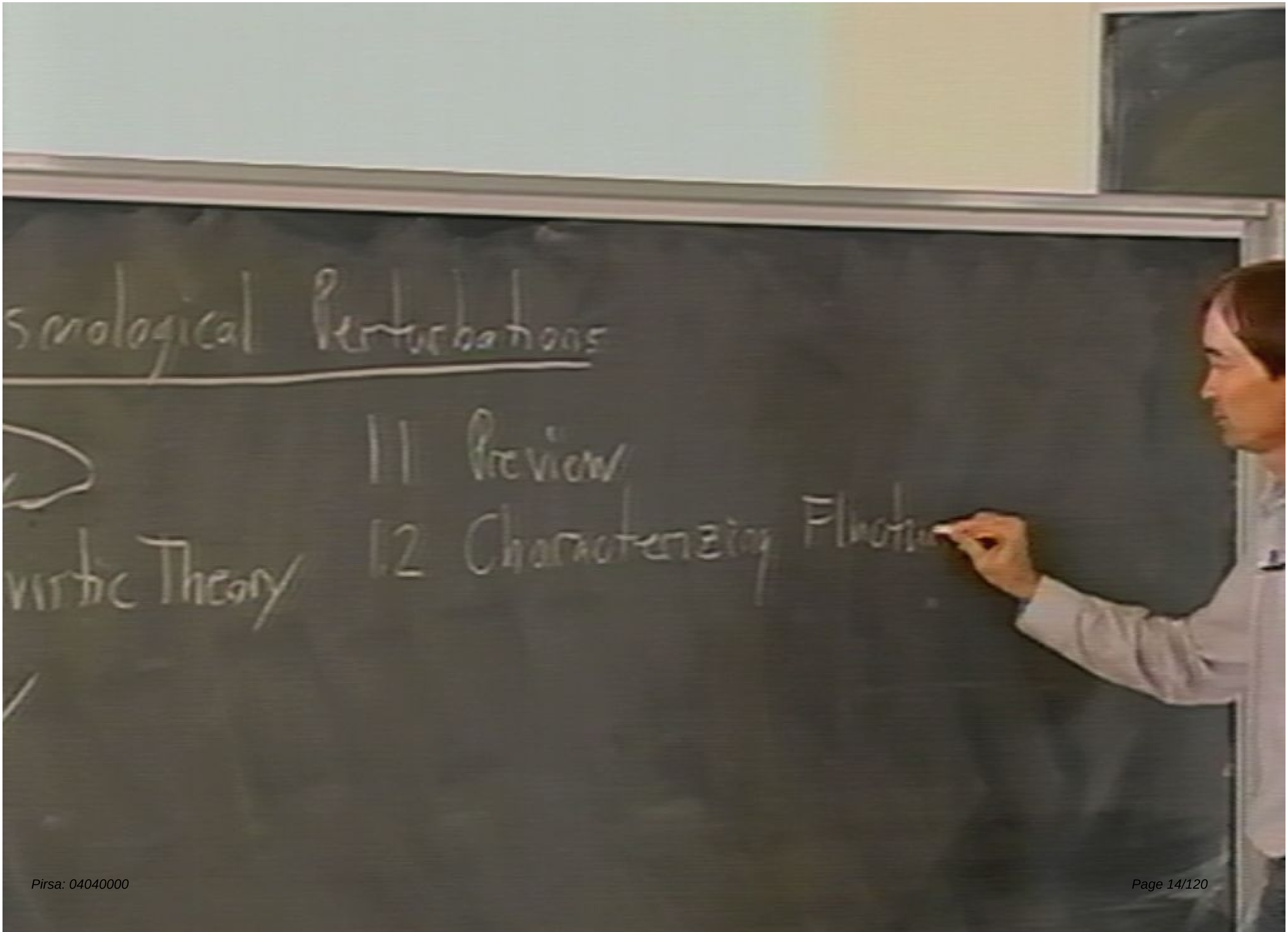


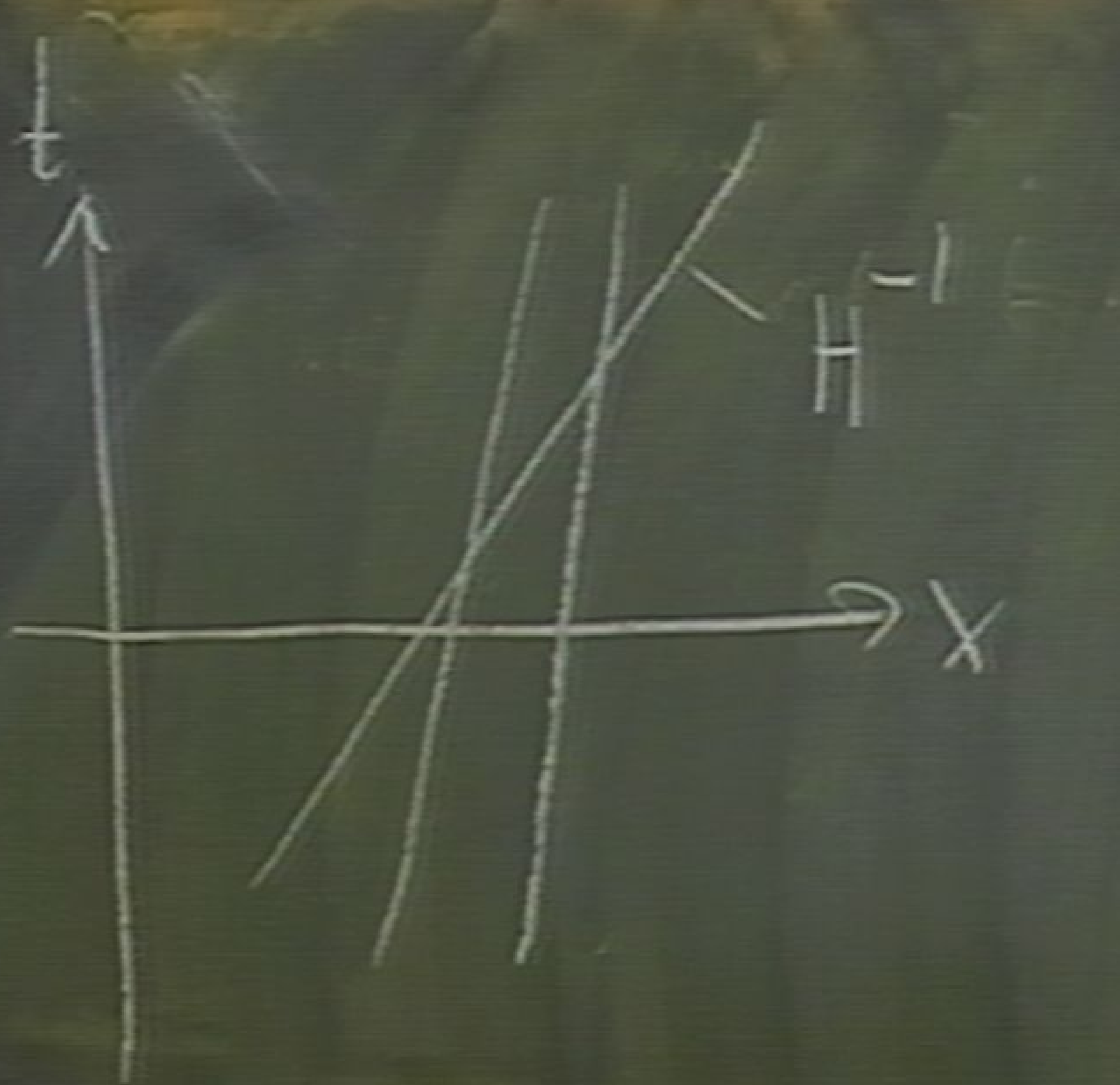
$$H\dot{\delta\rho} + \ddot{\delta\rho} \sim G\,\delta\rho$$

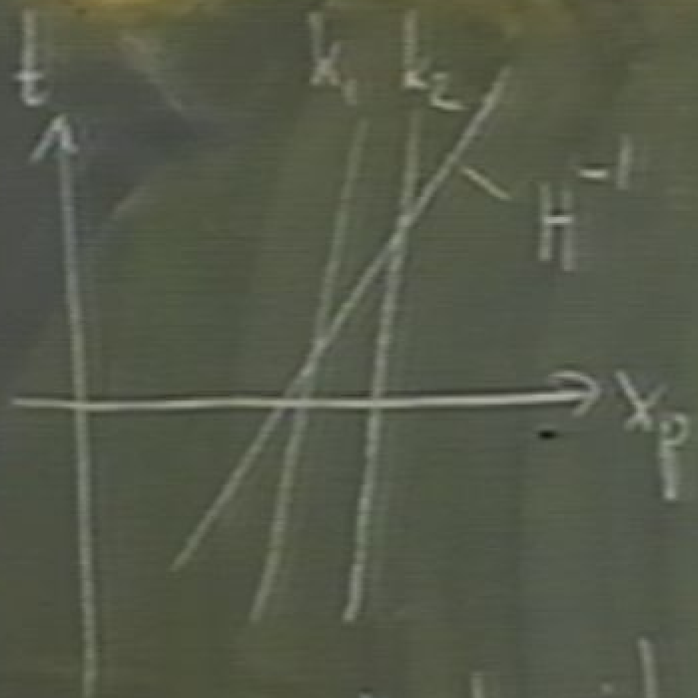
exponential instability

power law " "


smological Perturbations
1.1 Preview
1.2 Characterizing
Theory


Perturbations
1.1 Preview
1.2 Characterizing
Theory


t
x
H
H^{-1}

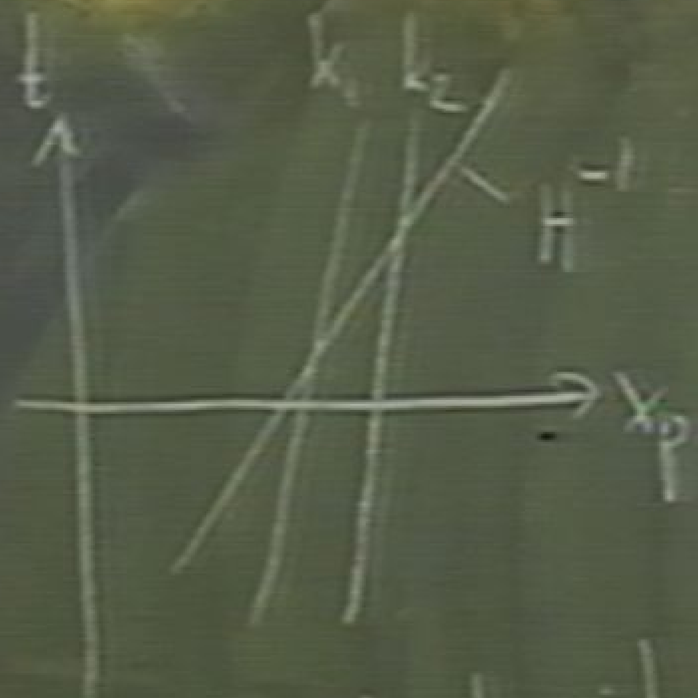


$\delta_\varepsilon(\underline{x},t) = \int d^3k$

$\varepsilon(\underline{x},t)$ density

$\delta\varepsilon(\underline{x},t)$ " pert.

$$\delta_\varepsilon = \frac{\delta\varepsilon(\underline{x},t)}{\varepsilon}$$



$$\delta_\varepsilon(\underline{x},t) = \int d^3k$$

$\varepsilon(\underline{x},t)$ density

$\delta\varepsilon(\underline{x},t)$ " pert.

$$\delta_\varepsilon(\underline{x},t) = \frac{\delta\varepsilon(\underline{x},t)}{\varepsilon_0(t)}$$

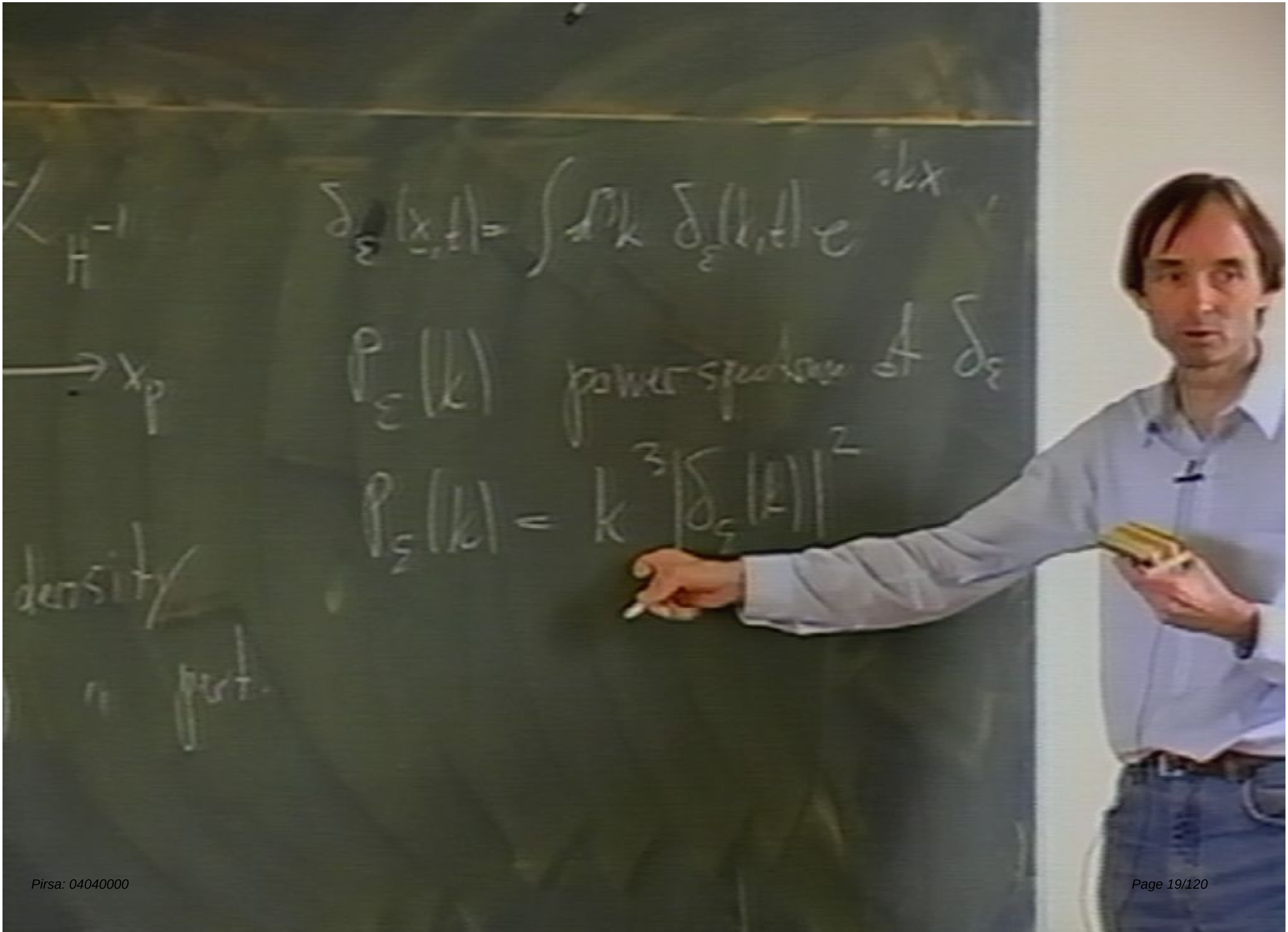
$/$
H^{-1}

$\delta_\varepsilon(\underline{x},t) = \int d^3k \, \delta_\varepsilon(k,t)\, e^{ikx}$

$\longrightarrow x_p$

density

) " pert.



$\delta_\varepsilon(x,t) = \int d^3k\, \delta_\varepsilon(k,t)\, e^{ikx}$
H^{-1}
$\rightarrow x_p$
$P_\varepsilon(k)$ power spectrum of δ_ε
$P_\varepsilon(k) = k^3 |\delta_\varepsilon(k)|^2$
density
n part.

$< H^{-1}$

$\delta_\epsilon(\underline{x},t) = \int d^3k\, \delta_\epsilon(k,t)\, e^{ikx}$

$\rightarrow x_p$

$P_\epsilon(|k|)$ power spectrum of δ_ϵ

$P_\epsilon(|k|) = k^3 |\delta_\epsilon(k)|^2$

density

pert.

$\frac{\delta M}{M}$

H^{-1}

$\longrightarrow x_p$

density

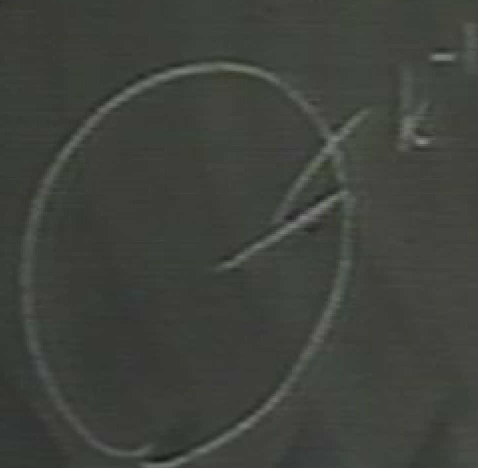
" pert.

$$\delta_\epsilon(\underline{x},t) = \int d^3k \, \delta_\epsilon(k,t)\, e^{ikx}$$

$P_\epsilon(k)$ power spectrum of δ_ϵ

$$P_\epsilon(k) = k^3 |\delta_\epsilon(k)|^2$$

$$\frac{\delta M}{M}$$



$$P_\varphi(k) \equiv k^3 |\delta_\varphi(k)|^2$$

$\varphi(x,t)$ Newt. gravit. potential

$$\delta_\varepsilon(\underline{x},t) = \int d^3k \, \delta_\varepsilon(k,t) e^{ikx}$$

H^{-1}

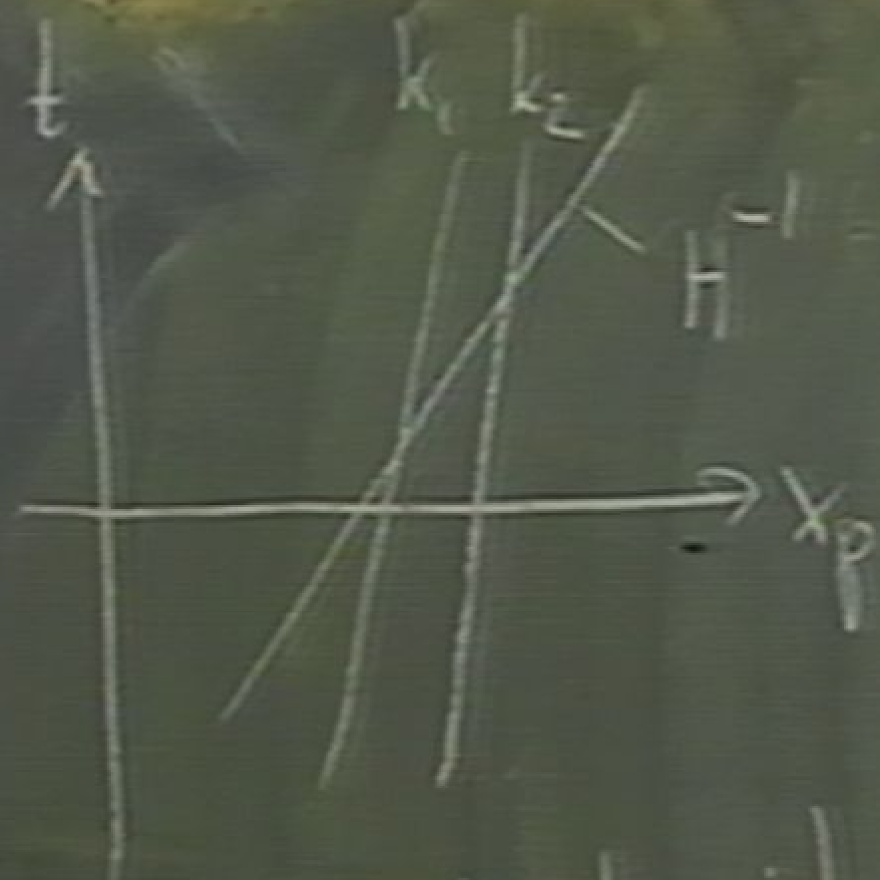
$P_\varepsilon(k)$ power spectrum of δ_ε

$\rightarrow x_p$

power spectrum of ψ

$P_\psi(k)$

$\psi(x,t)$



$\delta_\varepsilon(x$

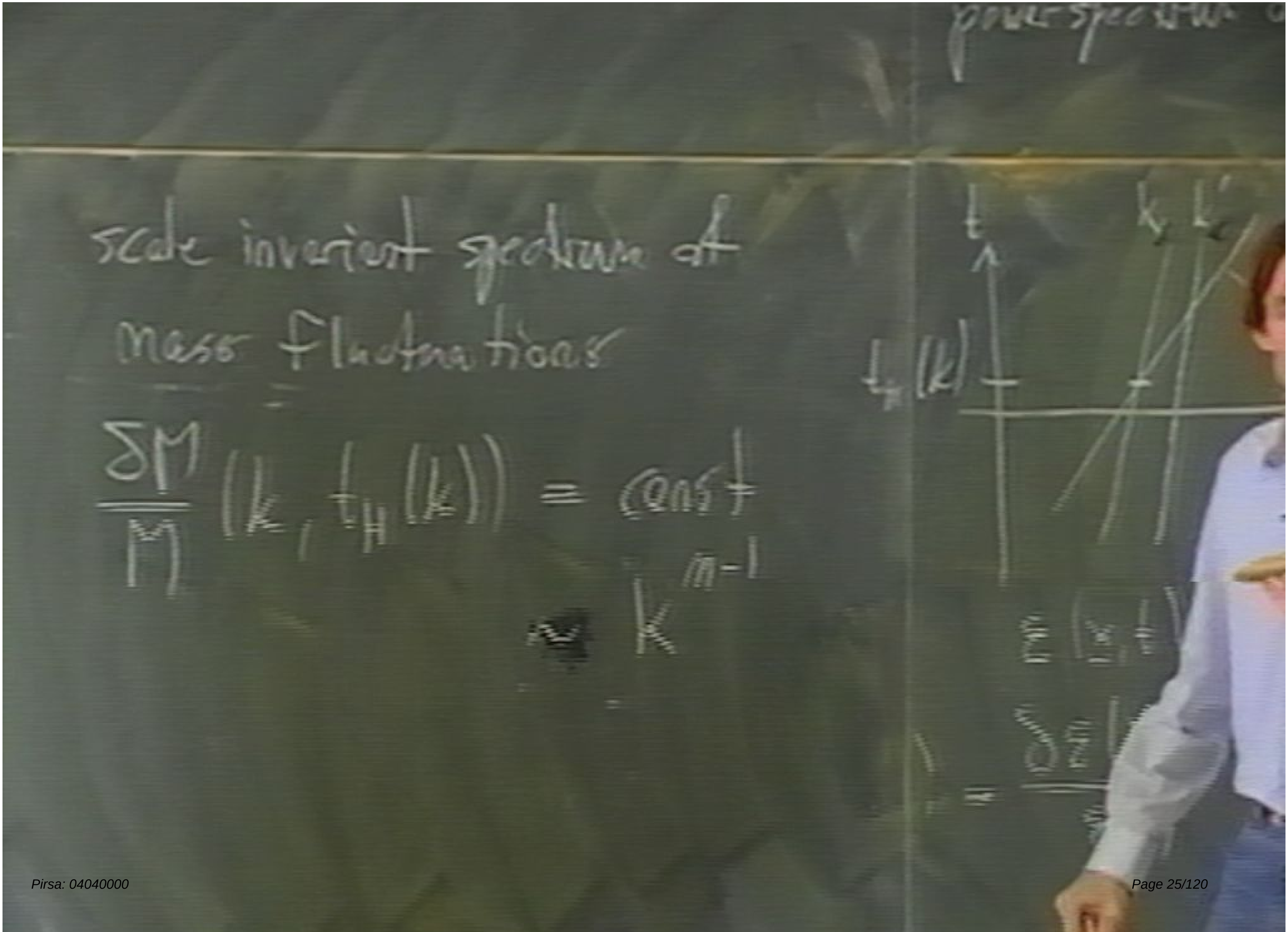
$P_\varepsilon($

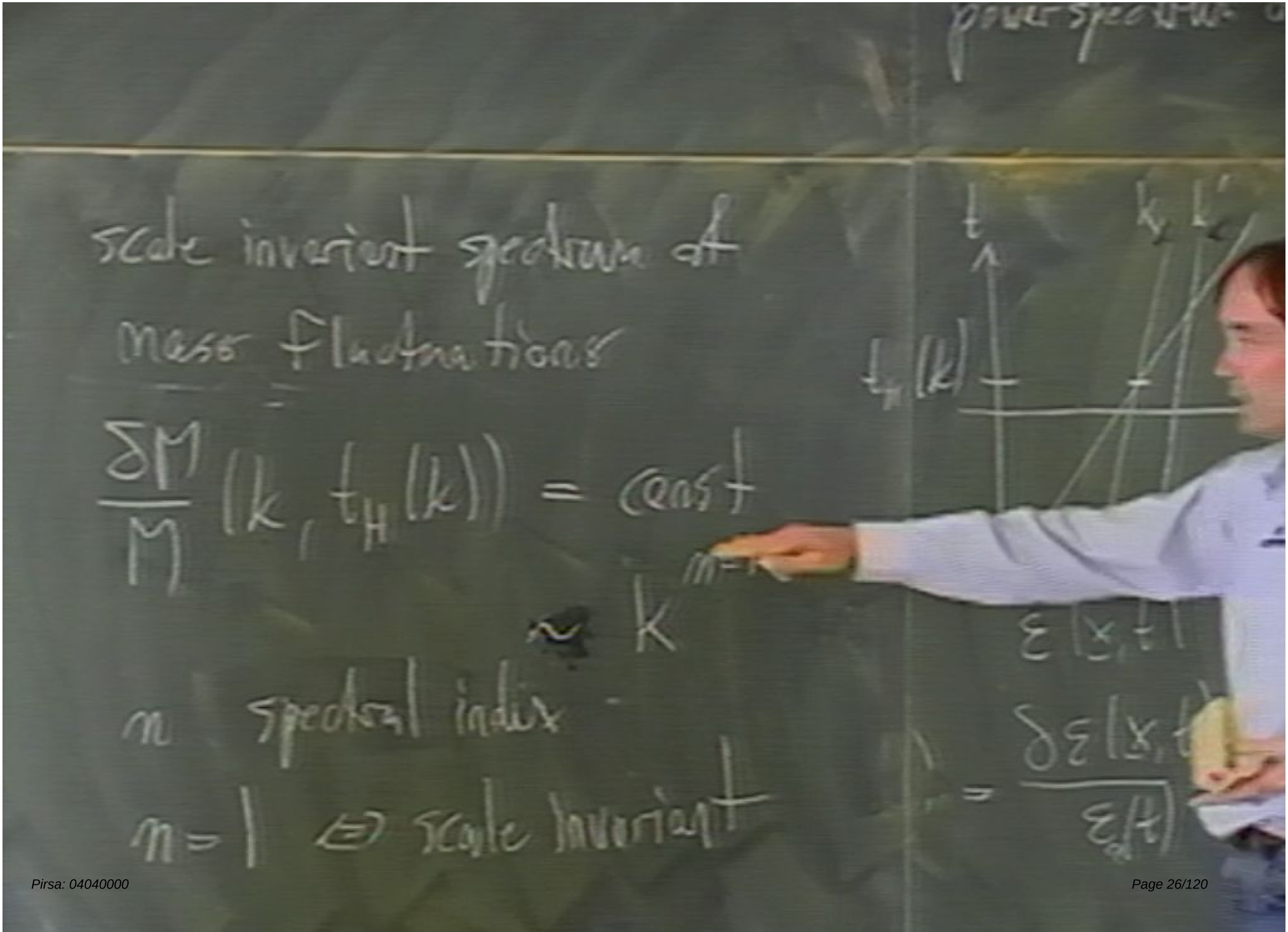
$P_\varepsilon($

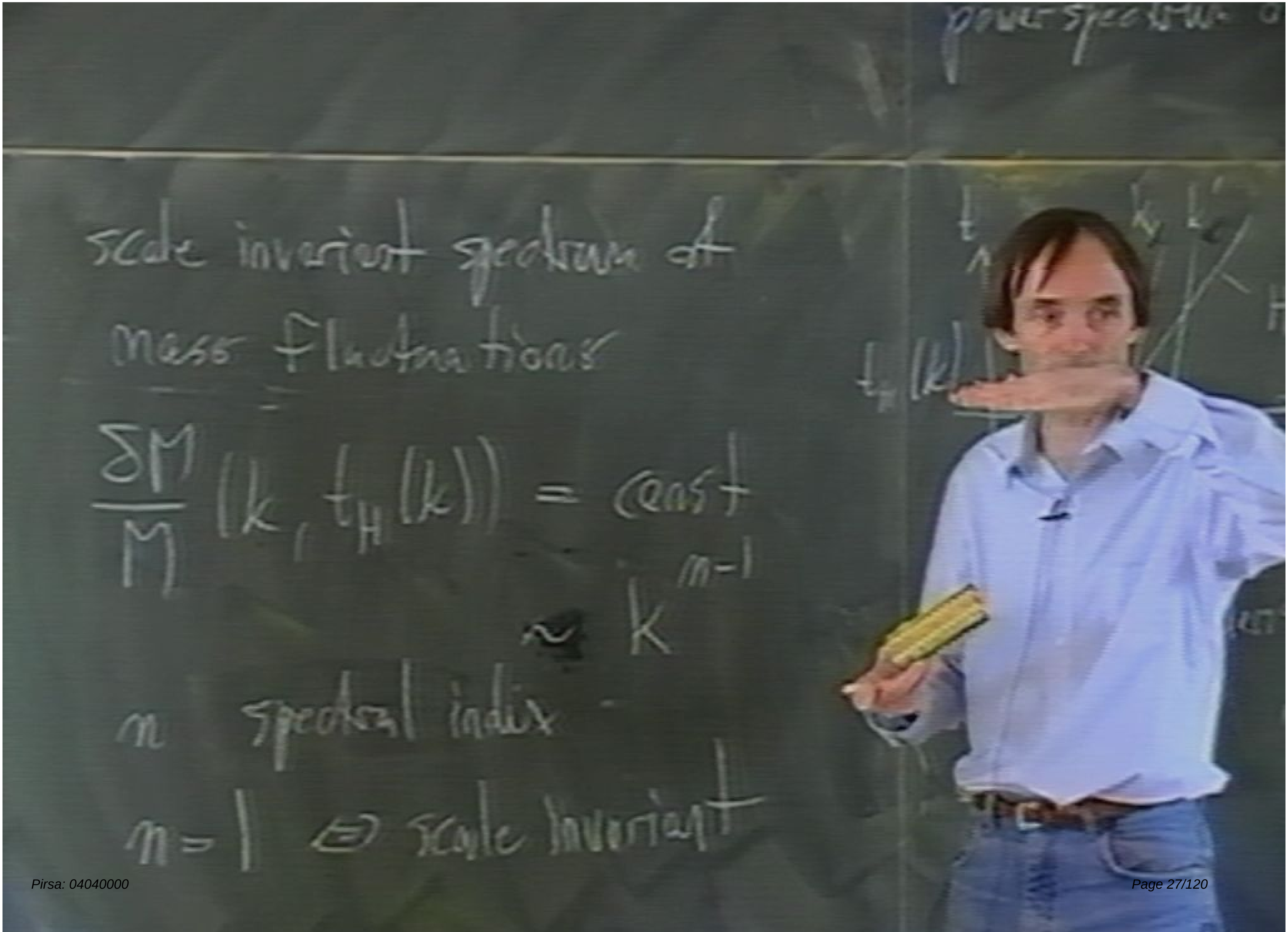
density

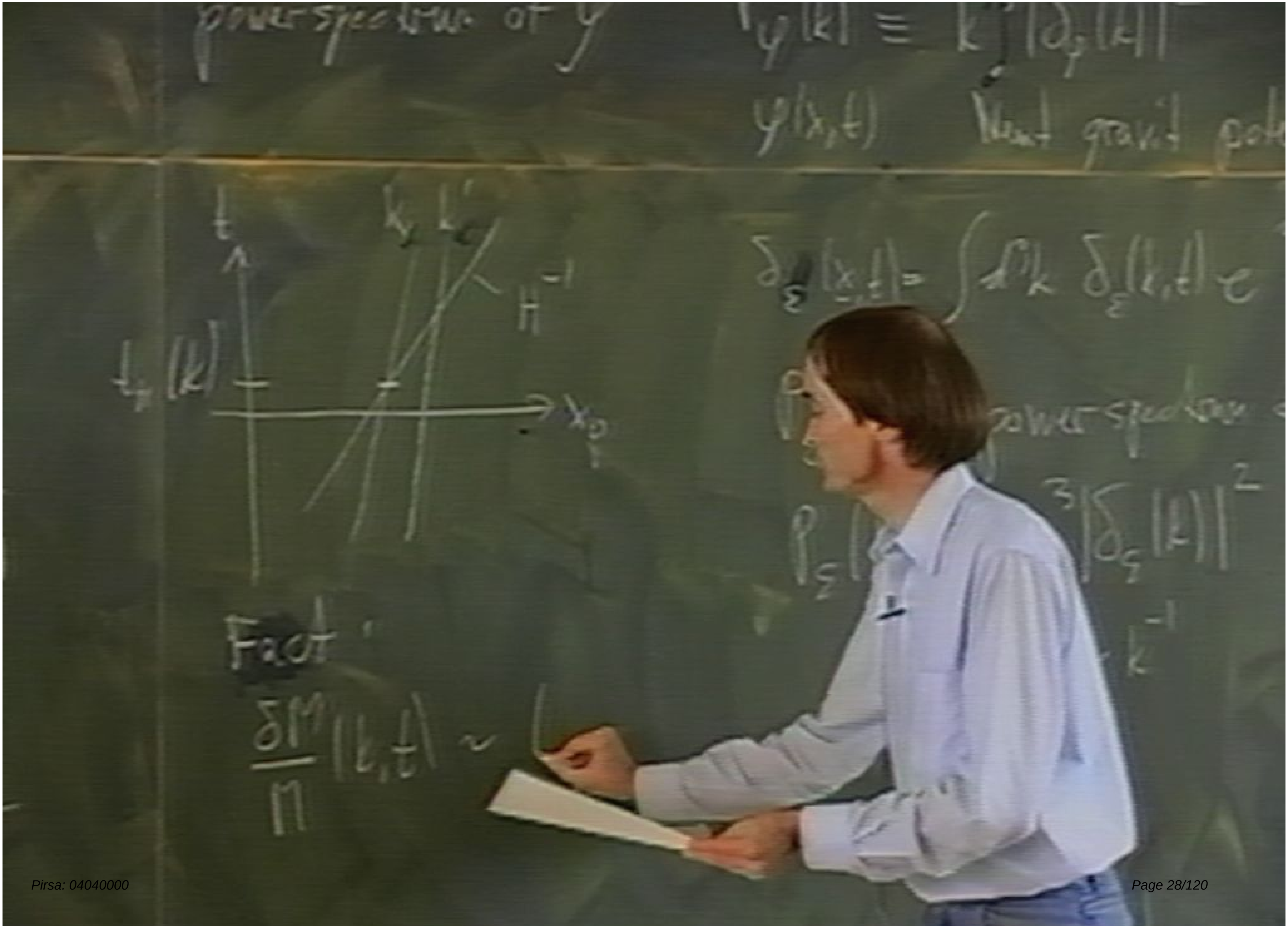
Def: scale invariant spectrum of mass fluctuations

$$\frac{\delta M}{M}(k,$$


scale invariant spectrum of
mass fluctuations
$\frac{\delta M}{M}(k, t_H(k)) = \text{const}$
$\sim k^{n-1}$
t
k
k'
$t_H(k)$
$\varepsilon(\underline{x}, t)$

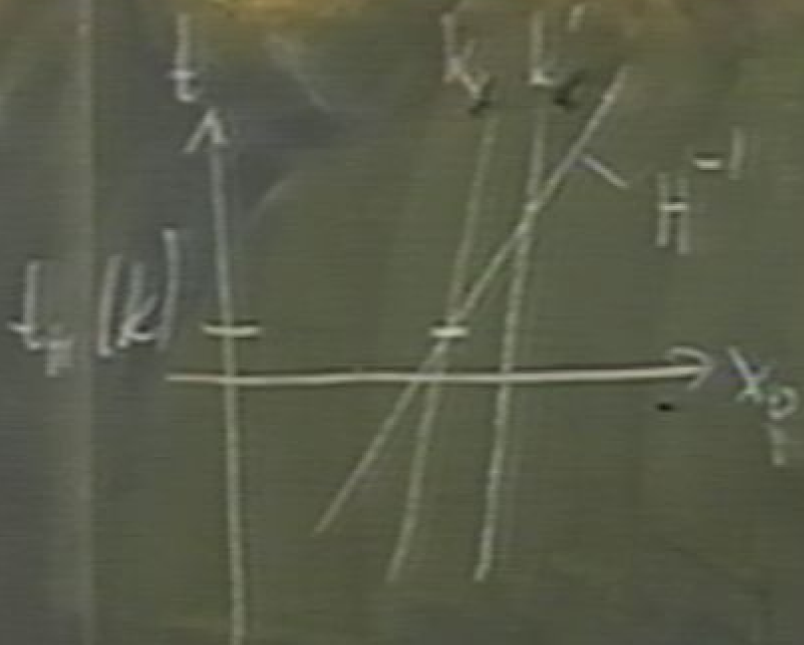

power spectrum
scale invariant spectrum of
mass fluctuations
δM/M (k, t_H(k)) = const
~ k^{n-1}
n spectral index
n = 1 ⟺ scale invariant
t
k, k'
t_H(k)
ε(x, t)
δε(x, t)/ε_0(t)


power spectrum
scale invariant spectrum of
mass fluctuations
$\frac{\delta M}{M}(k, t_H(k)) = \text{const}$
$\sim k^{n-1}$
n spectral index
$n = 1 \Leftrightarrow$ scale invariant
$t_H(k)$


power-spectrum of ψ
ψ(x,t) Newt gravit pot
t
k, k'
H^-1
t_H(k)
x_p
Fact :
δM/M (k,t) ~
power-spectrum
k^-1

power spectrum of φ $\qquad P_\varphi(k) \equiv k^3 |\delta_\varphi(k)|^2$

$\varphi(x,t)$ Newt gravit pot



$$\delta_\varepsilon(\underline{x},t) = \int d^3k\, \delta_\varepsilon(k,t)\, e$$

$P_\varepsilon(k)$ power spectrum

$$P_\varepsilon(k) \equiv k^3 |\delta_\varepsilon(k)|^2$$

Fact:

$$\frac{\delta M}{M}(k,t) \sim t^{2/3}$$

$\varphi(x,t)$ Newt gravit potential

$$\delta_\varepsilon(x,t) = \int d^3k \, \delta_\varepsilon(k,t) \, e^{ikx}$$

$P_\varepsilon(k)$ power spectrum of δ_ε

$$P_\varepsilon(k) \equiv k^3 |\delta_\varepsilon(k)|^2$$

$$\frac{\delta M}{M}(k,t) = \left(\frac{t}{t_H(k)}\right)^{2/3} \frac{\delta M}{M}(k, t_H(k))$$

$P_\varepsilon(k)$ power spectrum of δ_ε

$$P_\varepsilon(k) \equiv k^3 |\delta_\varepsilon(k)|^2$$

$$\frac{\delta M}{M}(k,t) = \left(\frac{t}{t_H(k)}\right)^{2/3} \frac{\delta M}{M}(k, t_H(k))$$

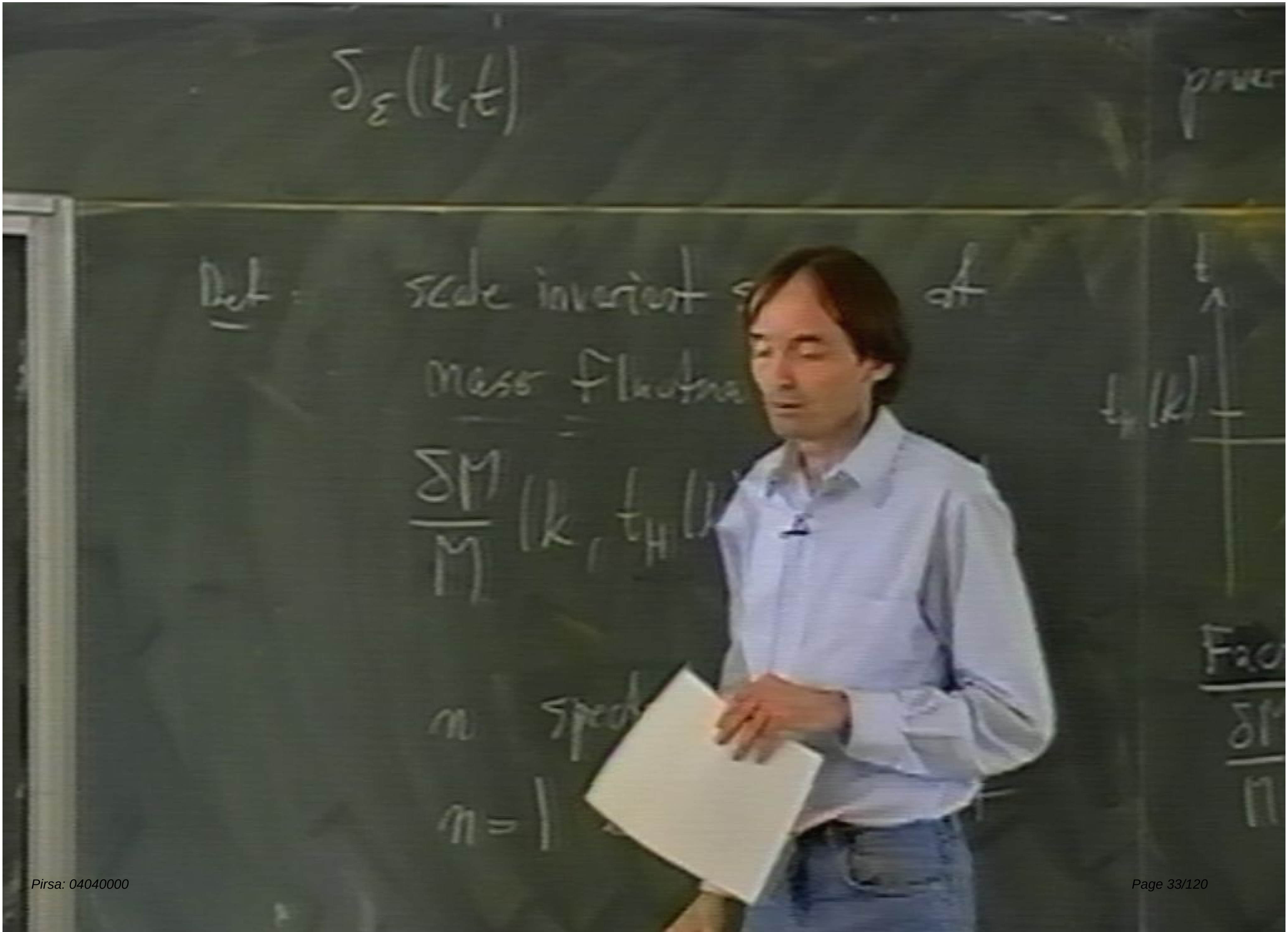
$t_H(k)$:

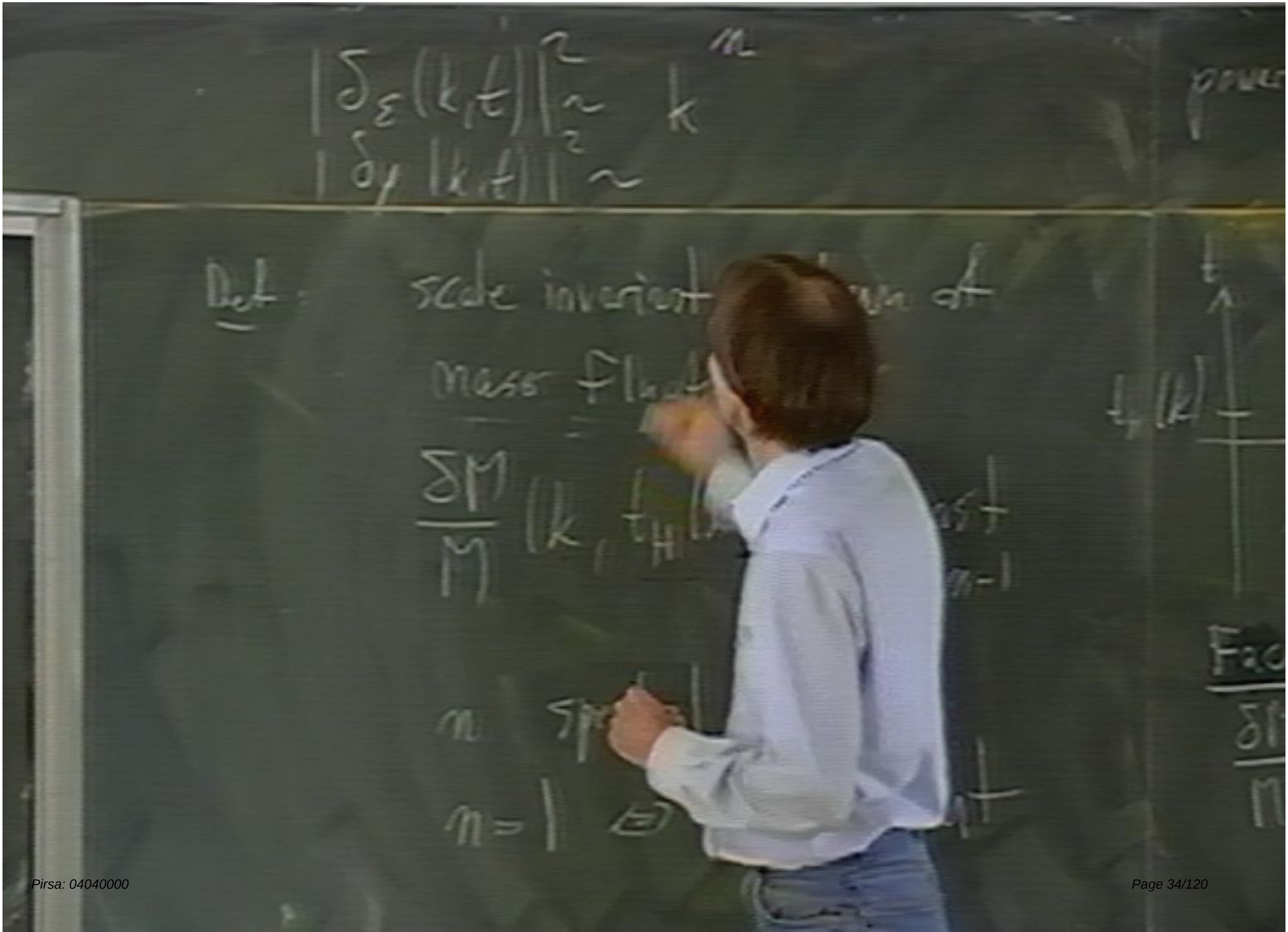
$P_\varepsilon(k)$ power spectrum of δ_ε

$$P_\varepsilon(k) \equiv k^3 |\delta_\varepsilon(k)|^2$$

$$\frac{\delta M}{M}(k,t) = \left(\frac{t}{t_H(k)}\right)^{2/3} \frac{\delta M}{M}(k, t_H(k))$$

$$t_H(k) : \quad H^{-1} = a(t_H(k))\, k^{-1}$$


δε(k,t)
Def:
scale invariant
of
mass fluctua
δM / M (k, tH(k)
n
n = 1
t
tH(k)



$|\delta_\varepsilon(k,t)|^2 \sim k^n$
$|\delta_\psi(k,t)|^2 \sim$
Def.
scale invariant
of
mass fl
$\frac{\delta M}{M}(k, t_H($
$n-1$
n spe
n = 1 ⇔
power
$t_H(k)$
t
Fac
$\frac{\delta M}{M}$

$$|\delta_\varepsilon(k,t)|^2 \sim k^{n}$$

$$|\delta_\varphi(k,t)|^2 \sim k^{-4+n}$$

power

Def: scale invariant spectrum of mass fluctuations

$$\frac{\delta M}{M}(k, t_H(k)) = \text{const}$$

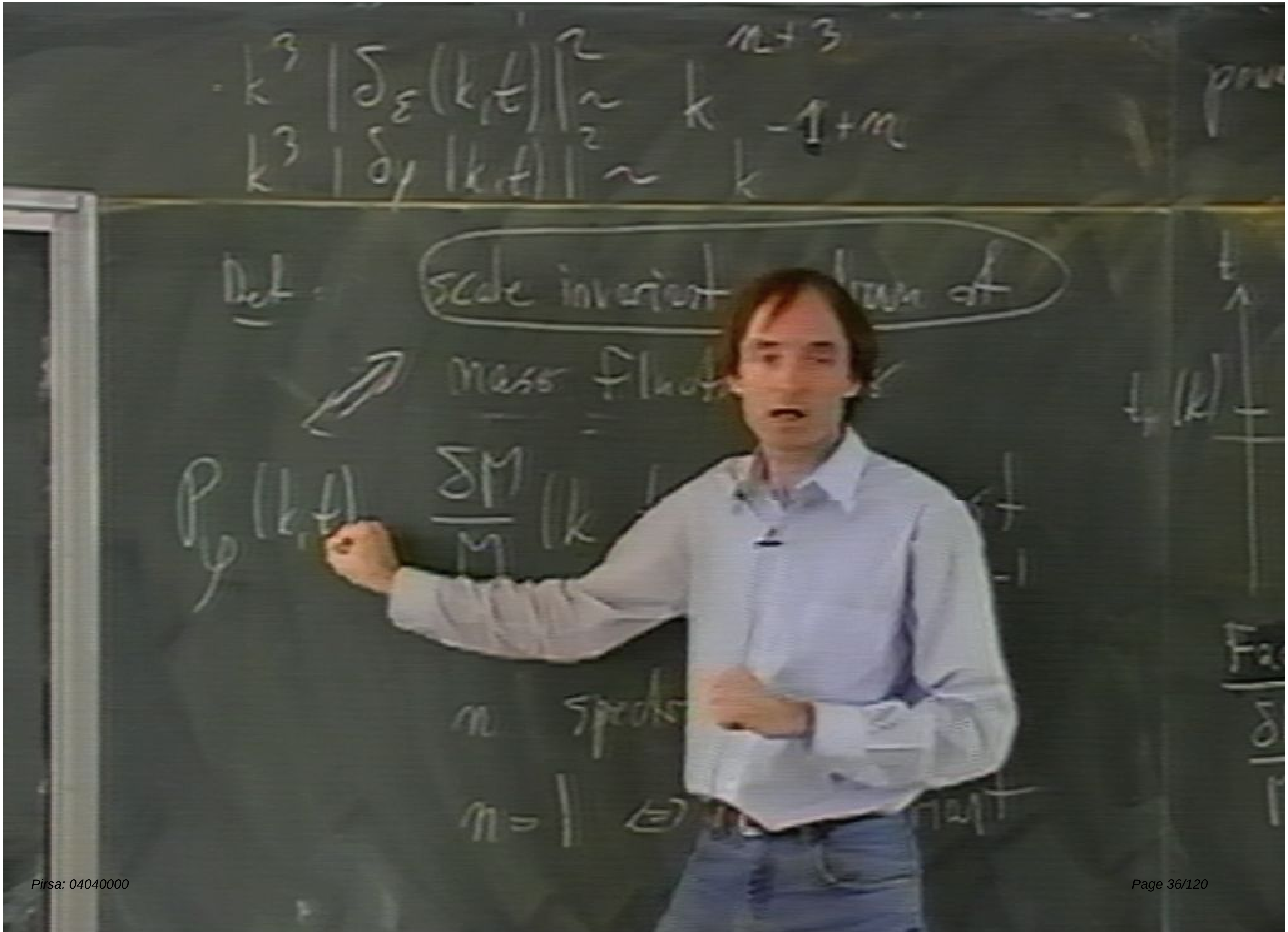
$$\sim k^{n-1}$$

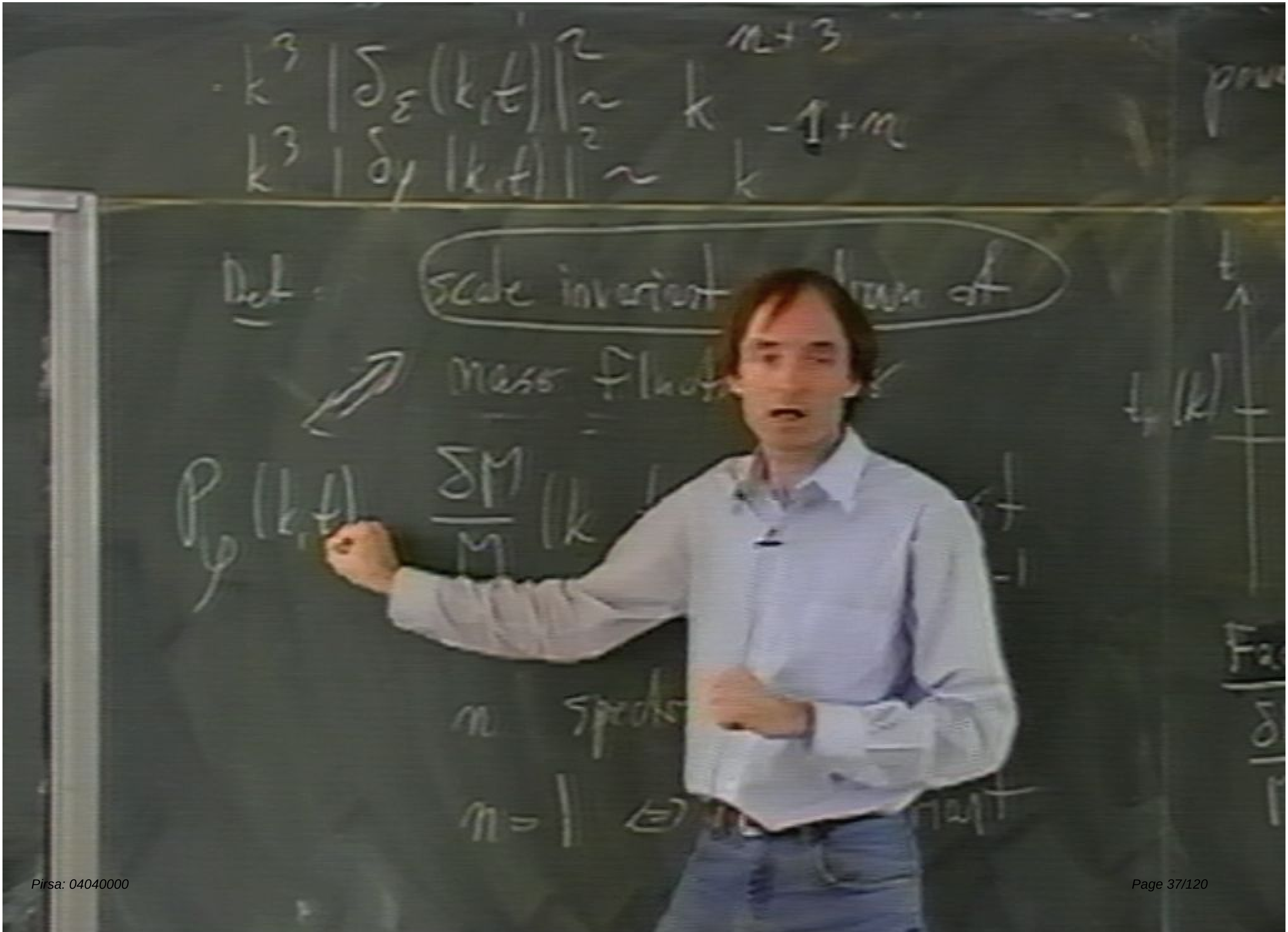
n spectral index

$n = 1 \Leftrightarrow$ scale invariant

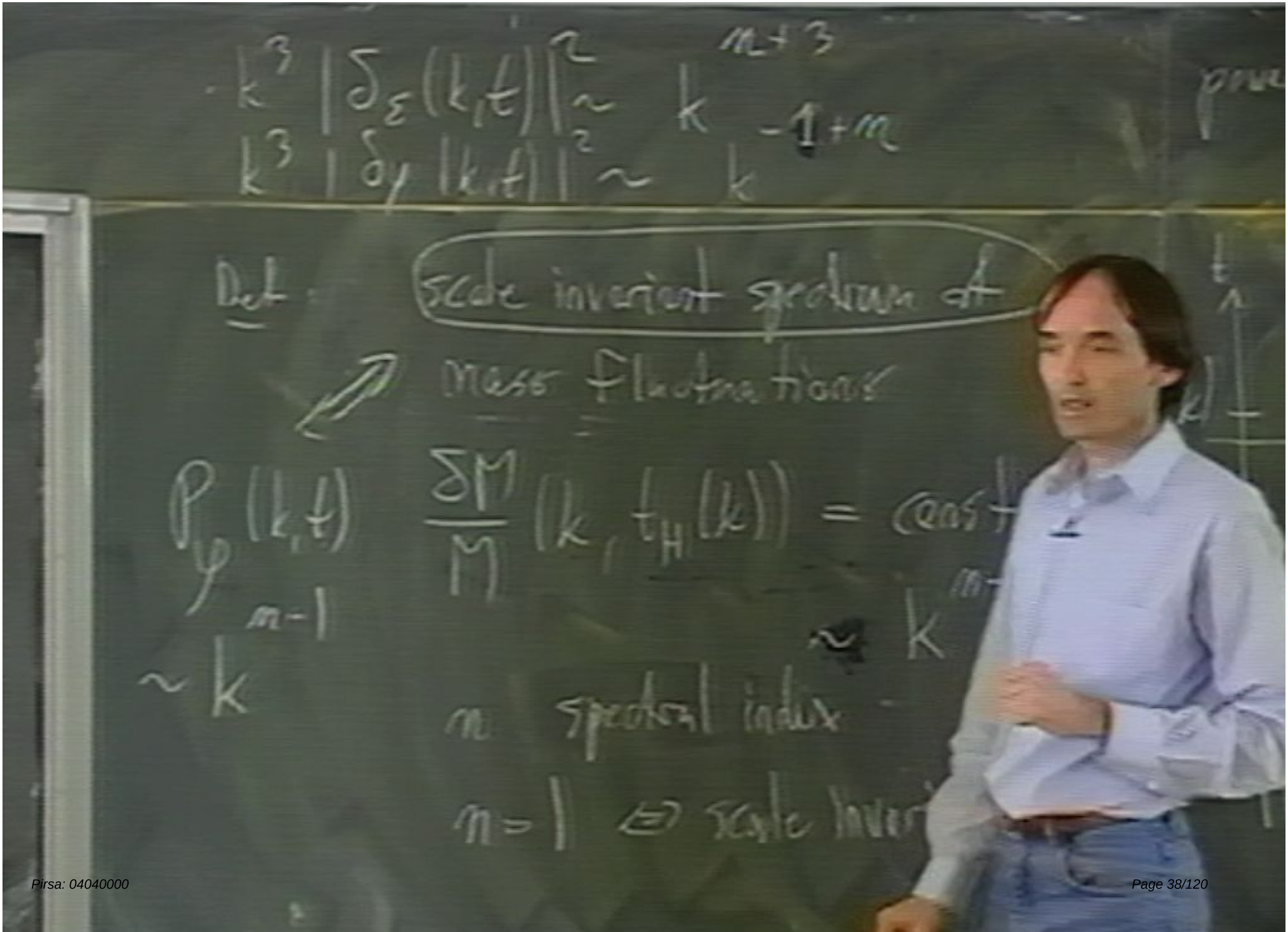
t

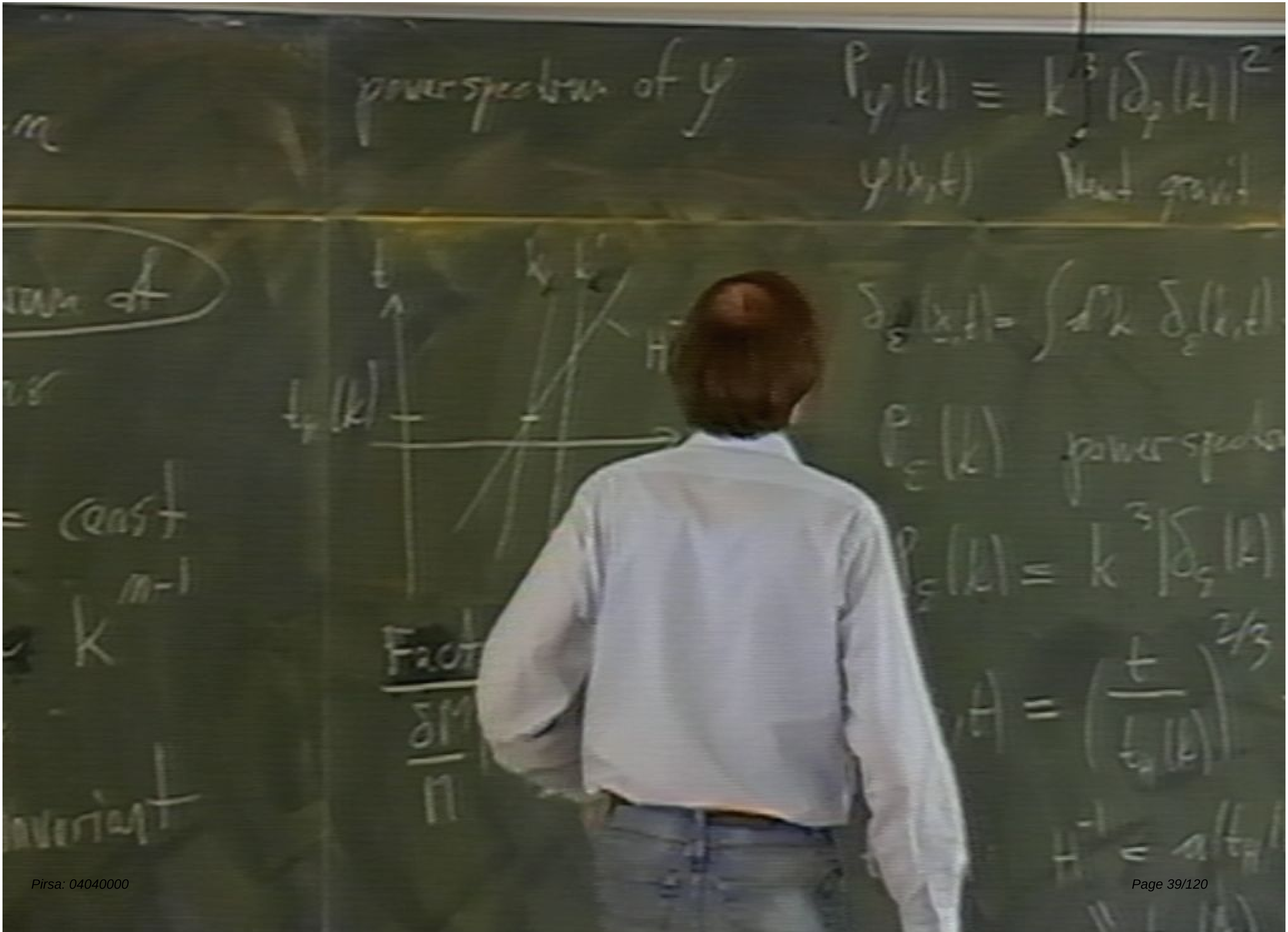
$t_H(k)$


Def.
scale invariant
mass fl
n spect
n = 1



$k^3 |\delta_\varepsilon(k,t)|^2 \sim k^{n+3}$
$k^3 |\delta_\varphi(k,t)|^2 \sim k^{-1+n}$
Def.
scale invariant
of
mass
$P_\varphi(k,t)$
$\frac{\delta M}{M}(k$
n
$n=1$


$k^3 |\delta_\varepsilon(k,t)|^2 \sim k^{n+3}$
$k^3 |\delta_\varphi(k,t)|^2 \sim k^{-1+n}$
Def:
scale invariant spectrum of
mass fluctuations
$P_\varphi(k,t) \sim k^{n-1}$
$\frac{\delta M}{M}(k, t_H(k)) = const$
$\sim k$
n spectral index
n = 1 ⟺ scale


power spectrum of ψ
P_ψ(k) ≡ k^3 |δ_ψ(k)|^2
ψ(x,t)
= const
k^{n-1}
invariant
t
t_H(k)
k
H
Fact
δ_ε(x,t) = ∫ d^3k δ_ε(k,t)
P_ε(k) power spectrum
P_ε(k) ≡ k^3 |δ_ε(k)|
H = (t / t_H(k))^{2/3}

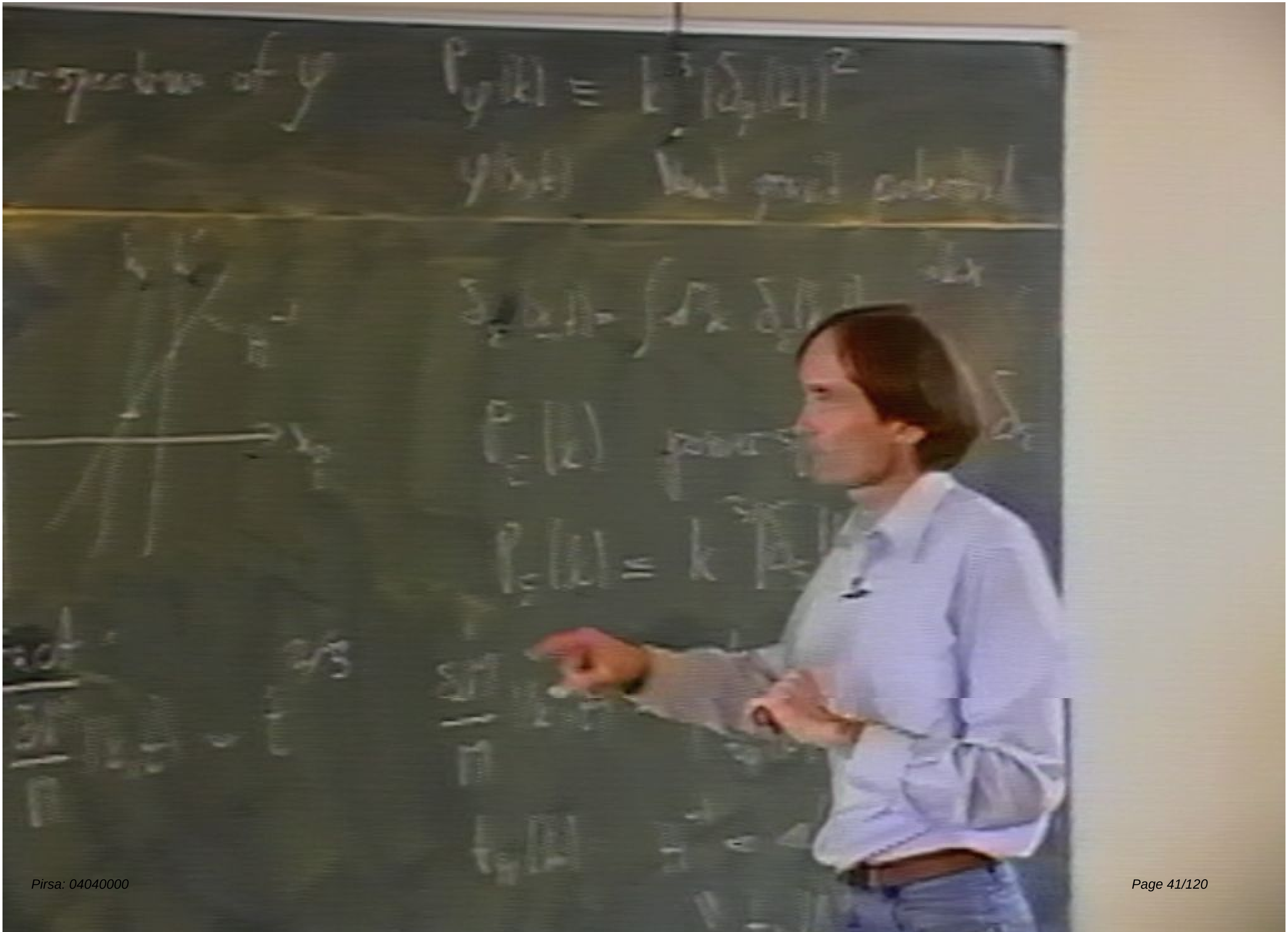
ogical Perturbations

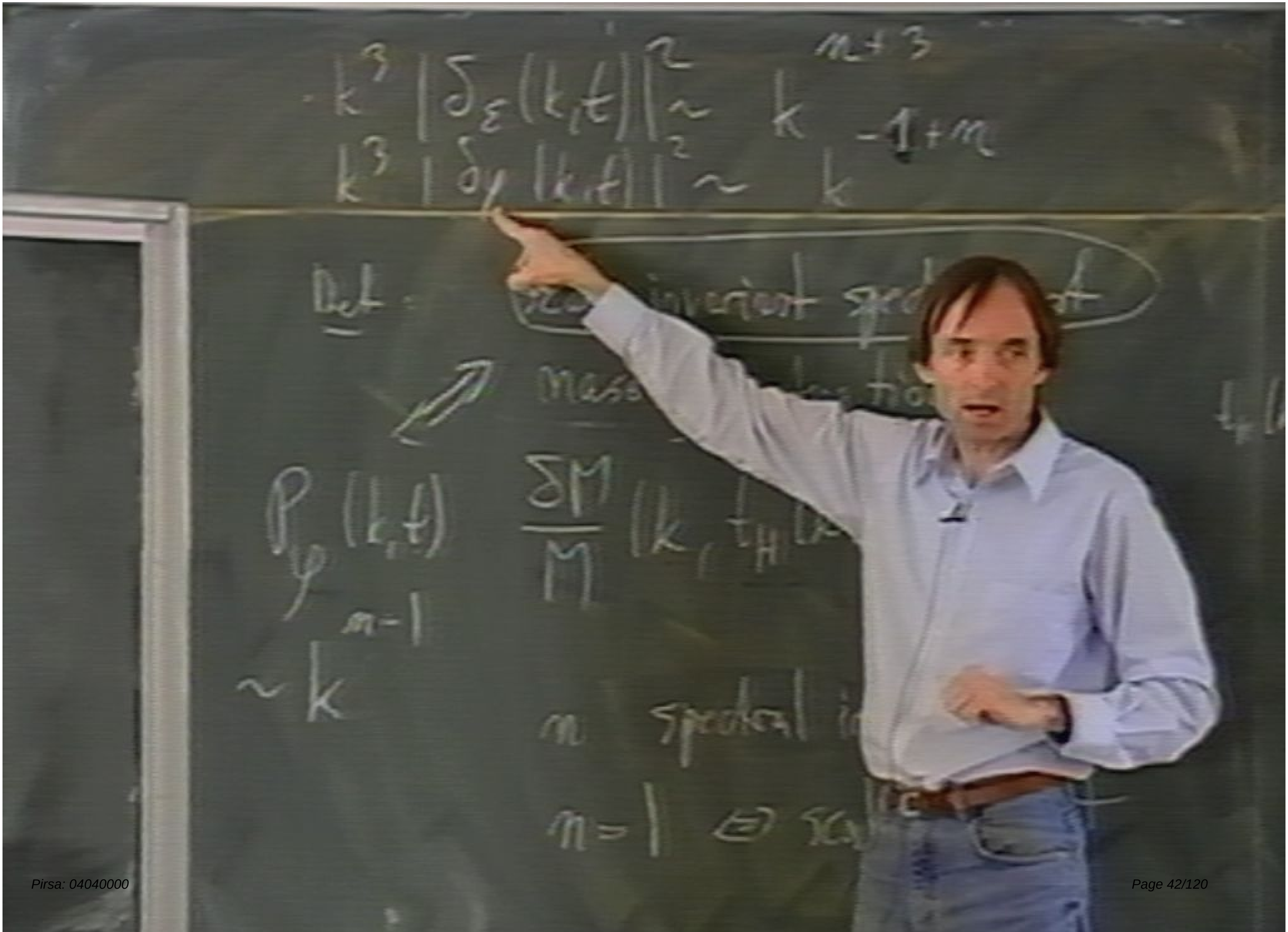
Theory

1.1 Preview

1.2 Characterizing Fluctuations

1.3




k^3 |δ_ε(k,t)|^2 ~ k^(n+3)
k^3 |δ_φ(k,t)|^2 ~ k^(-1+n)
Def.
invariant spect
mass
P_φ(k,t)
δM/M (k, t_H
~ k^(n-1)
n spectral i
n = 1

$$k^3 |\delta_\varepsilon(k,t)|^2 \sim k^{n+3}$$

$$k^3 |\delta_\varphi(k,t)|^2 \sim k^{-1+n}$$

Def. scale invariant spectrum of mass fluctuations

$$P_\varphi(k,t) \sim k^{n-1}$$

$$\frac{\delta M}{M}(k, t_H(k)) = \text{const}$$

$$\sim k^{n-1}$$

n spectral index

$n=1 \Leftrightarrow$ scale invariant

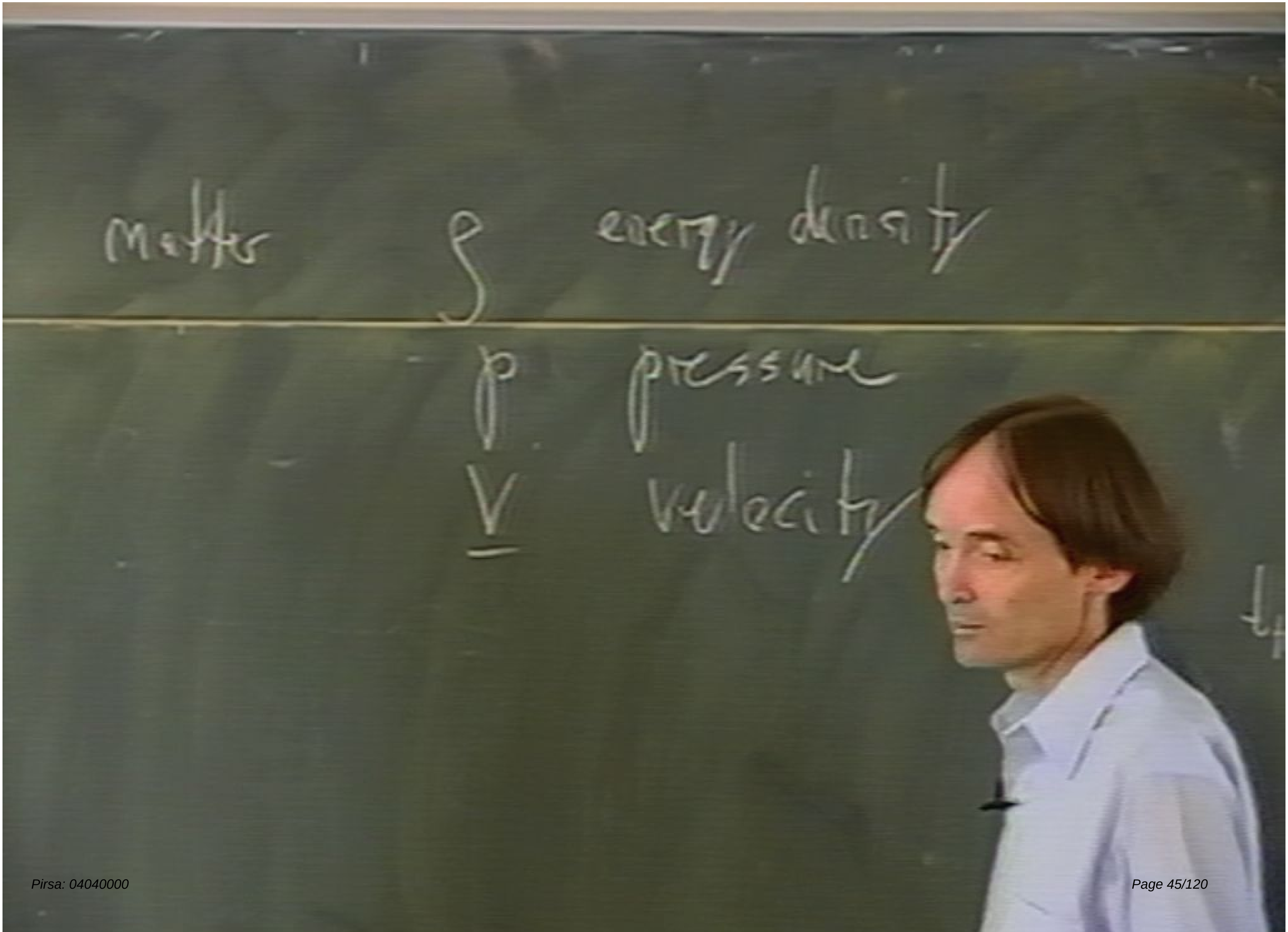
ological Perturbations

c Theory

1.1 Preview

1.2 Characterizing Fluctuations

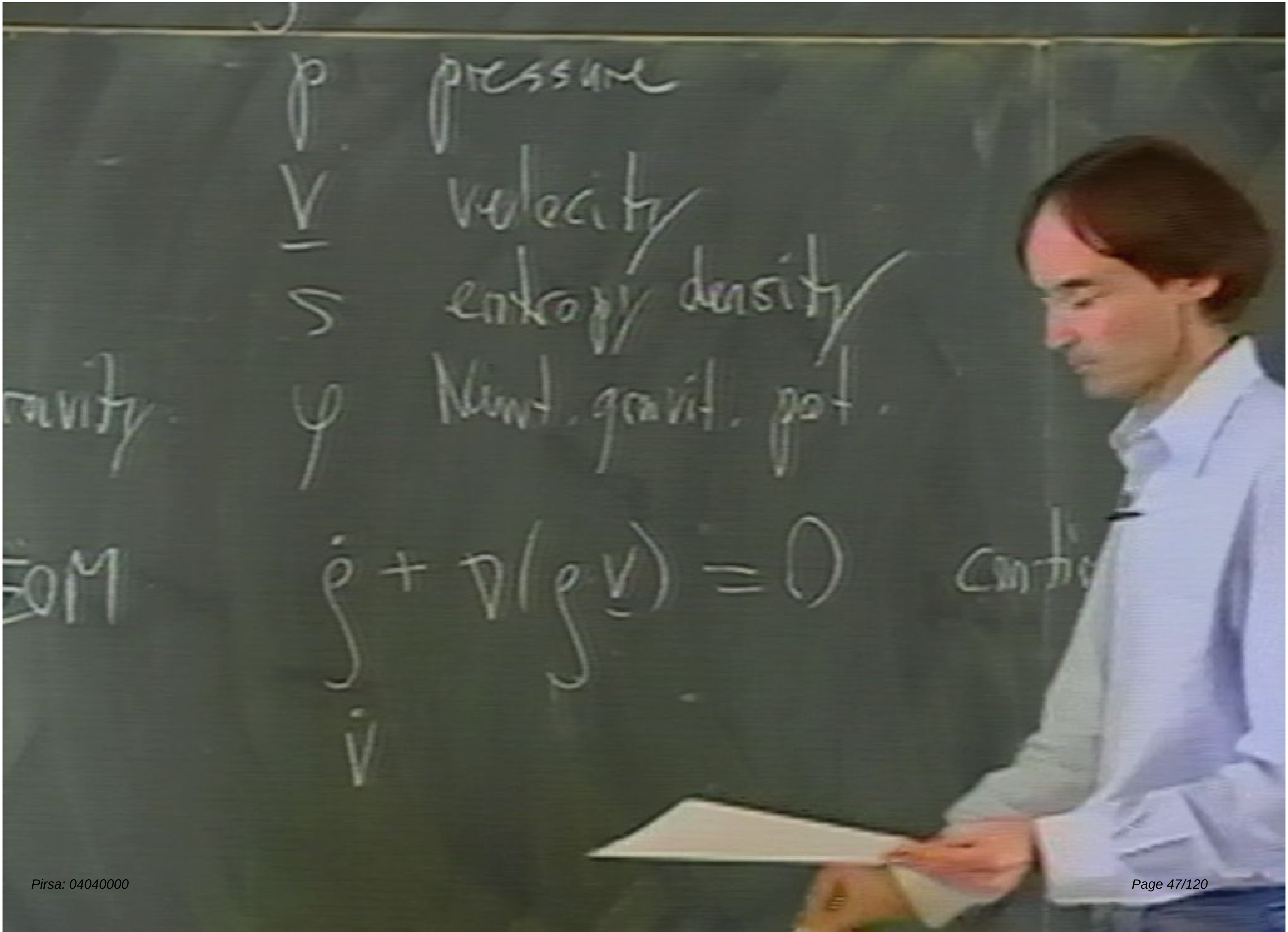
1.3 Newt. Pert. about M^4


matter
ρ energy density
p pressure
v velocity

$\underline{V}$ velocity

s entropy density

gravity: φ Newt. gravit. pot.


p pressure
v velocity
s entropy density
ravity φ Newt. gravit. pot.
EOM ρ̇ + ∇(ρ v) = 0 contin
v̇

p pressure

$\underline{v}$ velocity

s entropy density

...ravity. φ Newt. gravit. pot.

EOM $\dot{\rho} + \nabla(\rho \underline{v}) = 0$ continuity eq...

$\dot{\underline{v}} + (\underline{v} \cdot \underline{\nabla})\underline{v} -$

ρ pressure

$\underline{v}$ velocity

s entropy density

φ Newt. gravit. pot.

$$\dot{\rho} + \nabla(\rho \underline{v}) = 0 \quad \text{continuity eq.}$$

$$\dot{\underline{v}} + (\underline{v} \cdot \underline{\nabla}) \underline{v} + \frac{1}{\rho} \nabla p + \nabla \varphi = 0$$

ρ energy density

p pressure

$\underline{v}$ velocity

s entropy density

ty φ Newt. gravit. pot.

1 $$\dot{\rho} + \nabla(\rho \underline{v}) = 0 \quad \text{continuity eq.}$$

$$\dot{\underline{v}} + (\underline{v} \cdot \underline{\nabla})\underline{v} + \frac{1}{\rho}\nabla p + \nabla\varphi = 0 \quad \text{Euler eq}$$

ρ energy density

p pressure

$\underline{v}$ velocity

s entropy density

φ Newt. gravit. pot.

$$\dot{\rho} + \nabla(\rho \underline{v}) = 0 \quad \text{continuity eq.}$$

$$\dot{\underline{v}} + (\underline{v} \cdot \underline{\nabla})\underline{v} + \frac{1}{\rho}\nabla p + \nabla\varphi = 0 \quad \text{Euler}$$

$$\nabla^2 \varphi = 4\pi G \rho \quad \text{Poisson}$$

ρ energy density

p pressure

$\underline{v}$ velocity

s entropy density

φ Newt. gravit. pot.

$$\frac{\partial}{\partial t} s + (\underline{v} \cdot \nabla) s = 0$$

$$\dot{\rho} + \nabla(\rho \underline{v}) = 0$$ continuity eq.

$$\dot{\underline{v}} + (\underline{v} \cdot \nabla)\underline{v} + \frac{1}{\rho} \nabla p + \nabla \varphi = 0$$ Euler eq

$$\nabla^2 \varphi = 4\pi G \rho$$ Poisson

energy density

pressure

velocity

entropy density

Newt. gravit. pot.

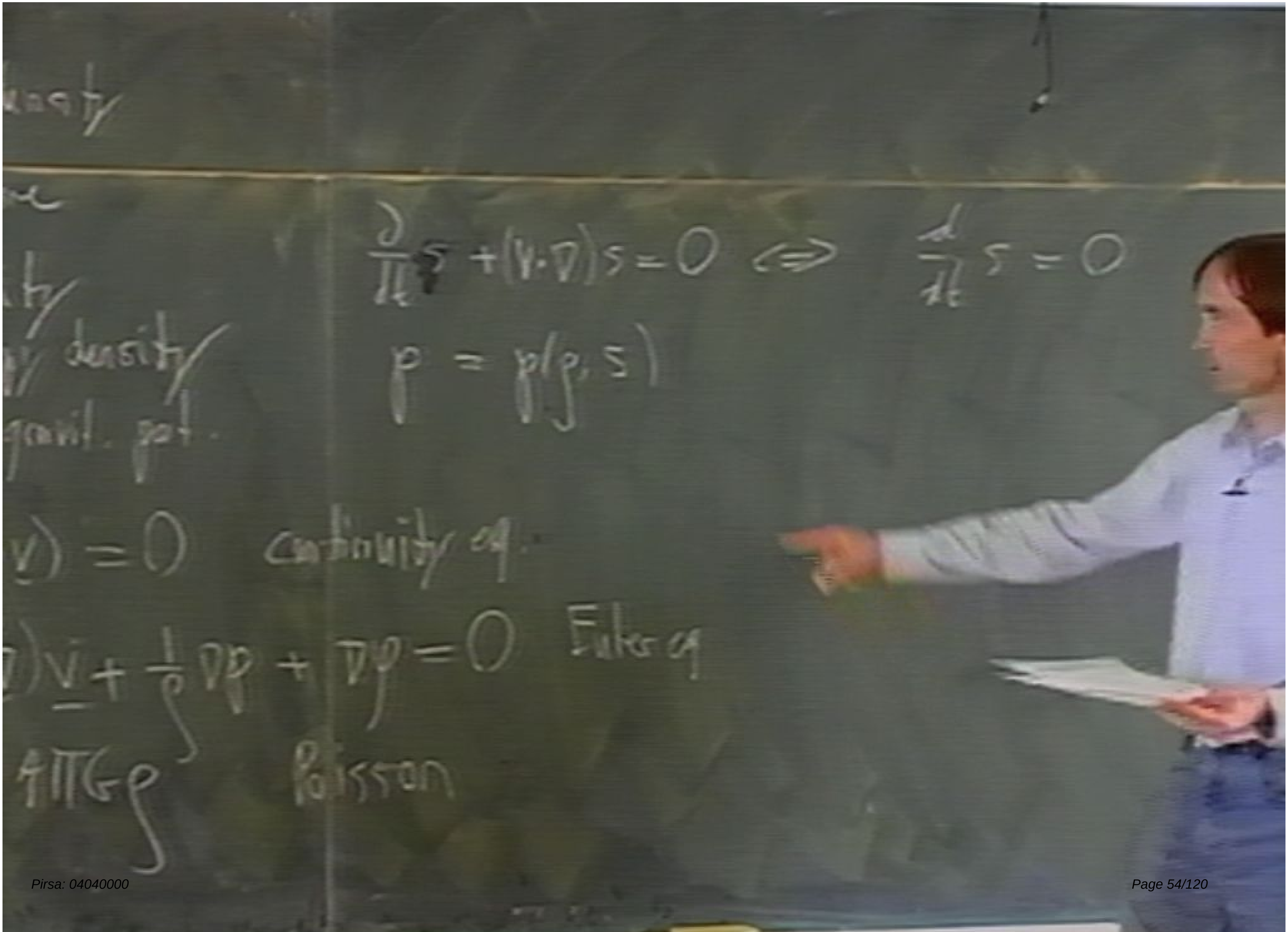
$$\frac{\partial}{\partial t} s + (v \cdot \nabla) s = 0$$

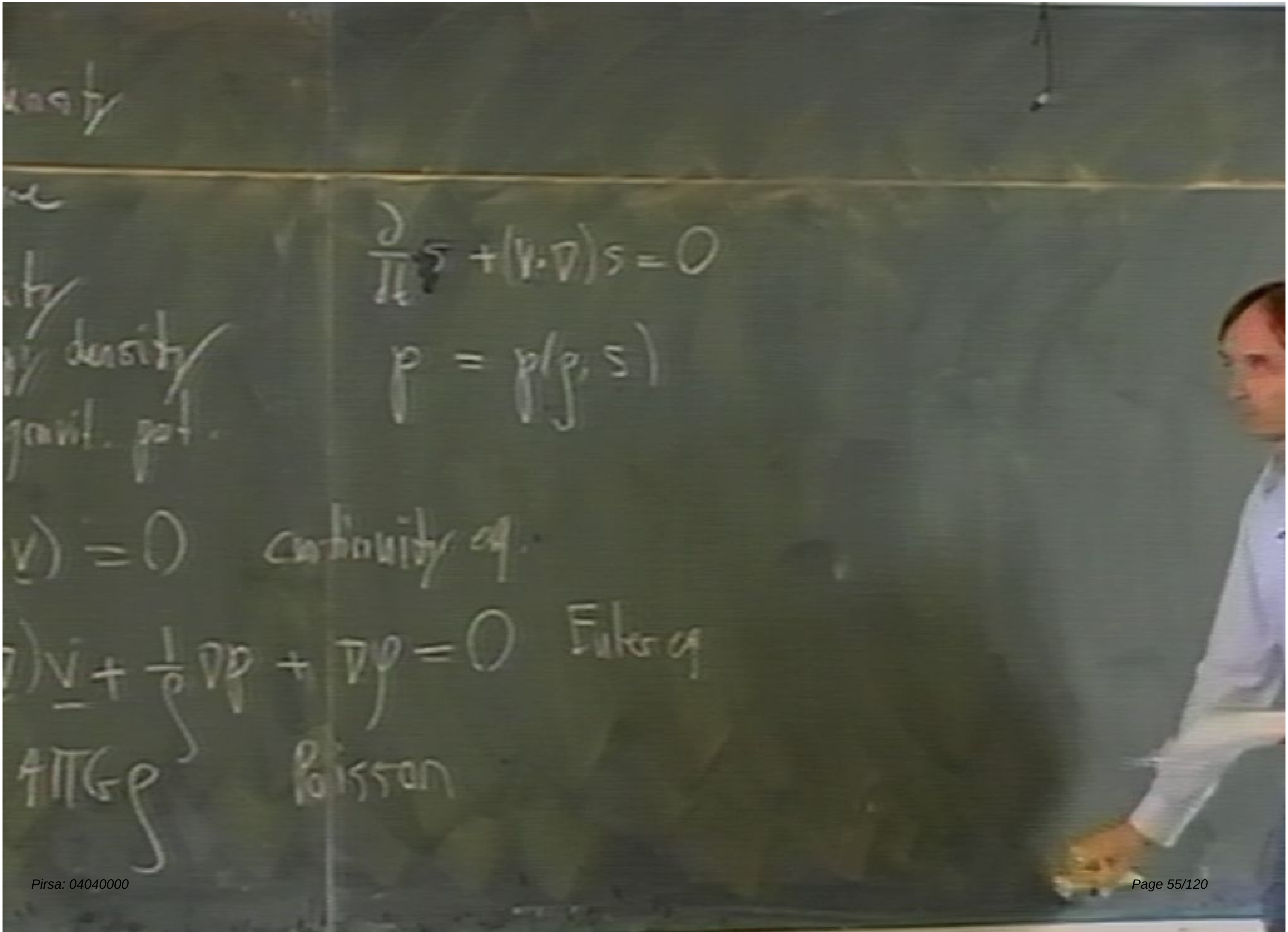
$$p = p(\rho, s)$$

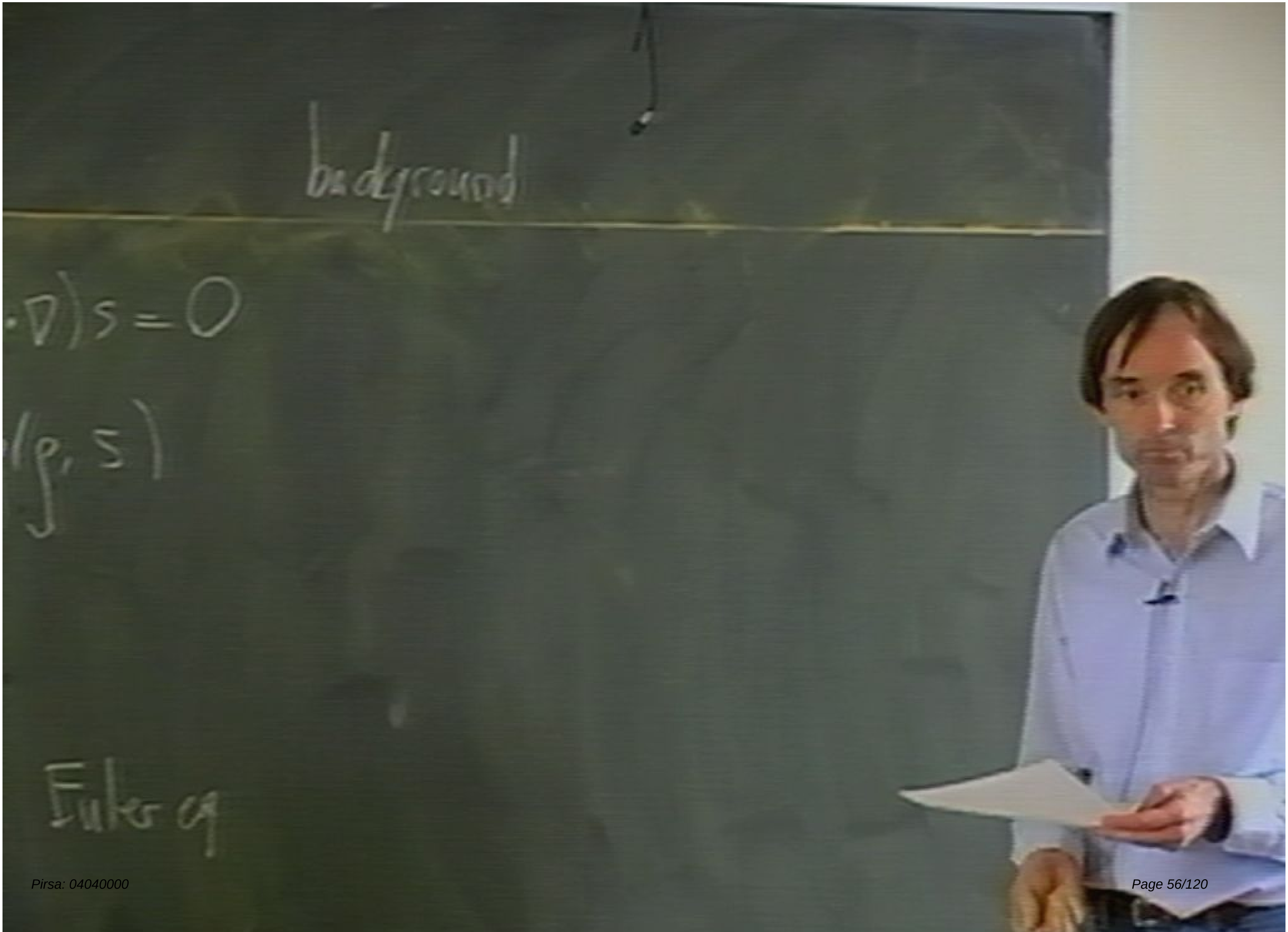
$$\dot{\rho} + \nabla(\rho \underline{v}) = 0$$ continuity eq.

$$\dot{\underline{v}} + (\underline{v} \cdot \underline{\nabla}) \underline{v} + \frac{1}{\rho} \nabla p + \nabla \varphi = 0$$ Euler eq

$$\nabla^2 \varphi = 4\pi G \rho$$ Poisson


density
gravit. pot.
$\frac{\partial}{\partial t} s + (v \cdot \nabla) s = 0 \iff \frac{d}{dt} s = 0$
$p = p(\rho, s)$
$v) = 0$ continuity eq.
$\nabla) v + \frac{1}{\rho} \nabla p + \nabla \varphi = 0$ Euler eq
$4\pi G \rho$ Poisson


density
density
gravit. pot.
$\frac{\partial}{\partial t} s + (v \cdot \nabla) s = 0$
$p = p(\rho, s)$
$v) = 0$ continuity eq.
$)\underline{v} + \frac{1}{\rho}\nabla p + \nabla \varphi = 0$ Euler eq
$4\pi G \rho$ Poisson


background
Euler eq

background: $\rho_0, p_0, \underline{v}=0, \psi_0, s_0$

inconsistent

$\cdot \nabla) s = 0$

(ρ, s)

Euler eq

background: $\rho_0,\ p_0,\ v=0,\ \psi_0,\ s_0$

$\cdot\nabla)s=0$

inconsistent

(ρ, s)

perturbations:

Euler eq

background : $\rho_0, p_0, \underline{v} = 0, \varphi_0, s_0$

$(\underline{v} \cdot \nabla) s = 0$

inconsistent

$p(\rho, s)$

perturbations :

$$\rho = \rho_0 + \delta\rho(\underline{x}, t)$$
$$p = p_0 + \delta p(\underline{x}, t)$$
$$\underline{v} = \delta \underline{v}(\underline{x}, t)$$
$$\varphi = \varphi_0 + \delta\varphi(\underline{x}, t)$$
$$s = s_0 + \delta s(\underline{x}, t)$$

insert into EOM

Euler eq

linearize

energy density

p pressure

$\underline{v}$ velocity

s entropy density

φ Newt. gravit. pot.

$$\dot{\rho} + \nabla(\rho\underline{v}) = 0$$ continuity eq.

$$\dot{\underline{v}} + (\underline{v}\cdot\underline{\nabla})\underline{v} + \frac{1}{\rho}\nabla p + \nabla\varphi = 0$$ Euler

$$\nabla^2\varphi = 4\pi G\rho$$ Poisson

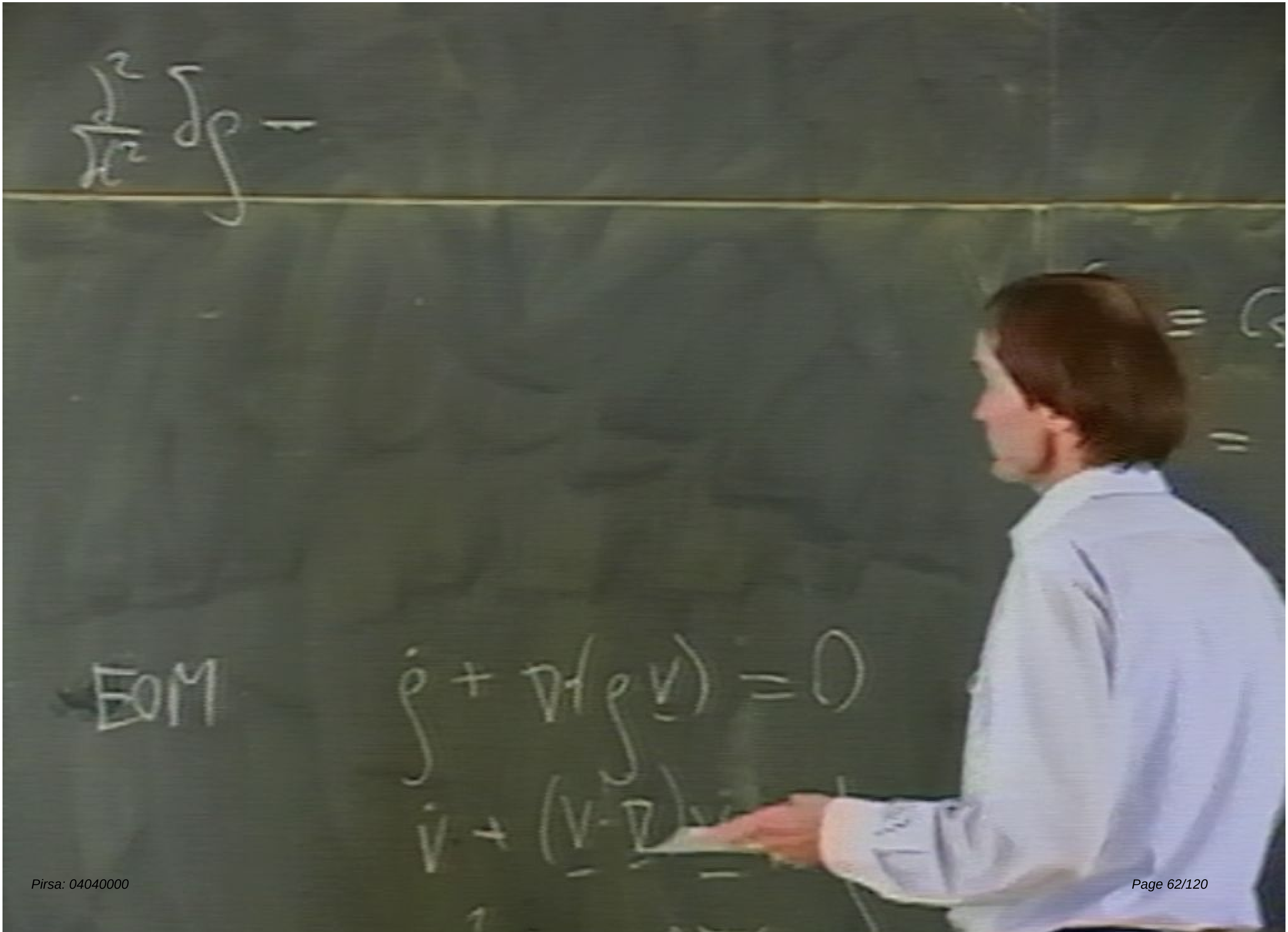
$$\frac{\partial}{\partial t}s + (v\cdot\nabla)s = 0$$

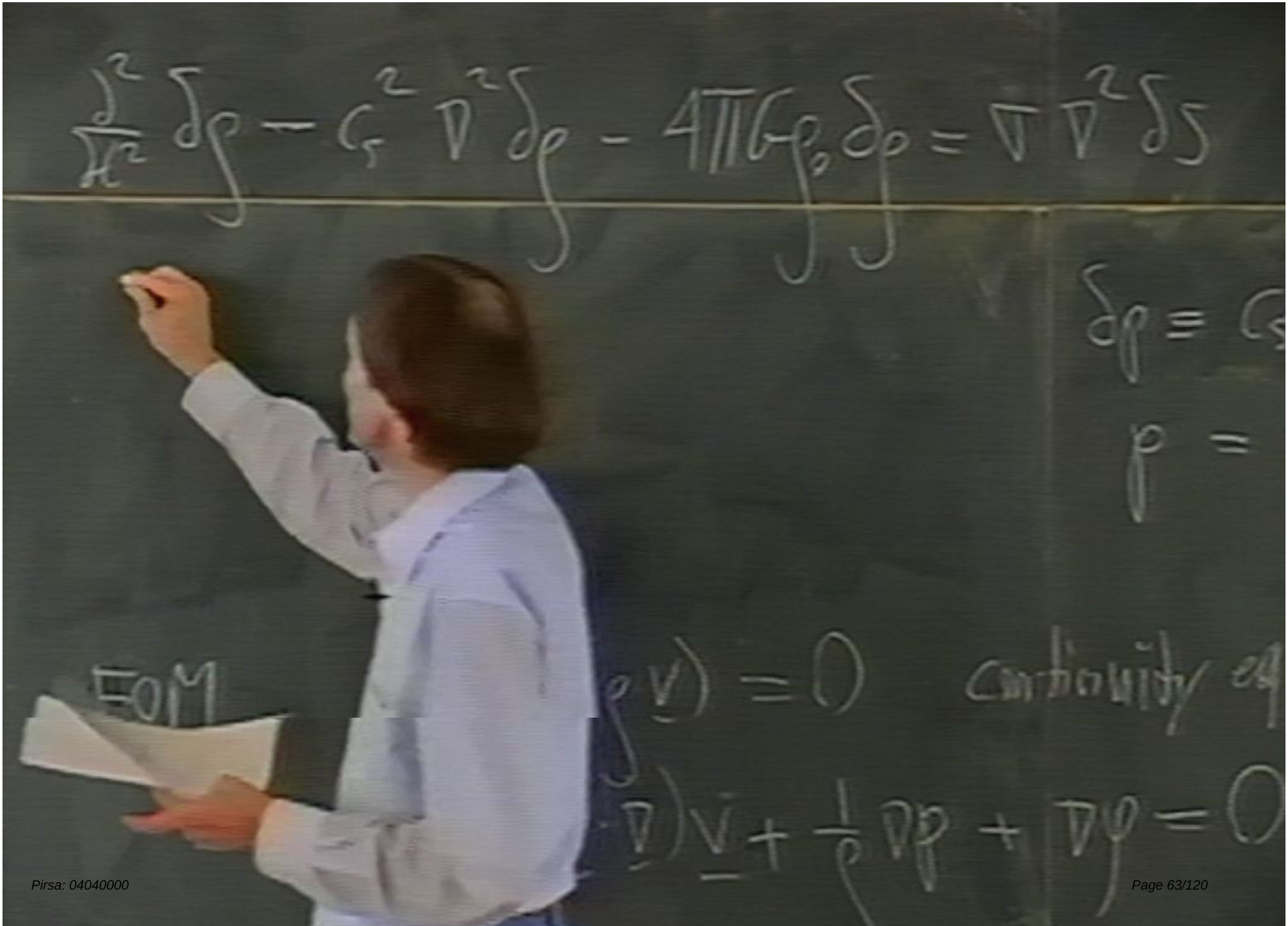
$$p = p(\rho, s)$$

$$\frac{\partial^2}{\partial t^2}\delta\rho \cdots$$

EOM $\dot{\rho} + \nabla\cdot(\rho\underline{v}) = 0$ continui

$\dot{\underline{v}} + (\underline{v}\cdot\underline{\nabla})\underline{v} + \frac{1}{\rho}\nabla p + \nabla\varphi$


$\frac{\partial^2}{\partial t^2}\delta\rho -$
EOM
$\dot{\rho} + \nabla\cdot(\rho\underline{v}) = 0$
$\dot{\underline{v}} + (\underline{v}\cdot\underline{\nabla})\underline{v}$


$\frac{\partial^2}{\partial t^2}\delta\rho - c_s^2\nabla^2\delta\rho - 4\pi G\rho_0\delta\rho = \nabla\nabla^2\delta s$
$\delta p = c_s$
$p =$
EOM
$\rho\underline{v}) = 0$ continuity eq
$\nabla)\underline{v} + \frac{1}{\rho}\nabla p + \nabla\varphi = 0$

$$\boxed{\frac{\partial^2}{\partial t^2}\delta\rho - c_s^2\nabla^2\delta\rho - 4\pi G\rho_0\delta\rho = \nabla\nabla^2\delta S}$$

$$\boxed{\frac{\partial}{\partial t}\delta S = 0}$$

$\delta p = c_s$

$p =$

EOM $\quad \dot{\rho} + \nabla(\rho\underline{v}) = 0 \quad$ continuity eq

$\dot{\underline{v}} + (\underline{v}\cdot\underline{\nabla})\underline{v} + \frac{1}{\rho}\nabla p + \nabla\varphi = 0$

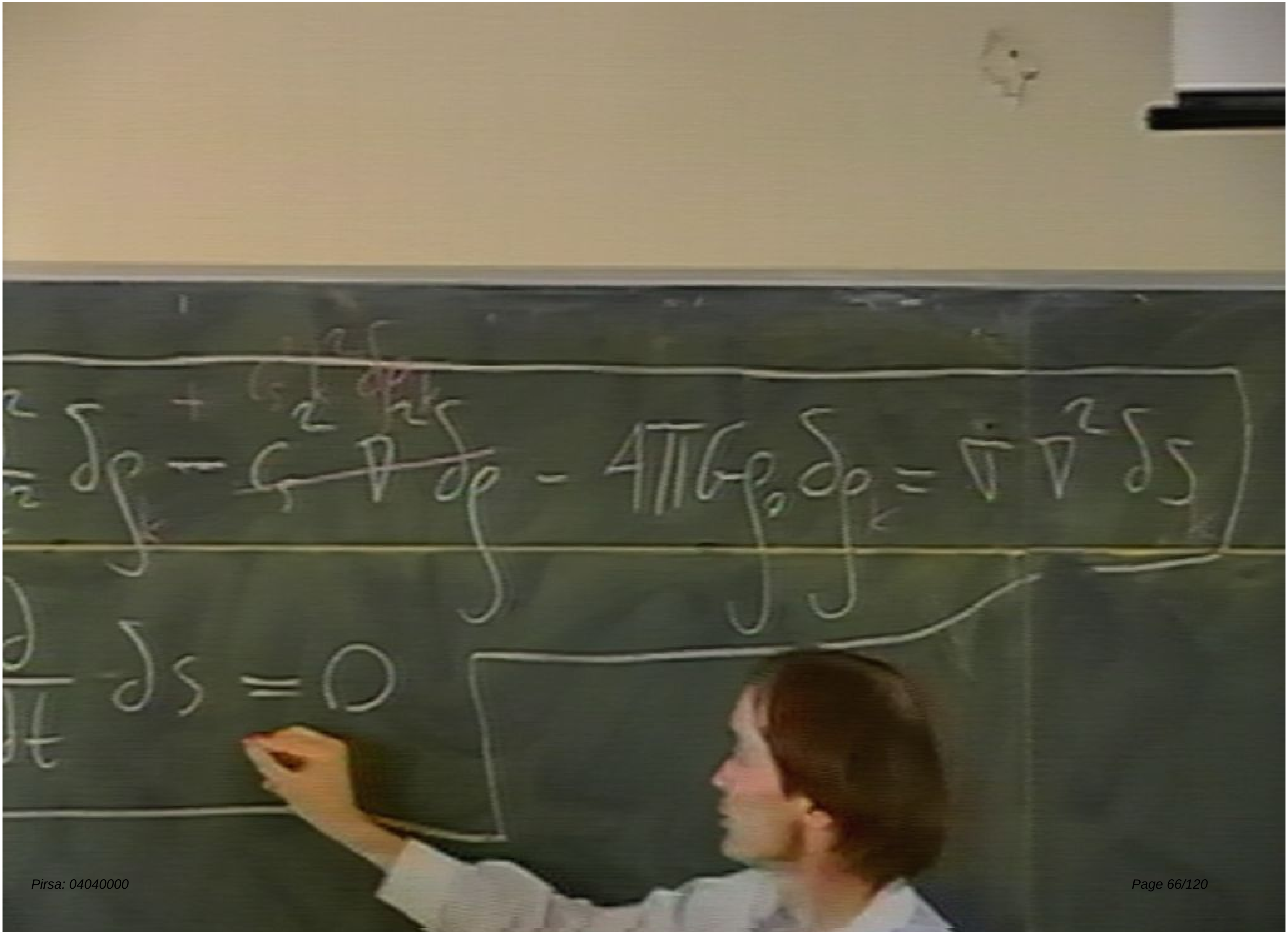
$$\boxed{\frac{\partial^2}{\partial t^2}\delta\rho - c_s^2\nabla^2\delta\rho - 4\pi G\rho_0\delta\rho = \dot{v}\,\nabla^2\delta S}$$

$$\boxed{\frac{\partial}{\partial t}\delta S = 0}$$

$$\frac{\delta\rho}{\rho} \equiv$$

EOM $\quad \dot{\rho} + \nabla\cdot(\rho\,\underline{v}) = 0$ continuity eq

$$\dot{\underline{v}} + (\underline{v}\cdot\underline{\nabla})\underline{v} + \frac{1}{\rho}\nabla p + \nabla\varphi = 0$$


$\delta\rho - c_s^2 \nabla^2 \delta\rho - 4\pi G \rho_0 \delta\rho_k = \dot{\nabla} \nabla^2 \delta S_k$
$\frac{\partial}{\partial t} \delta S = 0$

$$\frac{\partial^2}{\partial t^2}\delta\rho_k + c_s^2 k^2 \delta\rho_k \cancel{- c_s^2 \nabla^2 \delta\rho} - 4\pi G \rho_0 \delta\rho_k = \dot{r}\,\nabla^2 \delta S_k$$

$$\frac{\partial}{\partial t}\delta S_k = 0$$

classification: adiabatic pert. $\delta S = 0$

entropy pert. $\delta S \neq 0$

observations.

$$\boxed{\frac{\partial^2}{\partial t^2}\delta\rho_k + c_s^2 k^2 \delta\rho_k \cancel{- c_s^2 \nabla^2 \delta\rho} - 4\pi G \rho_0 \delta\rho_k = \dot{\nu}\, \nabla^2 \delta s_k}$$

$$\boxed{\frac{\partial}{\partial t}\delta s_k = 0}$$

classification: adiabatic pert. $\delta s = 0$

entropy pert. $\delta s \neq 0$

observations: entropy perts. do not grow

adiabatic pert. interesting time dep

$$\boxed{\begin{aligned} &\frac{\partial^2}{\partial t^2}\delta\rho_k \cancel{- c_s^2 \nabla^2 \delta\rho_k} + c_s^2 k^2 \delta\rho_k - 4\pi G \rho_0 \delta\rho_k = \dot{\tau}\, \nabla^2 \delta S_k \\ &\frac{\partial}{\partial t}\delta S_k = 0 \end{aligned}}$$

classification: adiabatic pert. $\delta S = 0$

entropy pert. $\delta S \neq 0$

observations: entropy perts. do not grow

adiabatic pert. interesting time dep

$$\boxed{\frac{\partial}{\partial t}\delta s_k = 0}$$

classification: adiabatic pert. $\delta s = 0$

entropy pert. $\delta s \neq 0$

observations. entropy pert. do not grow

adiabatic pert. interesting time dep

entropy pert. source a

$=0$

...on: adiabatic pert. $\delta s = 0$

entropy pert. $\delta s \neq 0$

...ns. entropy perts. do not grow

adiabatic pert. interesting time dep.

entropy pert. source adiabatic mode

$= 0$

adiabatic pert. $\delta s = 0$

on: entropy pert. $\delta s \neq 0$

ns. entropy perts. do not grow

adiabatic pert. interesting time dep.

entropy pert. source adiabatic mode

$$\boxed{\begin{aligned} &\frac{\partial^2}{\partial t^2}\delta\rho_k \cancel{- c_s^2 \nabla^2 \delta\rho} + c_s^2 k^2 \delta\rho_k - 4\pi G \rho_0 \delta\rho_k = \dot{\sigma}\, \nabla^2 \delta S_k \\ &\frac{\partial}{\partial t}\delta S_k = 0 \end{aligned}}$$

classification: adiabatic pert. $\delta S = 0$

entropy pert. $\delta S \neq$

observations: entropy perts. do not grow

adiabatic pert. interesting

$$\boxed{\frac{\partial^2}{\partial t^2}\delta\rho_k + c_s^2 \nabla^2 \delta\rho - 4\pi G \rho_0 \delta\rho_k = \dot{\nabla}\nabla^2 \delta S_k}$$

$$\frac{\partial}{\partial t}\delta S_k = 0$$

classification: adiabatic pert. $\delta S = 0$

entropy pert. $\delta S \neq 0$

observations. entropy pert. do not grow

adiabatic pert. interesting time dep.

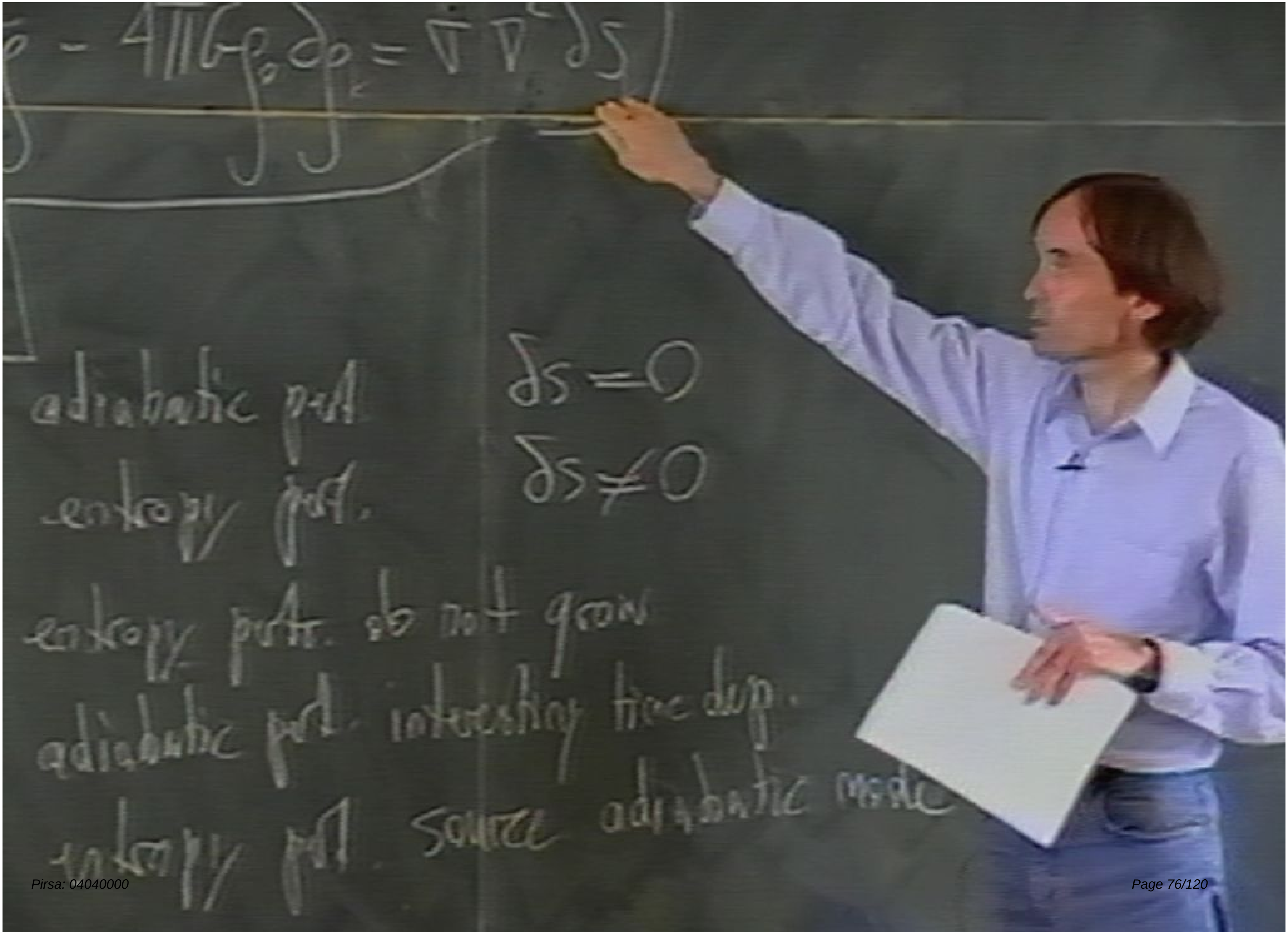
$$\boxed{\begin{aligned} &\frac{\partial^2}{\partial t^2}\delta\rho_k \cancel{- c_s^2 \nabla^2 \delta\rho_k} \;+ c_s^2 k^2 \delta\rho_k - 4\pi G \rho_0 \delta\rho_k = \dot{\sigma}\, \nabla^2 \delta S \\ &\frac{\partial}{\partial t}\delta S_k = 0 \end{aligned}}$$

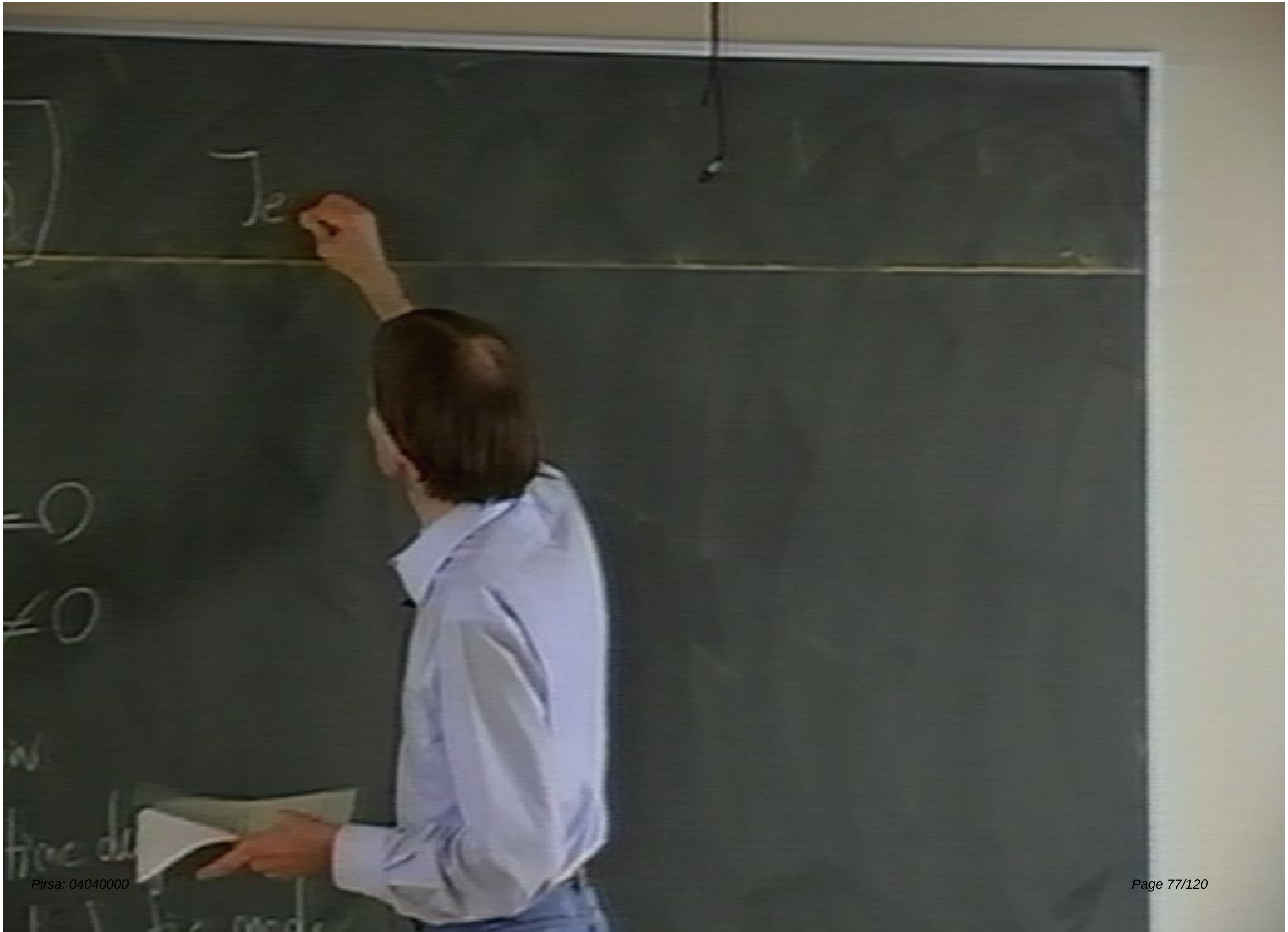
classification: adiabatic pert. $\delta S = 0$

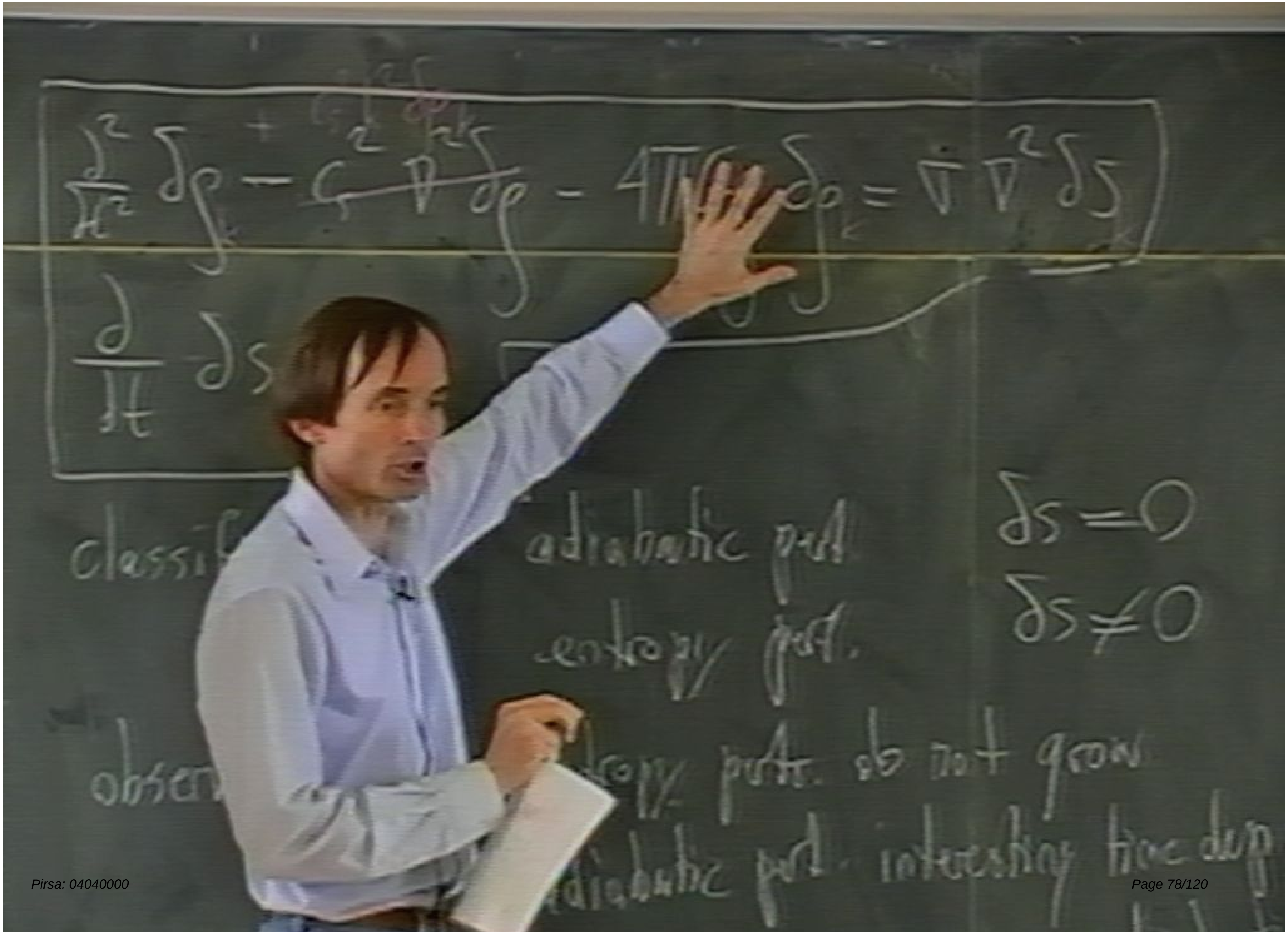
entropy pert. $\delta S \neq 0$

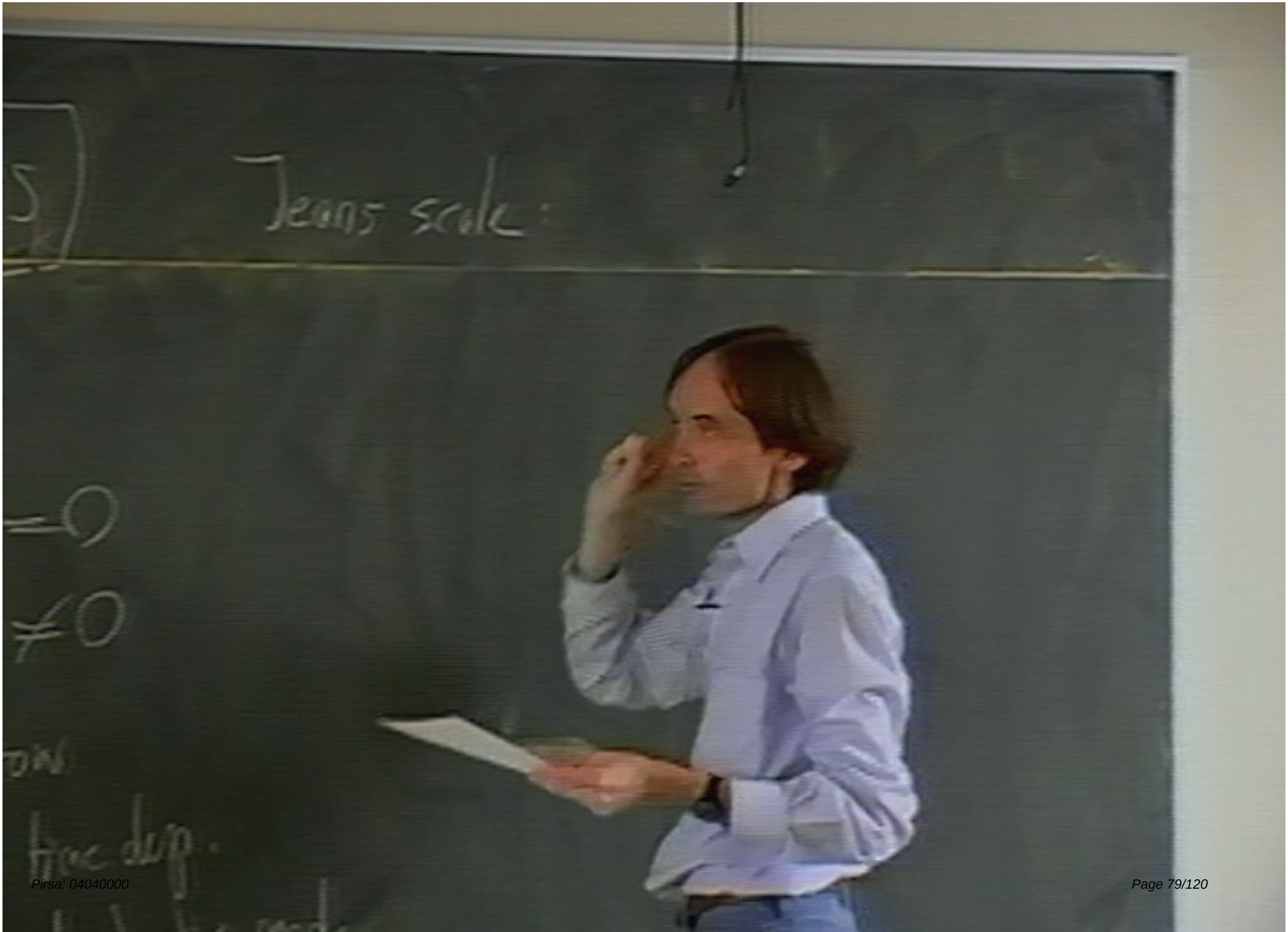
observations. entropy pert. do not grow

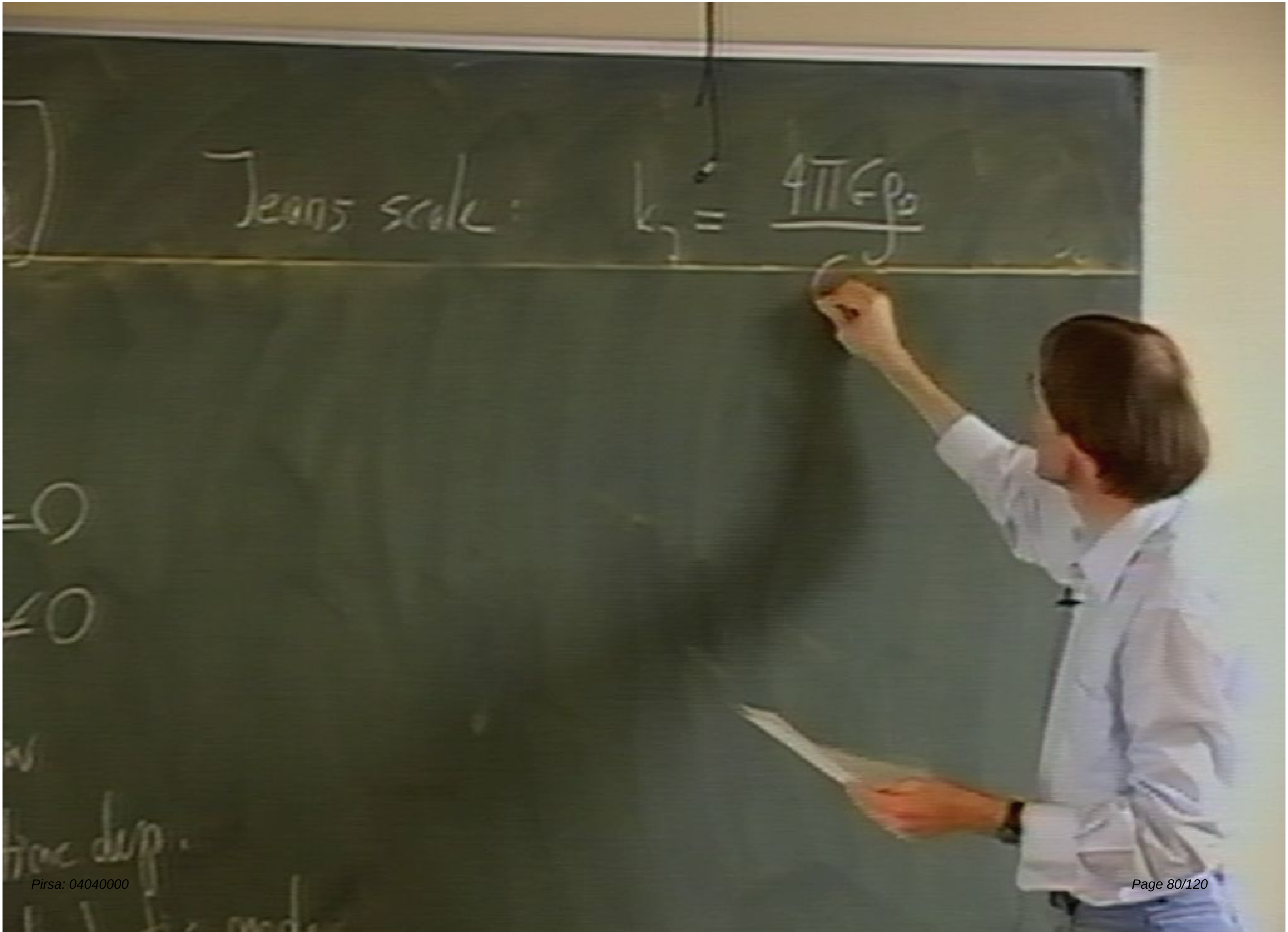
adiabatic pert. interesting time dep.


adiabatic pert.
δs = 0
entropy pert.
δs ≠ 0
entropy pert. do not grow
adiabatic pert. interesting time dep.
entropy pert. source adiabatic mode




adiabatic pert.
δs = 0
entropy pert.
δs ≠ 0
do not grow
adiabatic pert. interesting time dep


Jeans scale:


Jeans scale:
$k_J = \frac{4\pi G \rho_0}{\;}$

Jeans scale: $k_J = \left(\frac{4\pi G \rho_0}{c_s^2}\right)^{1/2}$

$k < k_J$ exponential growth of $\delta\rho$

$k > k_J$

$= 0$

≤ 0

w

tive dep.

Jeans scale: $k_J = \left(\frac{4\pi G\rho_0}{c_s^2}\right)^{1/2}$

$k < k_J$ exponential growth of $\delta\rho$

$k > k_J$ oscillations

$\omega_k \sim c_s^2 k$

$= 0$

≤ 0

w

tive dep.

$k < k_J$ exponential growth of $\delta\rho$

$k > k_J$ oscillations

$$\omega_k \sim c_s k$$

k_J depends on type of matter

radiation

$\ldots O$

$\ldots \leq O$

$\ldots$ time dep.

adiabatic mode

$k < k_J$ exponential growth of $\delta\rho$

$k > k_J$ oscillations

$$\omega_k \sim c_s\, k$$

k_J depends on type of matter

radiation $k_J \simeq H$

...dep.

...diabatic mode

$k < k_J$ exponential growth of $\delta\rho$

$k > k_J$ oscillations

$\omega_k \sim c_s k$

k_J depends on type of matter

radiation $k_J \simeq H$

Cold dark matter

$k < k_J$ exponential growth of $\delta\rho$

$k > k_J$ oscillations

$\omega_k \sim c_s k$

k_J depends on type of matter

radiation $k_J \simeq H$

Cold dark matter : $k_J = \infty$

...ical Perturbations

...heory

1.1 Preview

1.2 Characterizing Fluctuations

1.3 Newt. Pert. about M^4

1.4 Newt. Pert. " FRW

Theory of Cosmological Perturbatio

Background: $\rho_0(t)$, etc

1.1 Previ

1.2 Cl

1.3 Newt

→ 1.4 Newt

Theory of Cosmological Perturbatio…

Background: $\rho_0(t)$, etc

$\underline{V}_0 = H(t)\,\underline{x}$

1.1 Previ…

1.2 Char…

1.3 Newt…

→ 1.4 Newt…

Theory of Cosmological Perturbatio[...]

Background: $\rho_0(t)$, etc

$\underline{v}_0 = H(t)\,\underline{x}$

Pert. $\rho(\underline{x},t) = \rho_0(t)\left[1+\delta_\varepsilon(\underline{x},t)\right]$

$v(\underline{x},t) = \underline{v}_0 + \delta$

1.1 Previ[...]

1.2 Char[...]

1.3 Newt[...]

1.4 Newt[...]

Theory of Cosmological Perturbatio[...]

Background: $\rho_0(t)$, etc

$$\underline{v}_0 = H(t)\,\underline{x}$$

Pert. $\rho(\underline{x},t) = \rho_0(t)\left[1+\delta_\rho(\underline{x},t)\right]$

$$v(\underline{x},t) = \underline{v}_0 + \delta\underline{v}(\underline{x},t)$$

1.1 Previ[...]

1.2 Char[...]

1.3 Newt[...]

1.4 Newt[...]

Background: $\rho_0(t)$, etc

$$\underline{v}_0 = H(t)\,\underline{x}$$

1.1 Previo

1.2 Chara

1.3 Newt.

1.4 Newt.

Pert. $\rho(\underline{x},t) = \rho_0(t)\left[1+\delta_\rho(\underline{x},t)\right]$

$$\underline{v}(\underline{x},t) = \underline{v}_0 + \delta\underline{v}(\underline{x},t)$$

$$p(\underline{x},t) = p_0(t) + c_s^2\,\delta\rho(\underline{x},t) + \sigma\,\delta S(\underline{x},t$$

kground: $\rho_0(t)$, etc

$\underline{v}_0 = H(t)\,\underline{x}$

rt. $\rho(\underline{x},t) = \rho_0(t)\left[1+\delta_\varepsilon(\underline{x},t)\right]$

$v(\underline{x},t) = \underline{v}_0 + \delta\underline{v}(\underline{x},t)$

$p(\underline{x},t) = p_0(t) + c_s^2\,\delta\rho(\underline{x},t) + \sigma\,\delta s(\underline{x},t)$

1.1 Preview

1.2 Characteriz

1.3 Newt. Pert. a

1.4 Newt. Pert.

...kground: $\rho_0(t)$, etc

$$\underline{v}_0 = H(t)\,\underline{x}$$

...t. $\rho(\underline{x},t) = \rho_0(t)\left[1+\delta_\varepsilon(\underline{x},t)\right]$

$$\underline{v}(\underline{x},t) = \underline{v}_0 + \delta\underline{v}(\underline{x},t)$$

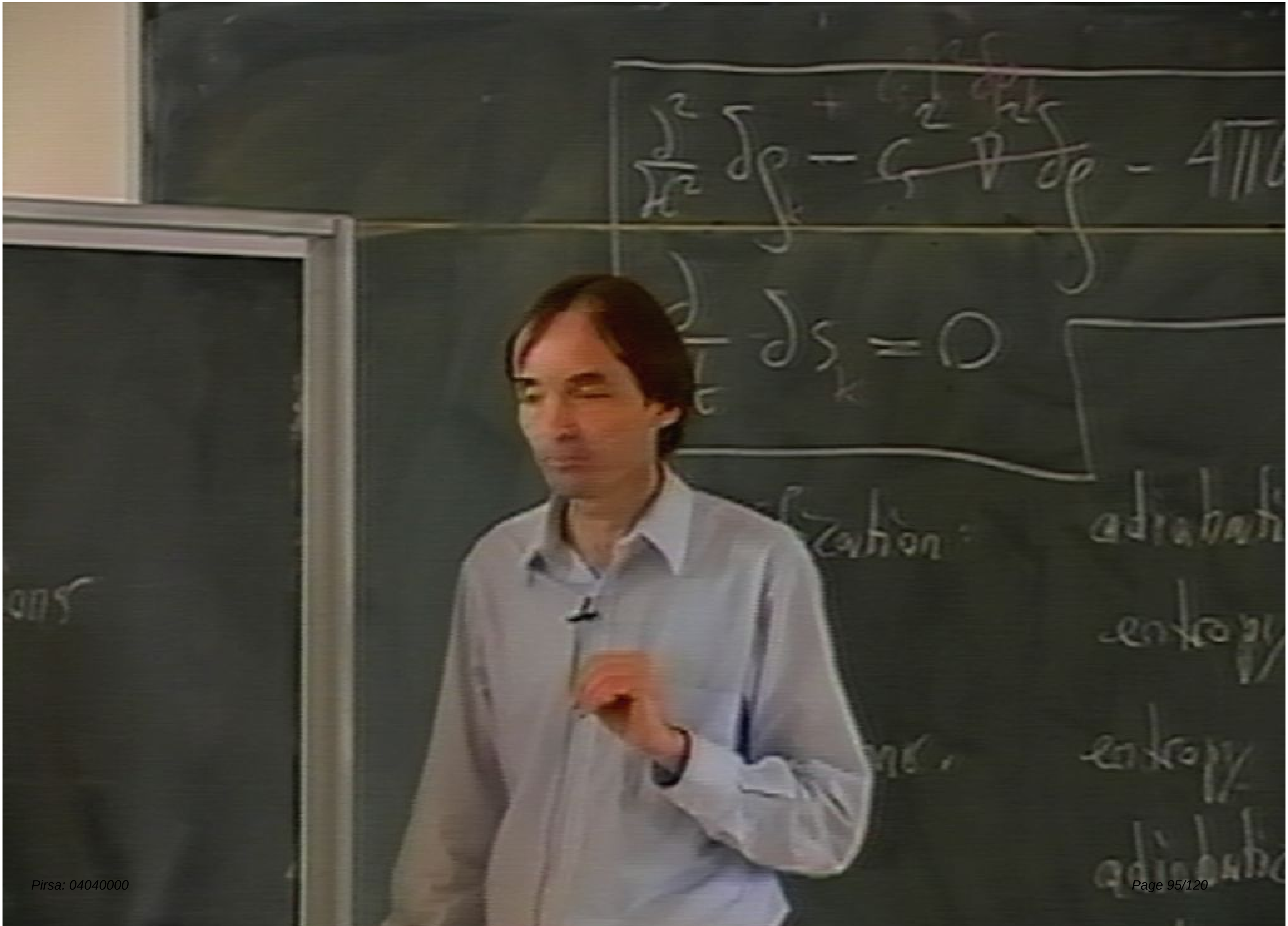
$$p(\underline{x},t) = p_0(t) + c_s^2\,\delta\rho(\underline{x},t) + \sigma\,\delta s(\underline{x},t)$$

1.1 Preview

1.2 Characteri...

1.3 Newt. Pert. ...

1.4 Newt. Pert. ...


entropy

rturbations

1.1 Preview

1.2 Characterizing Fluctuations

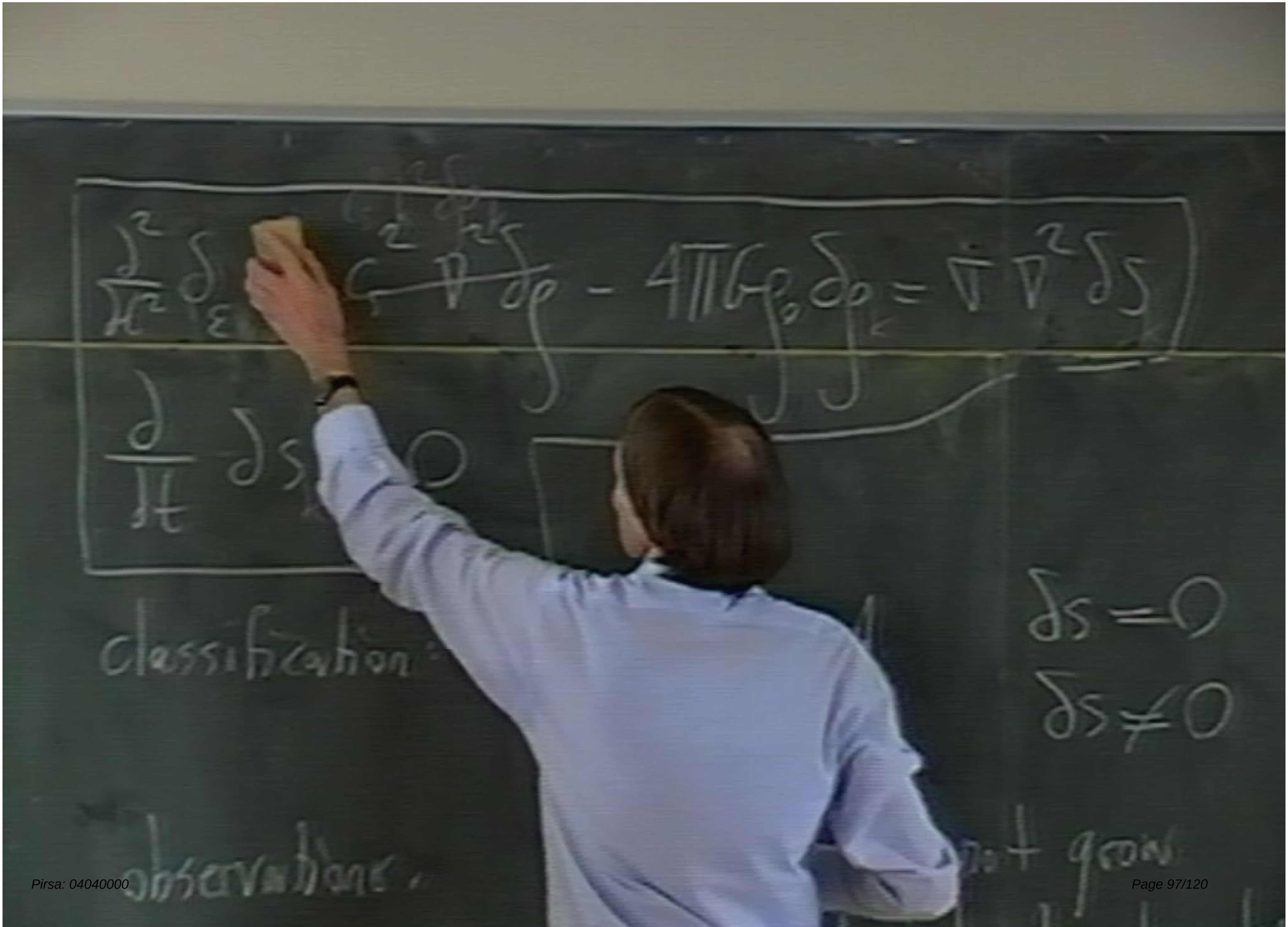
1.3 Newt. P.T. about M^4

1.4 Newt. P.T. " FRW

New step: Use $\underline{q}$ (comoving)

$+ \sigma\, \delta s(\underline{x}, t)$

$\underline{x}(t) = a(t)\, \underline{q}(t)$


classification:
δs = 0
δs ≠ 0
observations.

$$\ddot{\delta}_\varepsilon + 2\mathcal{H}\dot{\delta}_\varepsilon - \frac{c_s^2}{a^2}\nabla^2\delta_\varepsilon - 4\pi G\rho_0\delta\varepsilon = -$$

$$\frac{\partial}{\partial t}\delta s_k = 0$$

assification: adiabatic pert. $\delta s = 0$

entropy pert. $\delta s \neq 0$

bservations: entropy perts. do not

$$\ddot{\delta}_\varepsilon + 2H\dot{\delta}_\varepsilon - \frac{c_s^2}{a^2}\nabla^2\delta_\varepsilon - 4\pi G\rho_0\delta\varepsilon = \frac{\sigma}{\rho a^2}\nabla_a^2\delta s$$

$$\frac{\partial}{\partial t}\delta s = 0$$

$k < k_J$

classification: adiabatic pert. $\delta s = 0$ $k > k_J$

entropy pert. $\delta s \neq 0$

observations: entropy pert. do not grow

adiabatic pert. interesting time dep.

$$\ddot{\delta}_\varepsilon + 2H\dot{\delta}_\varepsilon - \frac{c_s^2}{a^2}\nabla_q^2\delta_\varepsilon - 4\pi G\rho_0\delta_\varepsilon = \frac{\sigma}{\rho a^2}\nabla_q^2\delta_S$$

$$\frac{\partial}{\partial t}\delta_S = 0$$

classification: adiabatic pert. $\delta_S = 0$

entropy pert. $\delta_S \neq 0$

observations: entropy pert. do not grow

adiabatic pert. interesting time dep.

adiabatic mode

$k <$

$k >$

$$\boxed{\ddot{\delta}_\varepsilon + 2H\dot{\delta}_\varepsilon - \frac{c_s^2}{a^2}\nabla_q^2\delta_\varepsilon - 4\pi G\rho_0\delta_\varepsilon = \frac{\sigma}{\rho a^2}\nabla_q^2\delta_S}$$

$$\boxed{\frac{\partial}{\partial t}\delta S_k = 0}$$

$k <$

$k >$

classification: adiabatic pert. $\delta S = 0$

entropy pert. $\delta S \neq 0$

observations: entropy pert. do not grow

adiabatic pert. interesting time dep.

adiabatic mode

$$\boxed{\ddot{\delta}_\varepsilon + 2H\dot{\delta}_\varepsilon - \frac{c_s^2}{a^2}\nabla_q^2\delta_\varepsilon - 4\pi G\rho_0\delta_\varepsilon = \frac{\tau}{\rho a^2}\nabla_q^2\delta_S}$$

$$\frac{\partial}{\partial t}\delta_{S_k} = 0$$

$k < k_J$

classification: adiabatic pert. $\delta_S = 0$ $k > k_J$

entropy pert. $\delta_S \neq 0$

observations: entropy pert. do not grow

adiabatic pert. interesting time dep.

$$- 4\pi G \rho_0 \delta\varepsilon = \frac{\sigma}{\rho a^2} \nabla_q^2 \delta_S$$

$$k_J = \left(\frac{4\pi G \rho_0}{c_s^2}\right)^{1/2}$$

$k < k_J$ ~~exponential~~ power law growth of δ_ρ

$k > k_J$ damped oscillations

$\omega_k \sim c_s k$

k_J depends on type of matter

radiation $k_J \simeq H$

cold dark matter : $k_J = \infty$

A. $\delta S = 0$

t. $\delta S \neq 0$

do not grow

I. interesting time dep.

II. source adiabatic mode

$$\boxed{\ddot{\delta}_\varepsilon + 2H\dot{\delta}_\varepsilon - \frac{c_s^2}{a^2}\nabla_q^2\delta_\varepsilon - 4\pi G\rho_0\delta_\varepsilon = \frac{\sigma}{\rho a^2}\nabla_q^2\delta S}$$

$$\boxed{\frac{\partial}{\partial t}\delta S_k = 0}$$

$k < k_J$

$k > k_J$

classification: adiabatic pert. $\delta S = 0$

entropy pert. $\delta S \neq 0$

observations: entropy pert. do not grow

adiabatic pert. interesting time dep.

entropy pert. source adiabatic mode

$$\boxed{\frac{\partial}{\partial t}\delta s_k = 0}$$

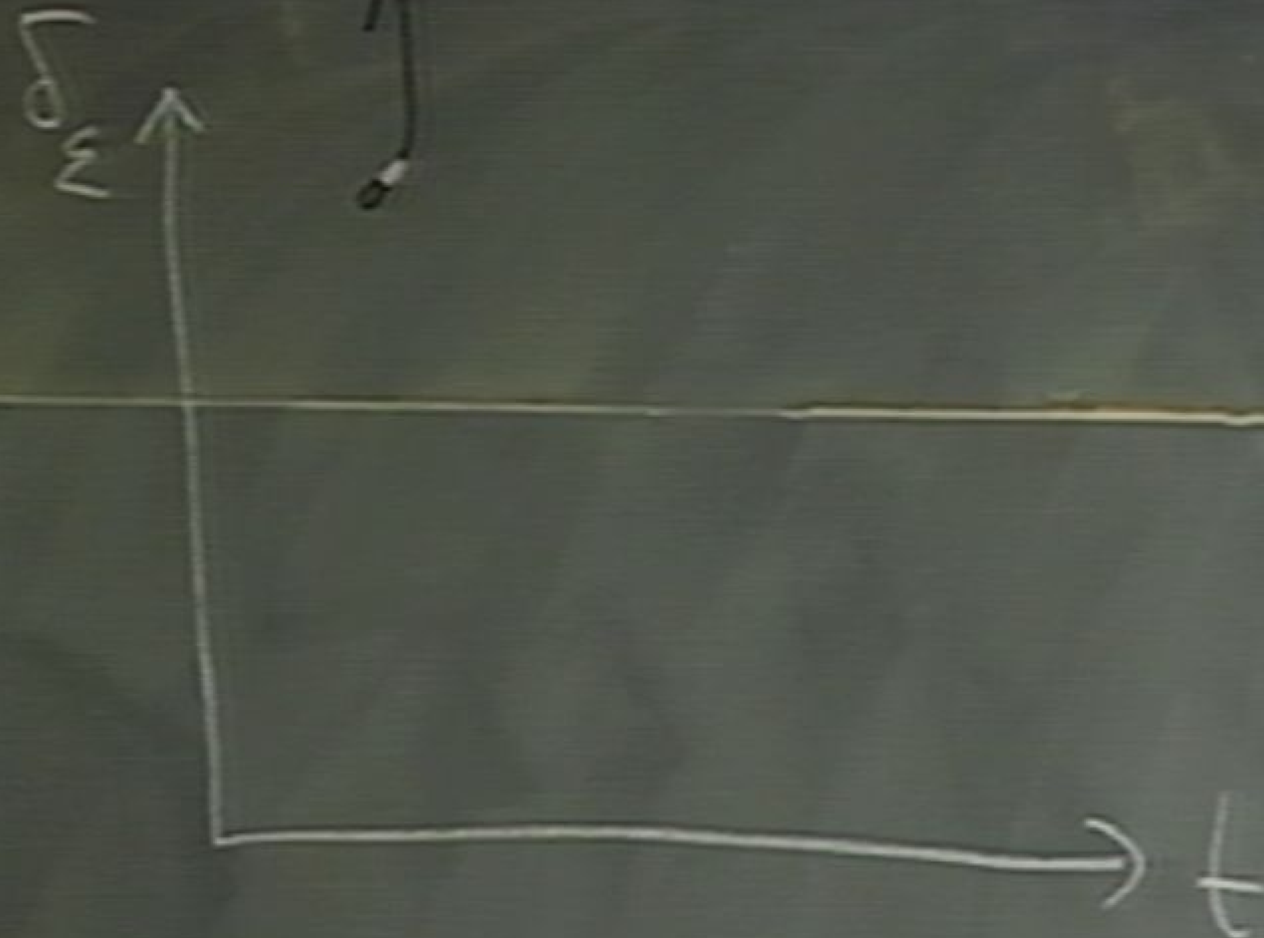
1995 dogma: $\Omega_{CDM} = \frac{\rho_{CDM}}{\rho_c} = 0.95$

$\Omega_{baryon} = 0.05$

2000 dogma: $\Omega_{CDM} \simeq 0.25$

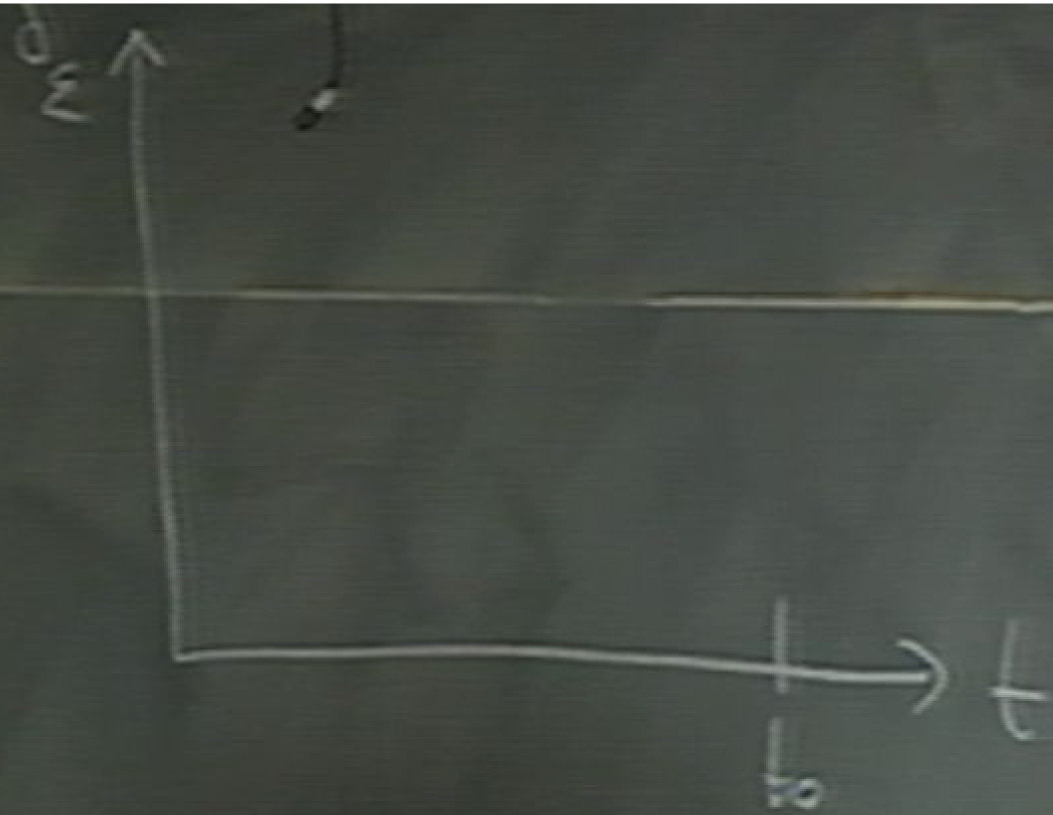
$\Omega_{baryon} \simeq 0.05$

$\Omega_\Lambda \sim 0.7$



(A) $\Omega_{CDM} = 0.95$, $\Omega_{baryon} = 0.05$

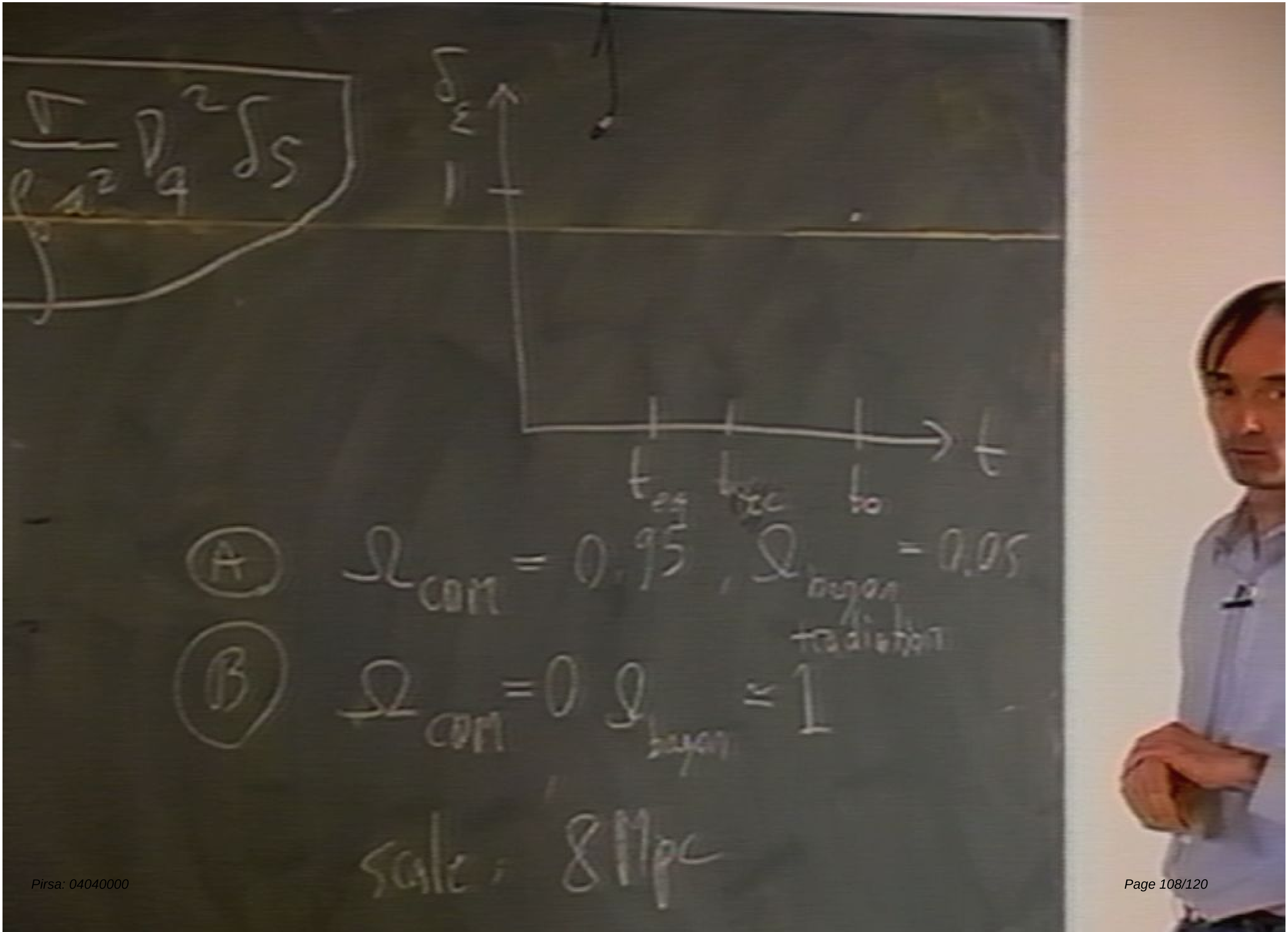
$$\frac{\quad}{a^2} D_q^2 \delta_S$$



(A) $\Omega_{CDM} = 0.95$, $\Omega_{\substack{\text{baryon} \\ \text{radiation}}} = 0.05$

(B) $\Omega_{CDM} = 0$ $\Omega_{baryon} \simeq 1$

scale: 8 Mpc


δε
1
t
teq trec t0
A Ωcdm = 0.95, Ωbaryon radiation = 0.05
B Ωcdm = 0 Ωbaryon ≃ 1
scale: 8 Mpc

δ_ε
1
t_{eq} t_{rec} t_0
t
= 0.95, Ω_{baryon} = 0.05
radiation
= 0 Ω_{baryon} ≃ 1
8 Mpc

$$2H\dot{\delta}_\varepsilon - \frac{c_s^2}{a^2}\nabla_q^2\delta_\varepsilon - 4\pi G\rho_0\delta_\varepsilon = \frac{\nabla}{\rho_0 a^2}\nabla_q^2$$

∇_x^2

$= 0$

$c_s^2 = 0 \qquad \delta_\varepsilon \sim t^{2/3} \sim a$

dogma: $\Omega_{CDM} = \frac{\rho_{CDM}}{\rho_c} = 0.95$

$\Omega_{bayon} \simeq 0.05$

dogma: $\Omega_{CDM} \simeq 0.25$

$\simeq 0.05$

$$\boxed{\ddot{\delta}_\varepsilon + 2H\dot{\delta}_\varepsilon - \frac{c_s^2}{a^2}\nabla_q^2 \delta_\varepsilon - 4\pi G \rho_0 \delta_\varepsilon = \quad \frac{\partial}{\partial t}\delta s_k = 0}$$

∇_x^2

Result: (for $\delta s = 0$)

$k < k_J$: $\delta_\varepsilon \sim a(t) \sim t^{2/3}$

$k > k_J$: $\delta_\varepsilon \sim a^{-1/2}(t)$

$$\boxed{\ddot{\delta}_\epsilon + 2H\dot{\delta}_\epsilon - \frac{c_s^2}{\cancel{a^2}}\cancel{\nabla_q^2}\,\nabla_x^2\,\delta_\epsilon - 4\pi G\rho_0\delta_\epsilon =}$$

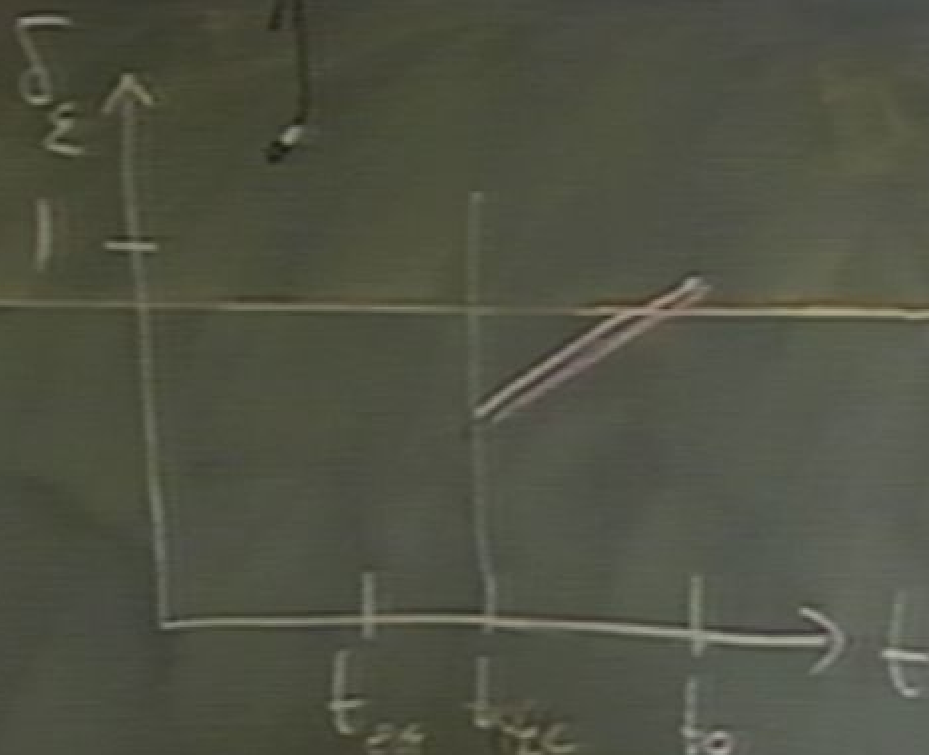
$$\boxed{\frac{\partial}{\partial t}\delta s_k = 0}$$

Results (for $\delta s = 0$)

$k < k_J$: $\delta_\epsilon \sim a(t) \sim t^{2/3}$

$k > k_J$: $\delta_\epsilon \sim a^{-1/2}(t)$ oscillatory

$$= \frac{\sigma}{\rho_0 a^2} \rho_q^2 \delta_S$$

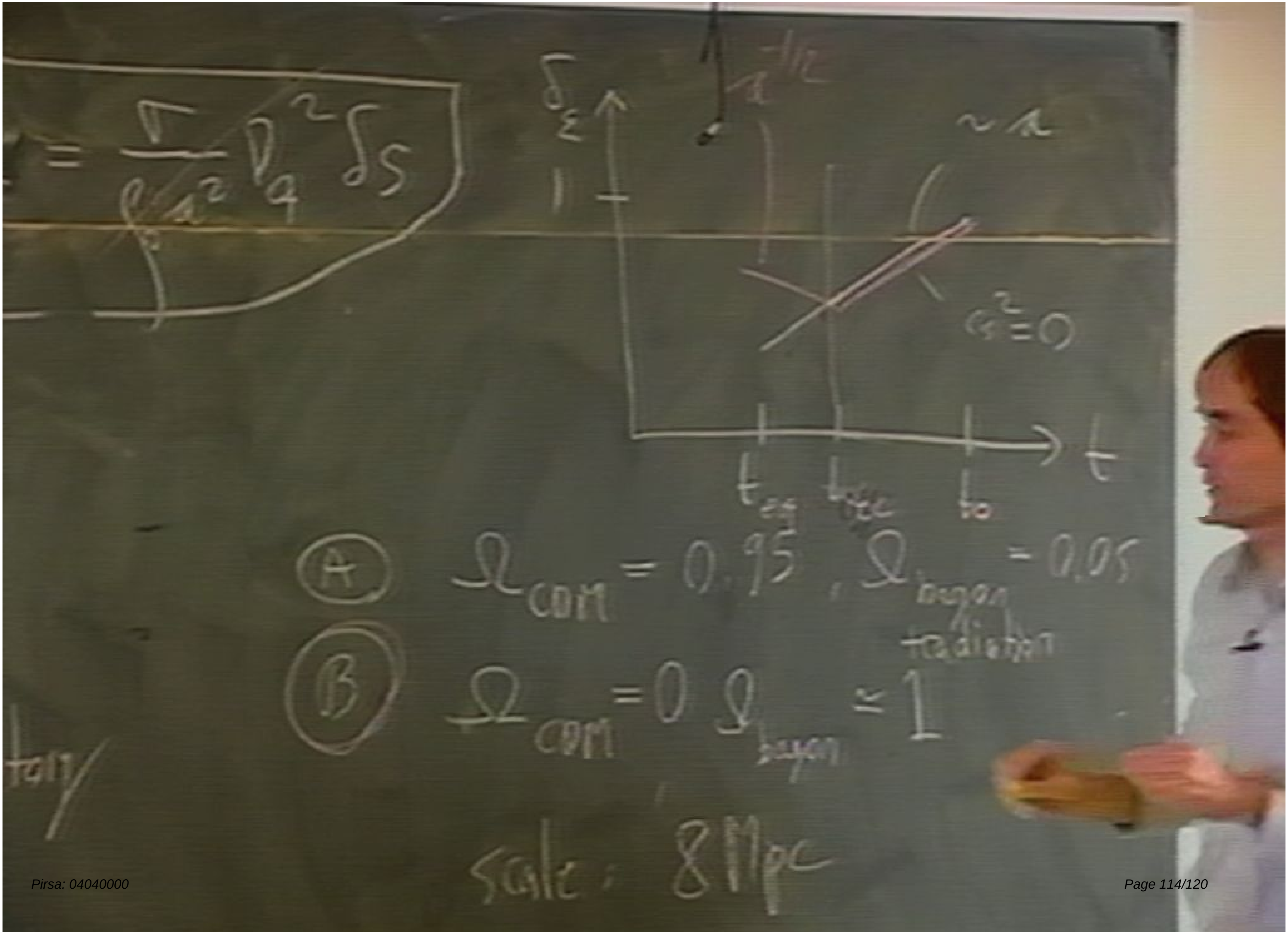


(A) $\Omega_{CDM} = 0.95$, $\Omega_{baryon} = 0.05$

(B) $\Omega_{CDM} = 0$ $\Omega_{\substack{baryon\\radiation}} \simeq 1$

tary

scale: 8 Mpc


δε
~ a
cs² = 0
t
teq tdec t0
(A) ΩCDM = 0.95 , Ωbaryon = 0.05
radiation
(B) ΩCDM = 0 Ωbaryon ≃ 1
scale : 8 Mpc

$= 0)$

$\delta_\varepsilon \sim a(t) \sim t^{2/3}$

$\delta_\xi \sim a^{-1/2}(t)$ oscillatory

$\delta_\varepsilon\Big|_B (t_{eq}) = \left(\frac{a(t_{\text{ex}})}{a(t_{eq})}\right)^{3/2}$

A Ω_{com}

B Ω_{com}

scale

$= 0)$

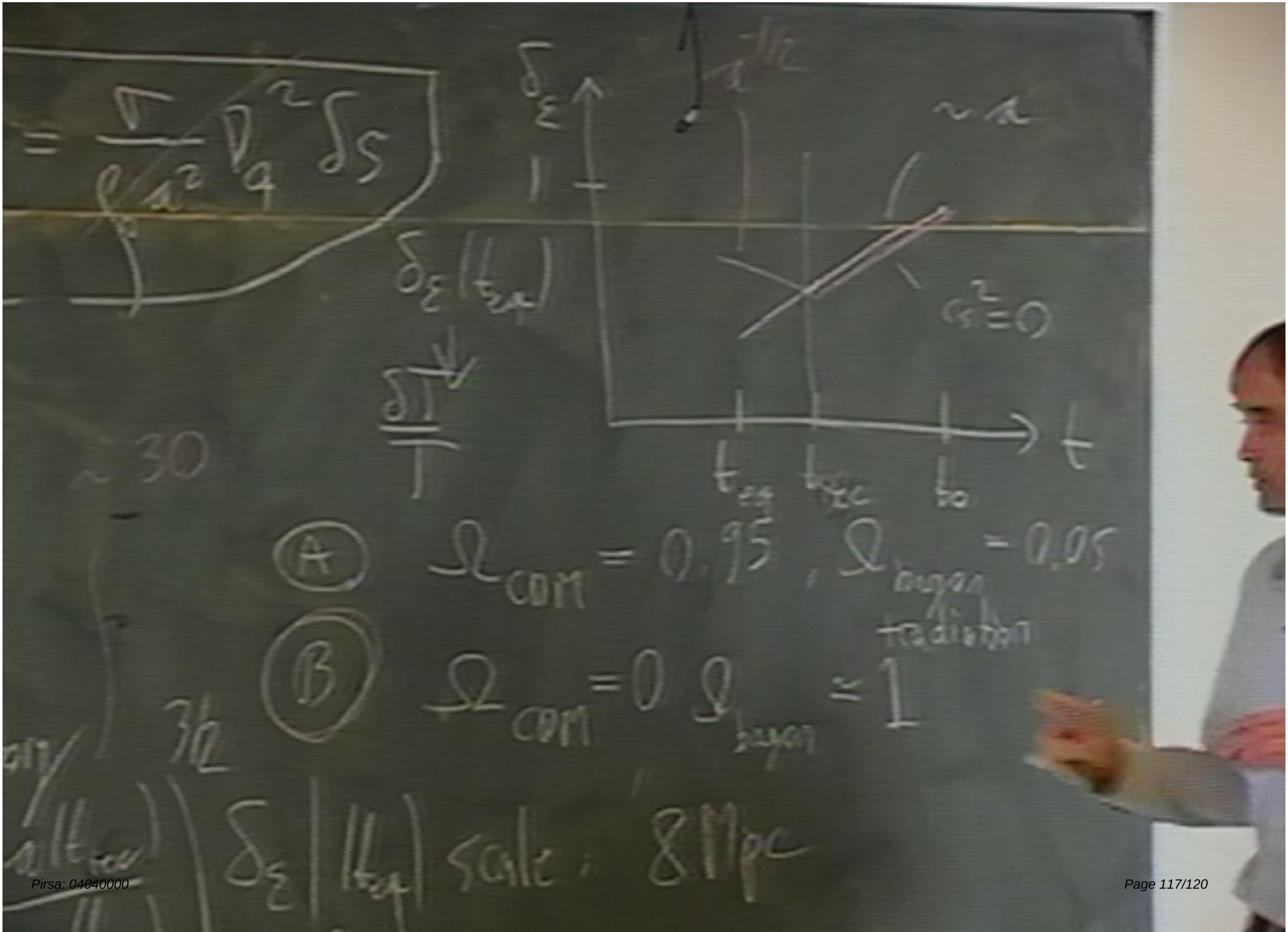
$\delta_\varepsilon \sim a(t) \sim t^{2/3}$

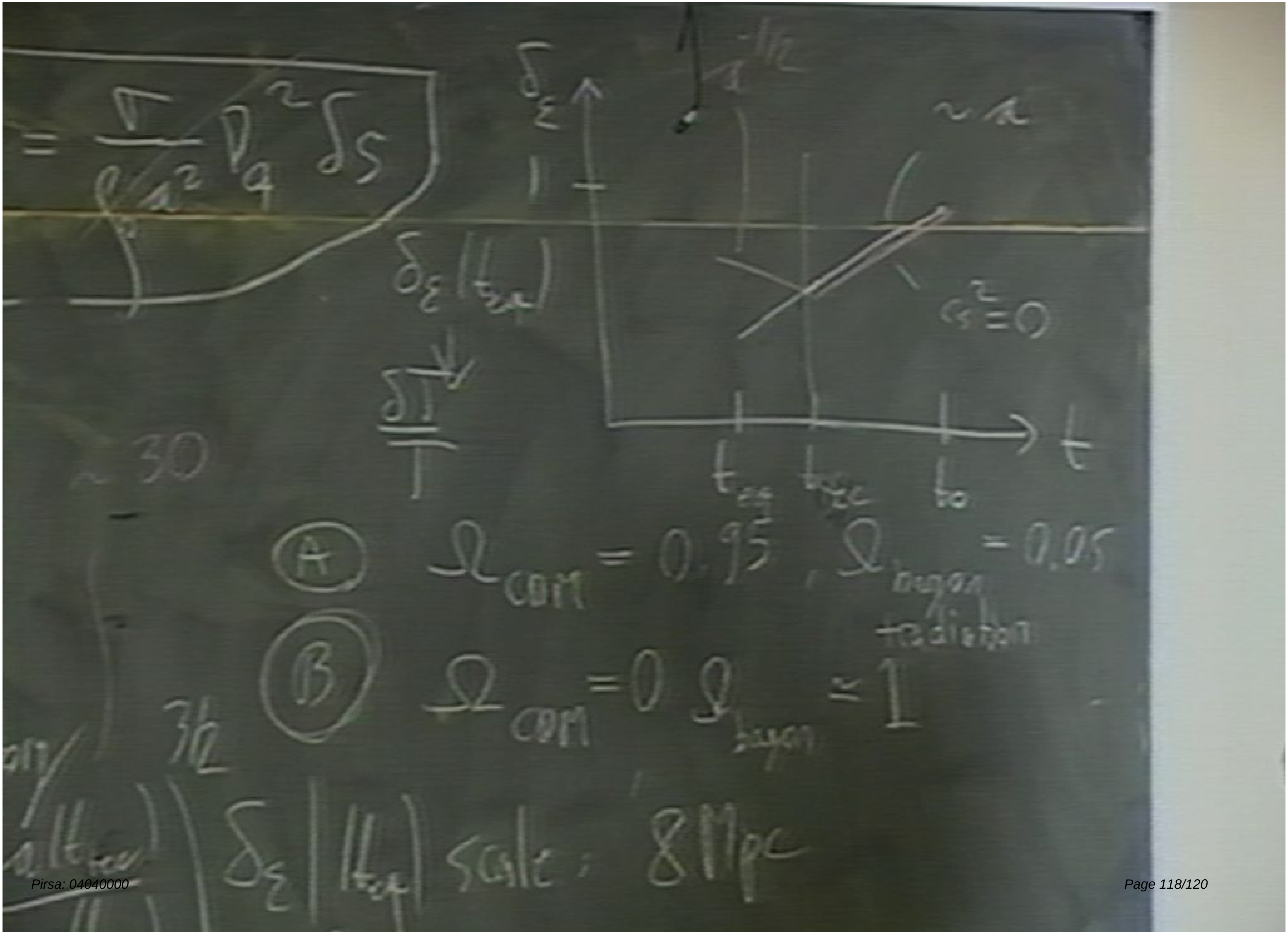
$\delta_\xi \sim a^{-1/2}(t)$ oscillatory

$$\delta_\varepsilon \Big|_B (t_{eq}) = \left(\frac{a(t_{rec})}{a(t_{eq})}\right)^{3/2} \delta_\varepsilon \Big|_A (t_{eq})$$ scale ;

A Ω_{com}

B $\Omega_{com} =$


~30
t_eq t_rec t_0
c_s^2 = 0
A Ω_CDM = 0.95, Ω_baryon = 0.05
radiation
B Ω_CDM = 0 Ω_baryon ≃ 1
scale: 8 Mpc


δε
~30
δε(t_eq)
δT/T
t_eq
t_0
t
cs² = 0
A Ω_CDM = 0.95, Ω_baryon+radiation = 0.05
B Ω_CDM = 0 Ω_baryon ≃ 1
δε(t_eq) scale: 8 Mpc

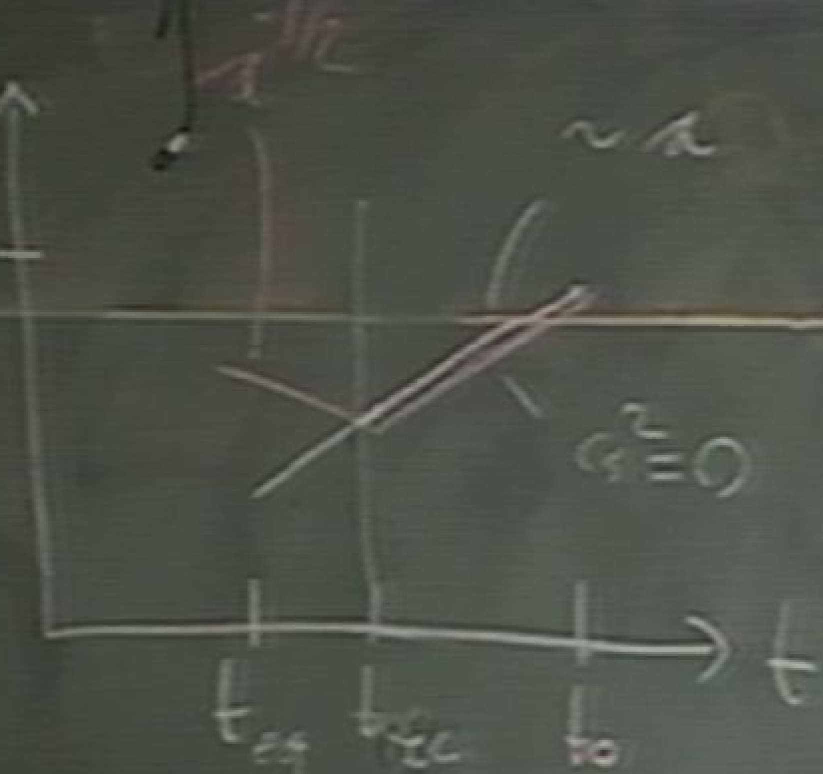
$$= \frac{\Gamma}{\rho\, a^2} \nabla_a^2 \delta_S$$

~ 30

$\delta_\varepsilon(t_{eq})$

$\Downarrow$

$\frac{\delta T}{T}$



(A) $\Omega_{CDM} = 0.95$, $\Omega_{baryon + radiation} = 0.05$

(B) $\Omega_{CDM} = 0$, $\Omega_{baryon} \simeq 1$

$3/2$

$\delta_\varepsilon(t_{eq})$ scale: 8 Mpc

Perturbations

1.1 Preview

1.2 Characterizing Fluctuations

1.3 Newt. Pert. about M^4

1.4 Newt. Pert. " FRW

New step: Use of (comoving)