

Title: Theory of Cosmological Perturbations - Final

Date: Feb 20, 2004 03:30 PM

URL: <http://pirsa.org/04020000>

Abstract:

DVD-R

REC

L3

MAIN

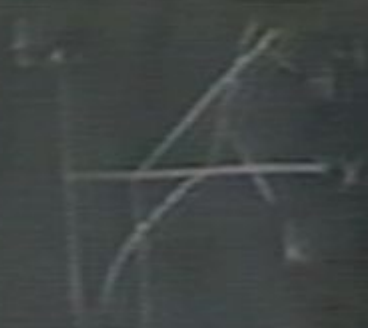
Relativistic Theory of Cosmology

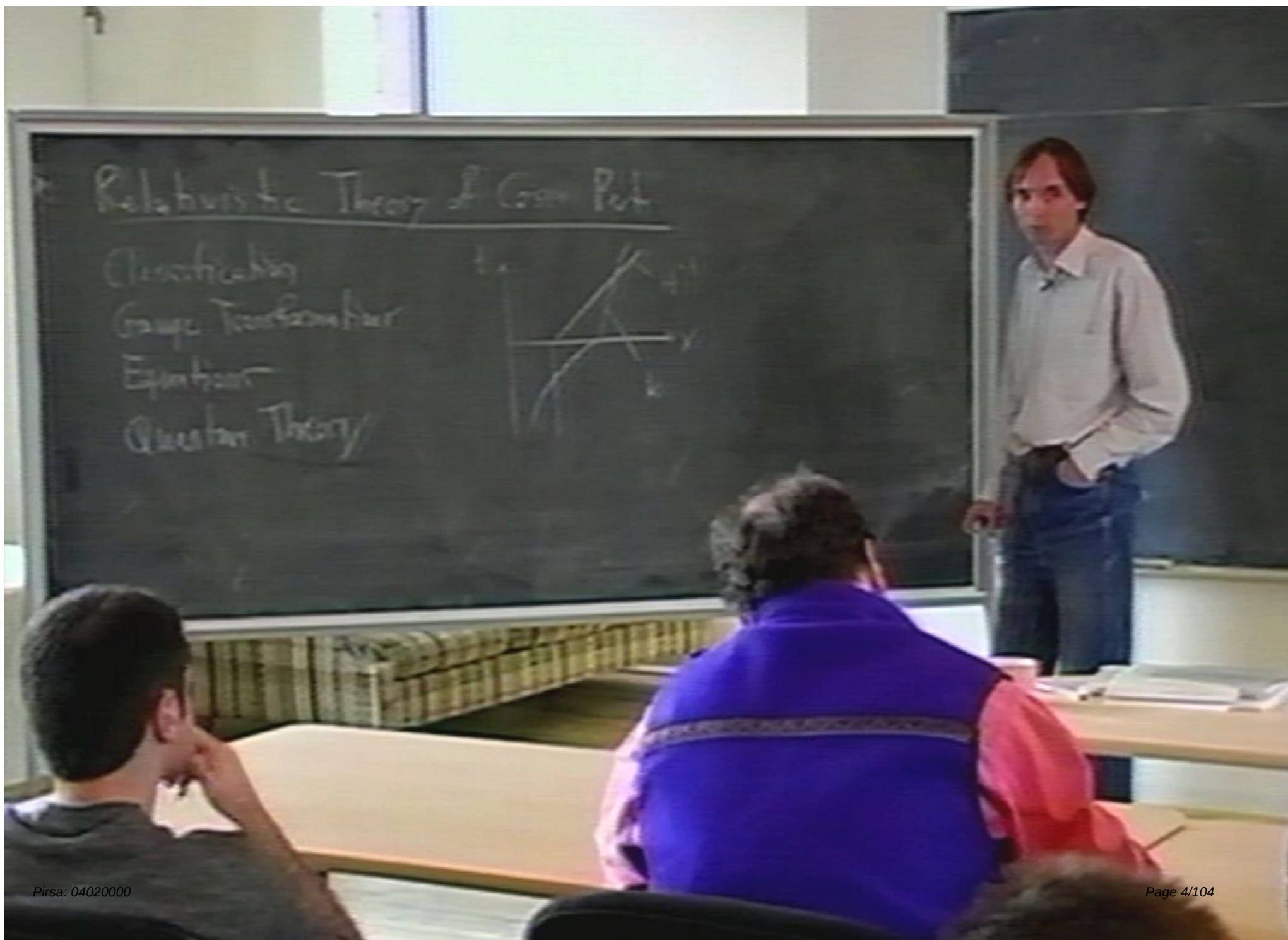
Classification

Gauge Transformation

Equation

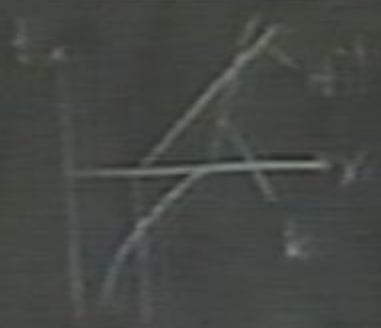
Quantum Theory

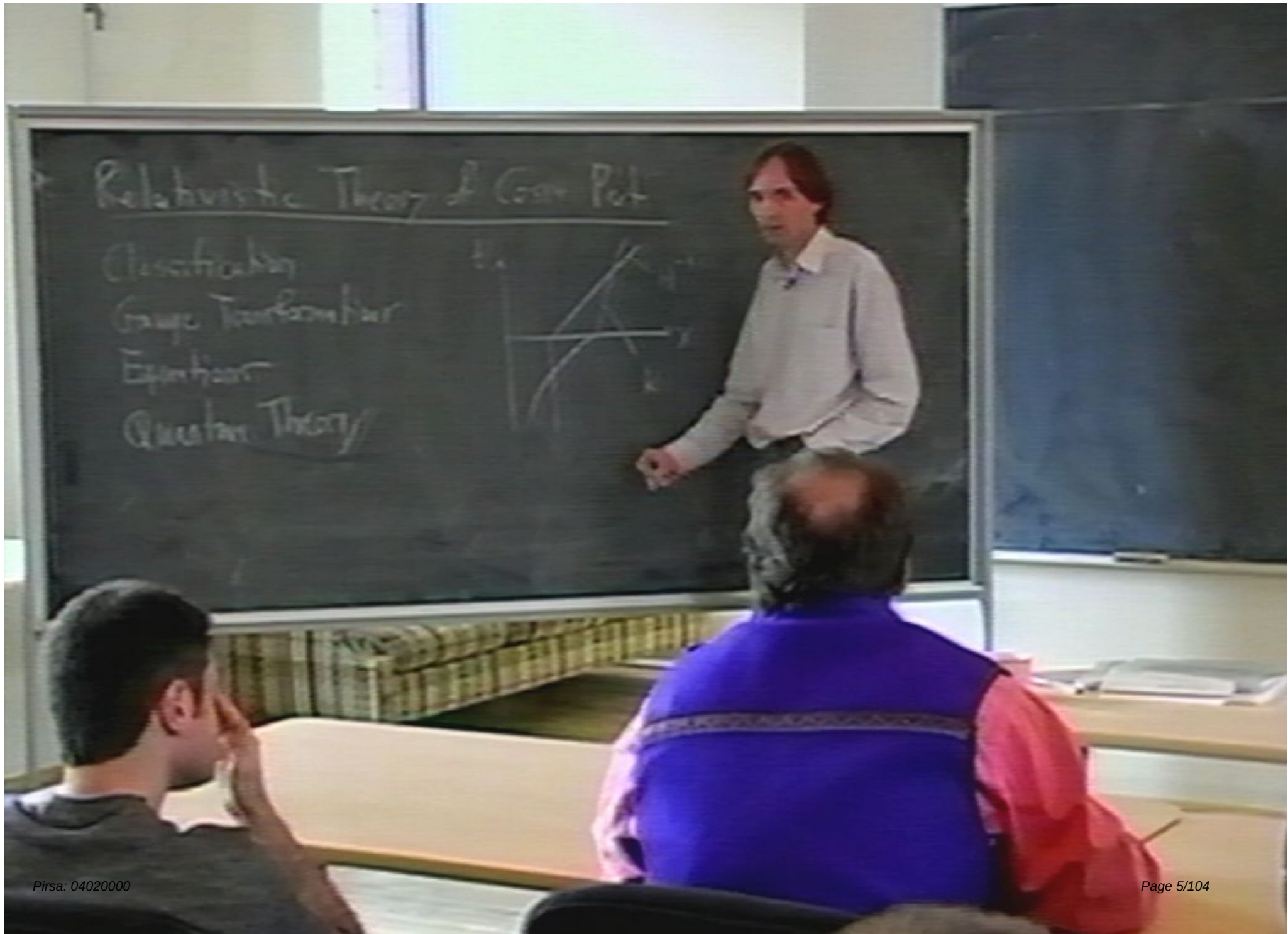




Relativistic Theory of Grav. Pot.

- Classification
- Gauge Transformation
- Equation
- Quantum Theory





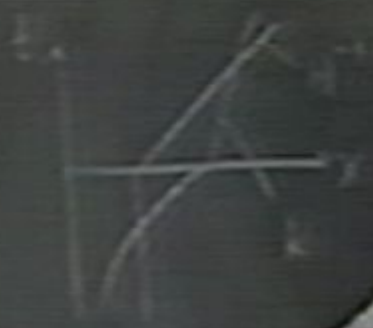
Relativistic Theory of Grav. Pot.

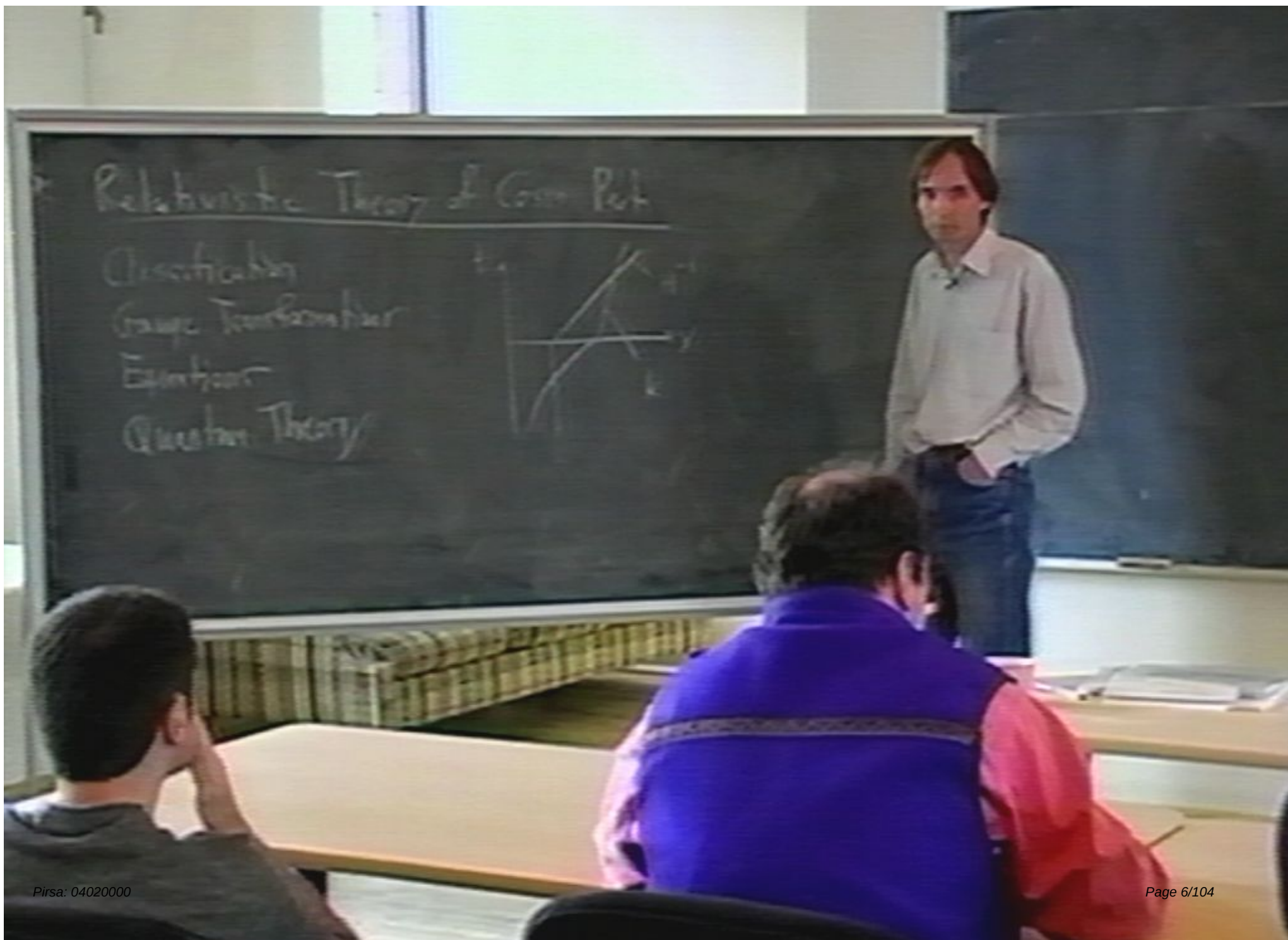
Classification

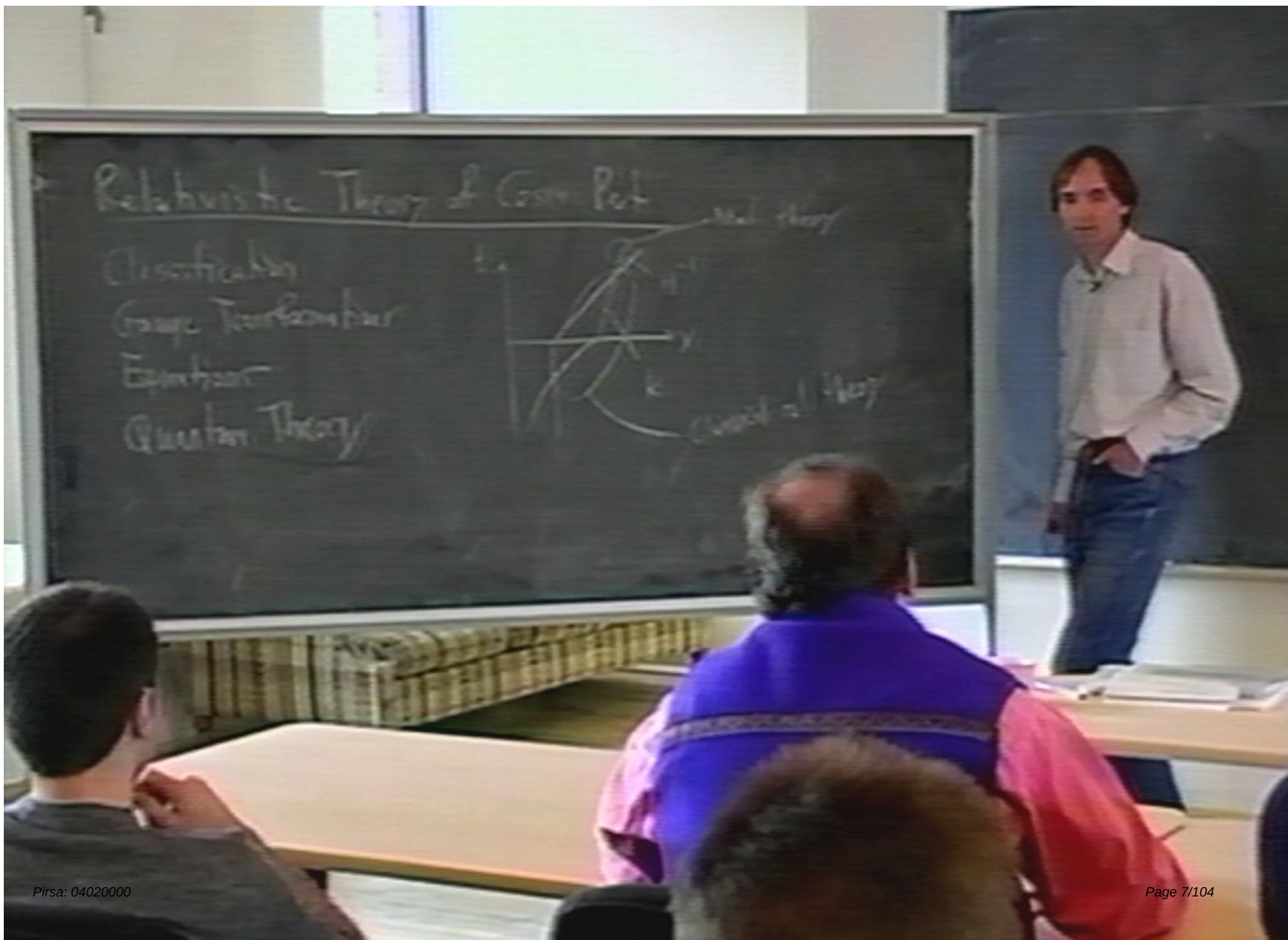
Gauge Transformation

Equations

Quantum Theory







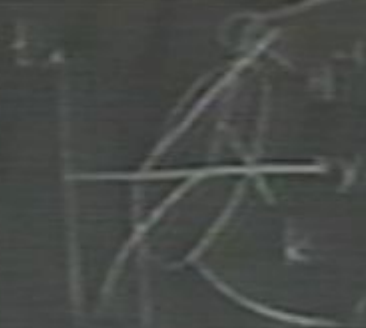
Relativistic Theory of Com. Part

Classification

Gauge Transformation

Equation

Quantum Theory

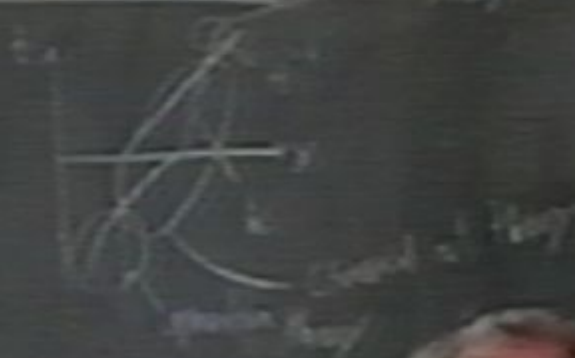


real theory

classical theory

Relativistic Theory of Cosmology

Classification
Group Transformation
Equations
Quantum Theory



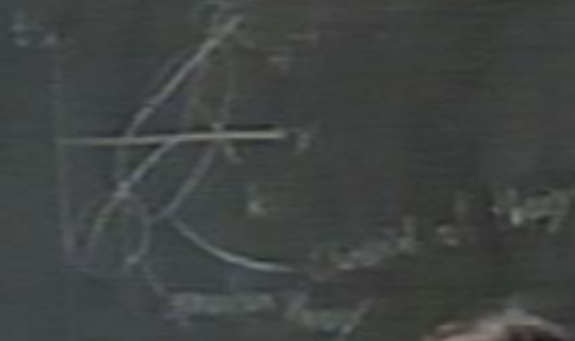
Relationships Theory of Comm. Prob. and Theory

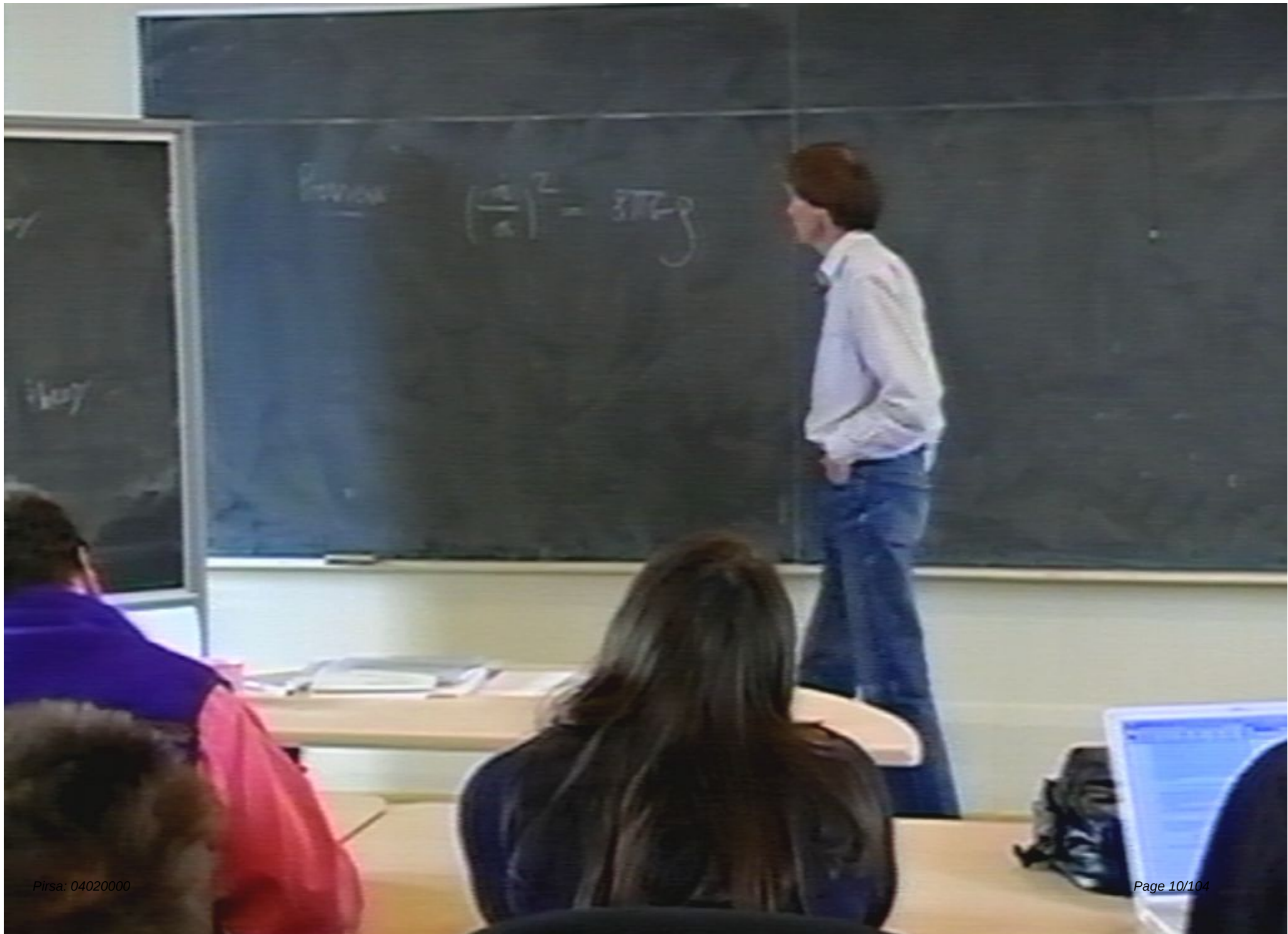
Construction

Gauge Transformation

Equation

Quantum Theory





Review

$$\left(\frac{a}{a}\right)^2 = 3\pi^2 g$$

Review

$$\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G \rho$$

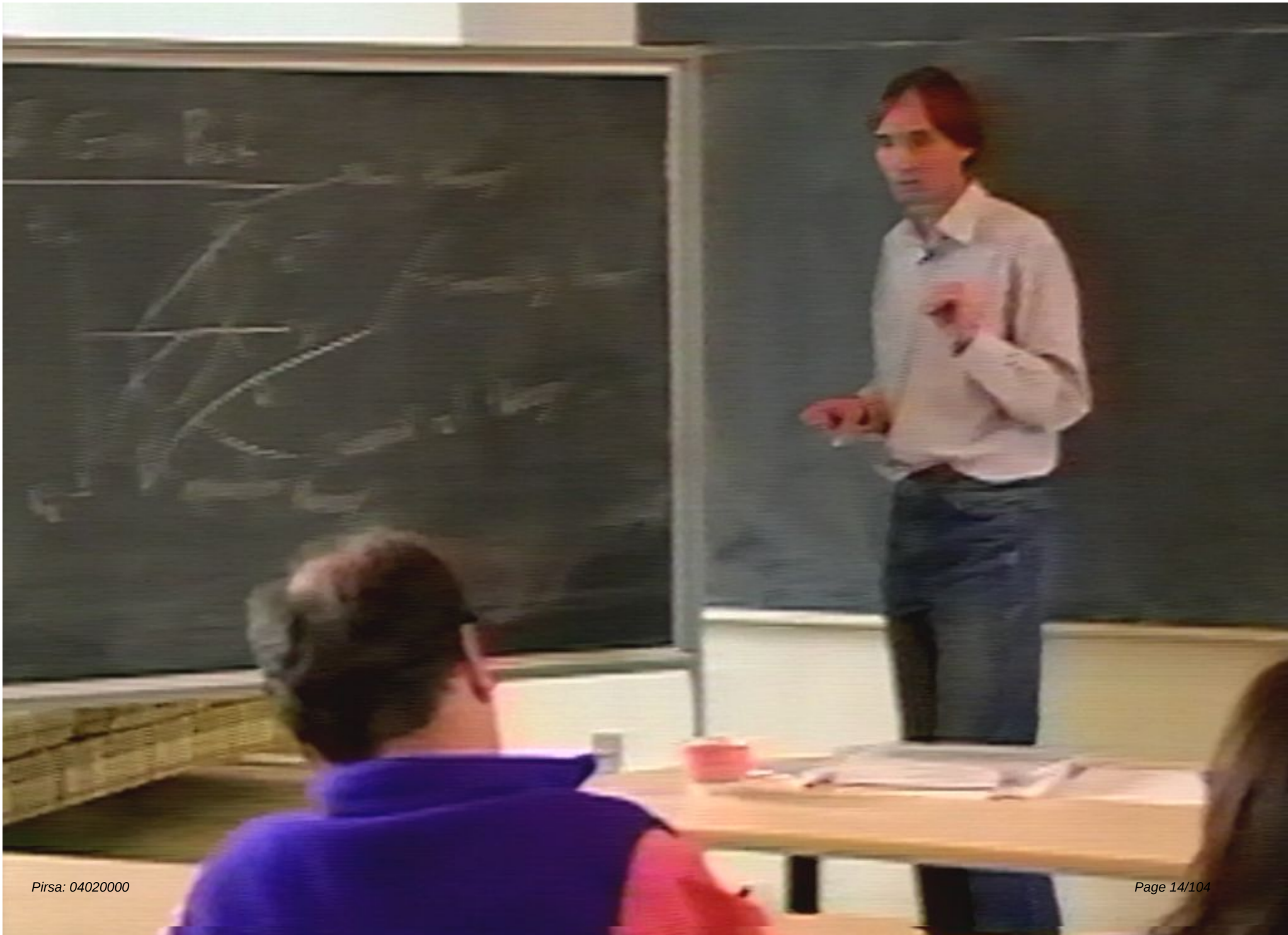
metric Photo grow on
Super-Hubble scales

Review

$$\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G \rho$$

also
metric fluctuations
super-Hubble scales





background $ds^2 = a^2(\eta) [d\eta^2 - d\mathbf{x}^2]$

$dt = a d\eta$ η conf. time

ρ_{rad}

$$ds^2 = a^2(t) [dt^2 - dx^2] \quad \left\{ \begin{array}{l} \text{flat} \\ \eta_{\mu\nu} \end{array} \right.$$

$$dt = a \, d\eta \quad \eta \text{ const time}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta g_{\mu\nu}$$

$$ds^2 = a^2(\eta) [d\eta^2 - d\mathbf{x}^2] \quad \left. \vphantom{\frac{d}{dt}} \right\} \frac{1}{a^2}$$

$$\frac{d}{dt} = a \frac{d}{d\eta} \quad \eta \text{ const. time}$$

$$g_{\mu\nu} = g_{\mu\nu}^{\text{flat}} + \delta g_{\mu\nu}$$

$$\psi = \psi^{\text{flat}} + \delta\psi$$

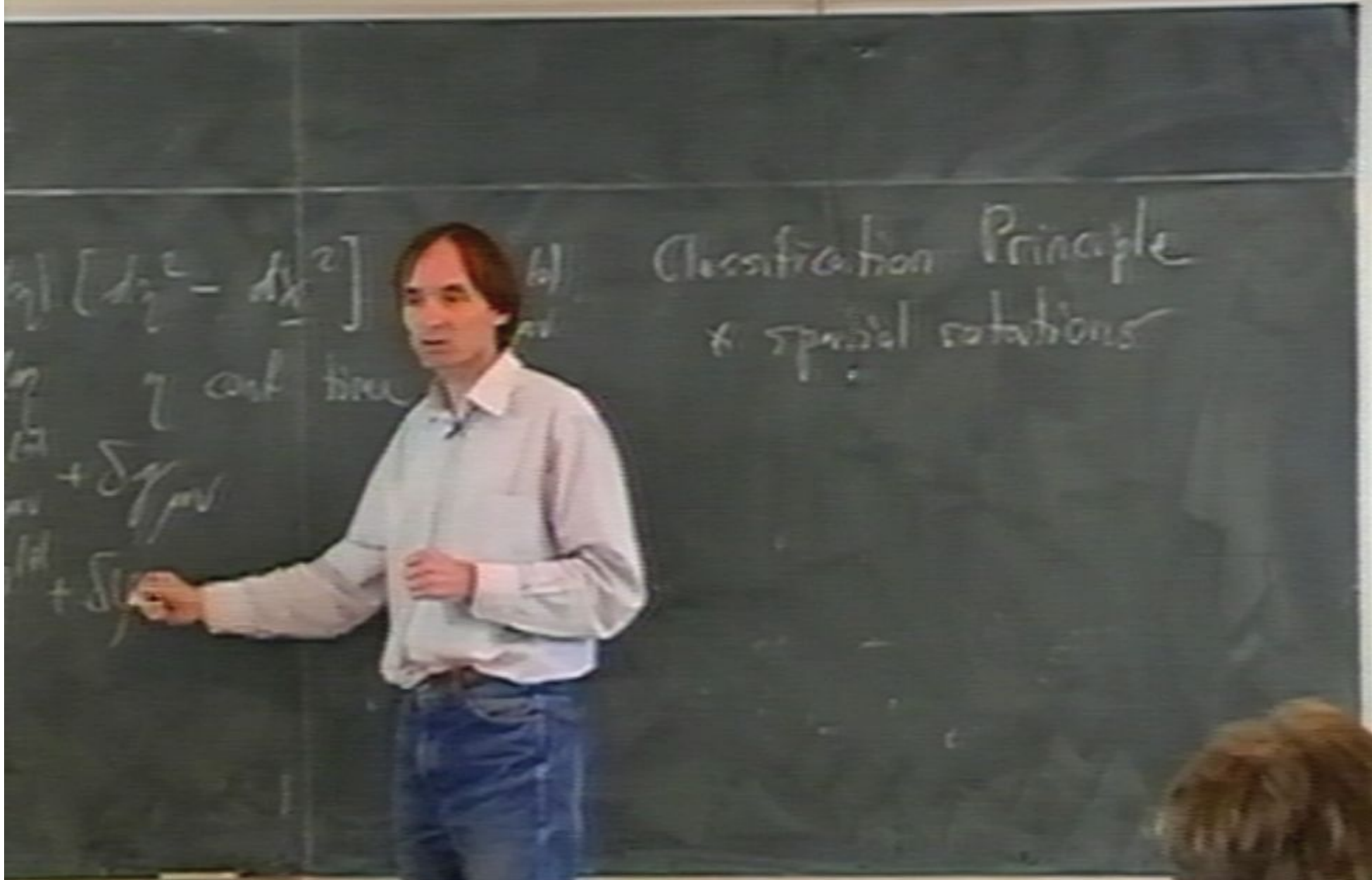
background

$$ds^2 = a^2(\eta) [d\eta^2 - d\mathbf{x}^2] \quad \left\{ \begin{array}{l} \text{flat} \\ \eta \text{ conf. time} \end{array} \right.$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta g_{\mu\nu}$$

$$\psi = \psi^{(0)} + \delta\psi$$





$\eta) [dx^2 - dt^2]$ b)

η out time

$+ \delta \eta_{\mu\nu}$

$+ \delta \eta_{\mu\nu}$

Classification Principle
x special solutions

$\left\{ \begin{array}{l} b) \\ \gamma_{\mu\nu} \end{array} \right.$ Classification Principle
 * spatial rotations

scalar $\delta g_{\mu\nu} = \begin{pmatrix} 0 & -B_{,i} \\ \delta g_{ij} + E_{,ij} \end{pmatrix}$

$\phi(\underline{x}, t), \psi(\underline{x}, t)$

Classification Principle & spatial rotations

scalar $\delta g_{\mu\nu} = \frac{2}{\sqrt{-g}} \delta B_{\mu\nu}$

$$\phi(x,t), \psi(x,t), \chi(x,t), E, B$$

Classification Principle

* spatial rotations

scalar $\delta\varphi = \frac{1}{2} \left(\begin{matrix} 0 & -B_{ij} \\ \delta_{ij} & E_{ij} \end{matrix} \right)$

$\phi(x,t), \beta(x,t), \eta(x,t), E(x,t)$

vector part. $S_{g/m} = a^2 \begin{pmatrix} 0 \end{pmatrix}$

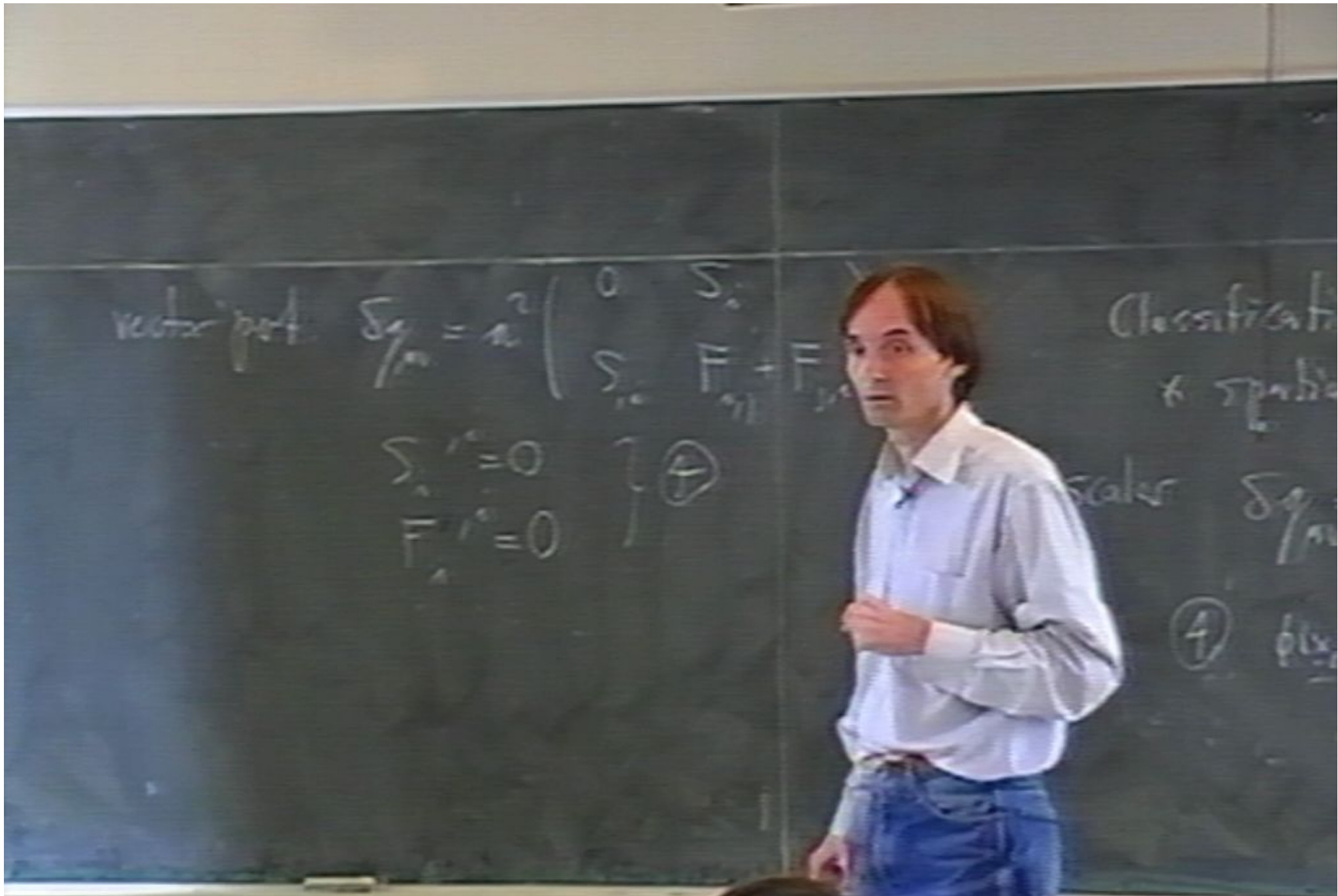
Class

x

scalar

vector part $\delta g_{\mu\nu} = a^2 \begin{pmatrix} 0 & S_{\mu} \\ S_{\nu} & \end{pmatrix}$

$$\sum_{\mu} S_{\mu}^{\mu} = 0$$



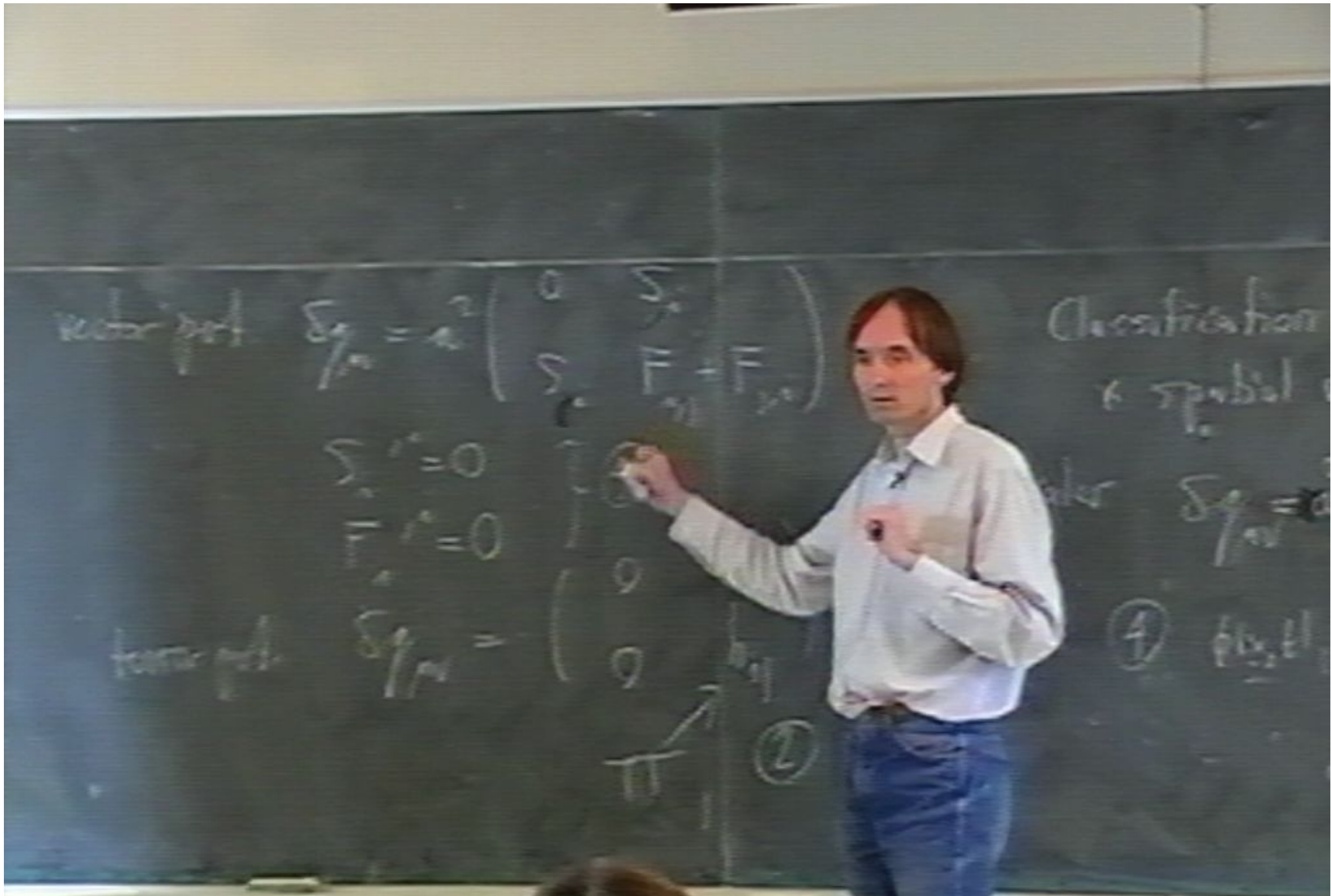
vector pot. $S_{\mu\nu} = a^2 \begin{pmatrix} 0 & S_{\mu i} \\ S_{\mu i} & F_{\mu\nu} + F_{\nu\mu} \end{pmatrix}$

$S_{\mu\nu}{}^{;\rho} = 0$
 $F_{\mu\nu}{}^{;\rho} = 0$

tensor pot. $S_{\mu\nu\rho\sigma} = \begin{pmatrix} 0 & 0 \\ 0 & h_{\mu\nu} \end{pmatrix}$

Class

scalar



Relativistic Theory of Cosm Part

Classification

Gauge Transformations

Equations

Quantum Theory

$$x_i^\mu = x^\mu + f^\mu$$

2 scalar

ξ^0
 ξ^1

4 coord
of freedom

Relativistic Theory of Cosm Pert

Classification

Gauge Transformation

Equations

Quantum Theory

$$x^{\mu'} = x^{\mu} + \xi^{\mu}$$

2 scalar

$$\xi^0$$

$$\xi^1$$

2 vector

4 coord deg
of freedom

$$\xi^i =$$

Theory of Cosm. Pert

$$X^{\mu} = \bar{X}^{\mu} + \xi^{\mu}$$

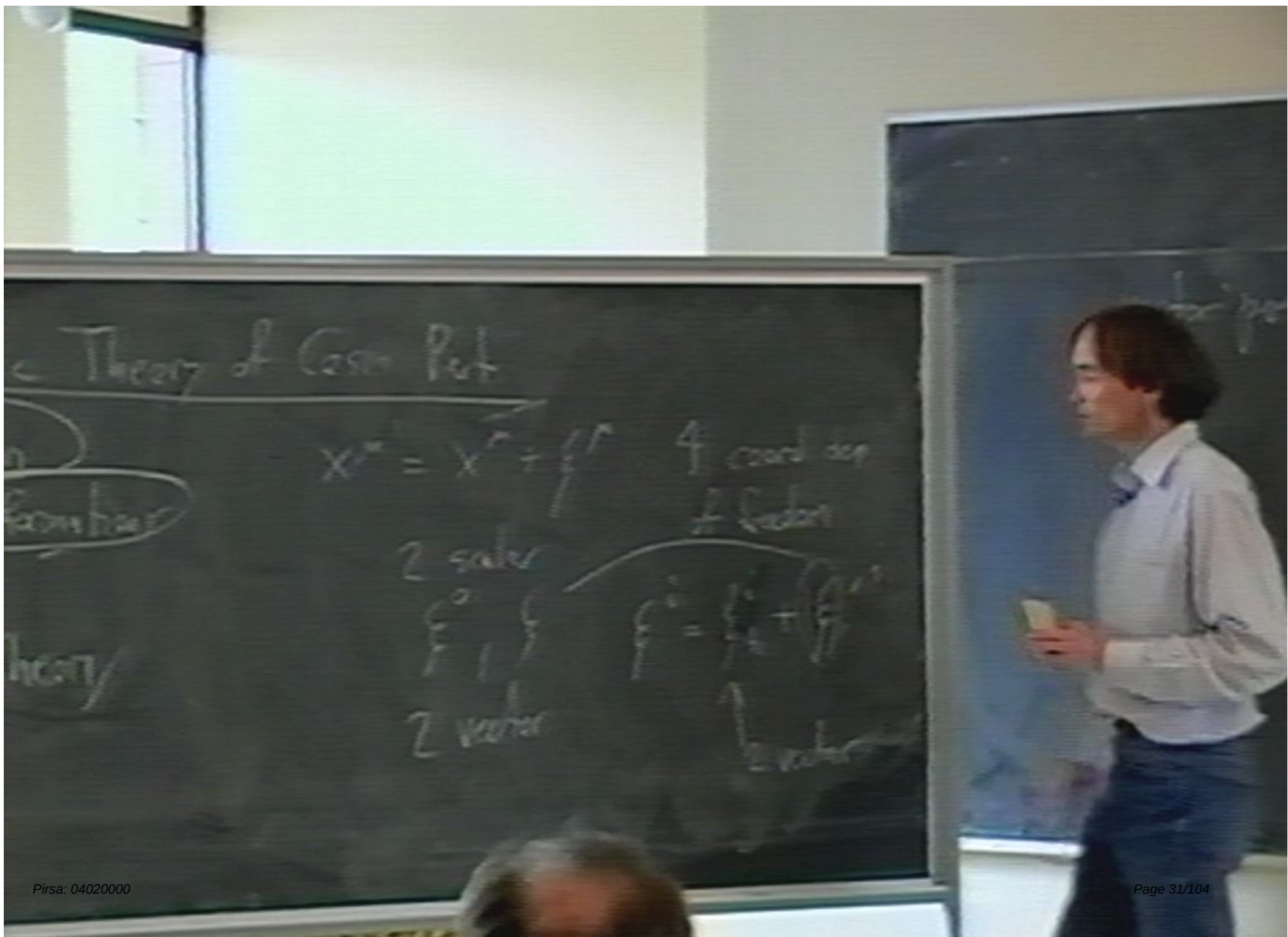
4 coord des
of Sectors

2 scalar

ξ^0
 ξ^1

2 vector

$$\xi^i = \xi_{\perp}^i + \xi_{\parallel}^i$$



Theory of Cosm Pert

formalism

theory

$$X^{(2)} = X^{(1)} + f^{(2)}$$

4 coord deg
of freedom

2 scalar

$$f^{(2)} = \begin{Bmatrix} f^{(2)}_h \\ f^{(2)}_p \end{Bmatrix}$$

2 vector

$$f^{(2)} = f^{(2)}_h + f^{(2)}_p$$

1 vector

$$\begin{pmatrix} 0 & S_{ij} \\ S_{ji} & F_{ij} + F_{ji} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & h_{ij} \end{pmatrix}$$

π $\nearrow$ $\textcircled{2}$

Classification Principle
 & spatial rotations

scalar $\delta g_{\mu\nu} = a \begin{pmatrix} 0 & -B_{\mu i} \\ -B_{i\mu} & \delta g_{ij} + E_{ij} \end{pmatrix}$

$\textcircled{4} \quad \phi(x,t), \beta(x,t), \gamma(x,t), E(x,t)$

2 choices ④ g.i. variables

⑤ completely gauge fix

2 dimer: ② gauge variable

③ completely gauge fix

$$\bar{\mathcal{L}} = \phi + \frac{1}{2} [(\mathcal{B} - \vec{E})a]'$$

Class

*

scalar

④

$$I = \frac{d}{dt}$$

2 choices: ② gauge variables

③ completely gauge fix

$$\bar{\psi} = \psi + \frac{1}{2} [(\beta - E')a]''$$

$$\bar{\psi} = \psi - \frac{1}{2} (\beta - E')$$

scal

Relativistic Theory of Gravity Part

- Classification
- Gauge Transformation
- Equations
- Quantum Theory

$x^\mu = \begin{pmatrix} t \\ \mathbf{x} \end{pmatrix}$ 4 coord dep
 of freedom

2 scalars
 $\xi^\alpha = \begin{pmatrix} \xi^0 \\ \xi^i \end{pmatrix}$
 $\xi^i = \xi^i + \xi^i$
 2 vectors

$$\mathcal{L}_\text{grav} = \mathcal{L}_\text{BD}$$

Relativistic Theory of Cosm Pert

- Classification
- Gauge Transformations
- Equations
- Quantum Theory

$$x^{\mu} = x^{\mu} + \xi^{\mu}$$

4 coord. dep
of freedom

2 scalar

$$\xi^0, \xi^1$$

$$\xi^i = \xi^i + \xi^i$$

2 vector

$$\delta g_{\mu\nu} = \delta g_{\mu\nu}$$

$$' = \frac{\partial}{\partial \eta}$$

2 choices: ② g.i. variables

③ completely gauge fix

$$\bar{\Phi} = \phi + \frac{1}{a} [(\beta - E')a]'$$

$$\bar{\Psi} = \psi - \frac{a'}{a} (\beta - E')$$

eg. longitudinal
gauge

Classification
* spin

scalar spin

④

$$I = \frac{d}{dt}$$

eg. longitudinal gauge

Classification Principle

* spatial rotations

scalar $\delta g_{\mu\nu} = \begin{pmatrix} 0 & -\delta_{,i} \\ -\delta_{,i} & \delta_{ij} + \delta_{,i} \delta_{,j} \end{pmatrix}$

④ $\phi(x,t), \theta(x,t), \psi(x,t)$

$$-3\mathcal{H}(\mathcal{H}\phi + \mathcal{H}') + \nabla^2 \mathcal{N} = 4\pi G$$

$$-\nabla^2 \psi + \nabla^2 \psi = 4\pi G \rho^2 \delta T \psi$$

$$-3\mathcal{H}(\mathcal{H}\phi + \psi') + \nabla^2 \psi = 4\pi G a^2 \delta T$$

$$(\mathcal{H}\phi + \psi')_{,i} = 4\pi G a^2 \delta T_{,i}$$

$$-3\dot{H}(\mathcal{H}\phi + \psi') + \nabla^2 \psi = 4\pi G a^2 \delta T^{\lambda}$$

$$(\mathcal{H}\phi + \psi')_{,i} = 4\pi G a^2 \delta T^0_{,i}$$

$$(2\mathcal{H}' + \mathcal{H}^2)\phi + \mathcal{H}\phi' + \psi'' + 2\mathcal{H}\psi'$$

$$3\mathcal{H}(\mathcal{H}\phi + \mathcal{N}') + \nabla^2 \mathcal{N} = 4\pi G a^2 \delta T^i{}_i$$

$$(\mathcal{H}\phi + \mathcal{N}')_{,i} = 4\pi G a^2 \delta T^0{}_i$$

$$(2\mathcal{H}' + \mathcal{H}^2)\phi + \mathcal{H}\phi' + \mathcal{N}'' + 2\mathcal{H}\mathcal{N}'] \delta;$$

$$+ \nabla^2 \psi = 4\pi G \cdot a^2 \delta T^0_0$$

$$= 4\pi G a^2 \delta T^0_0$$

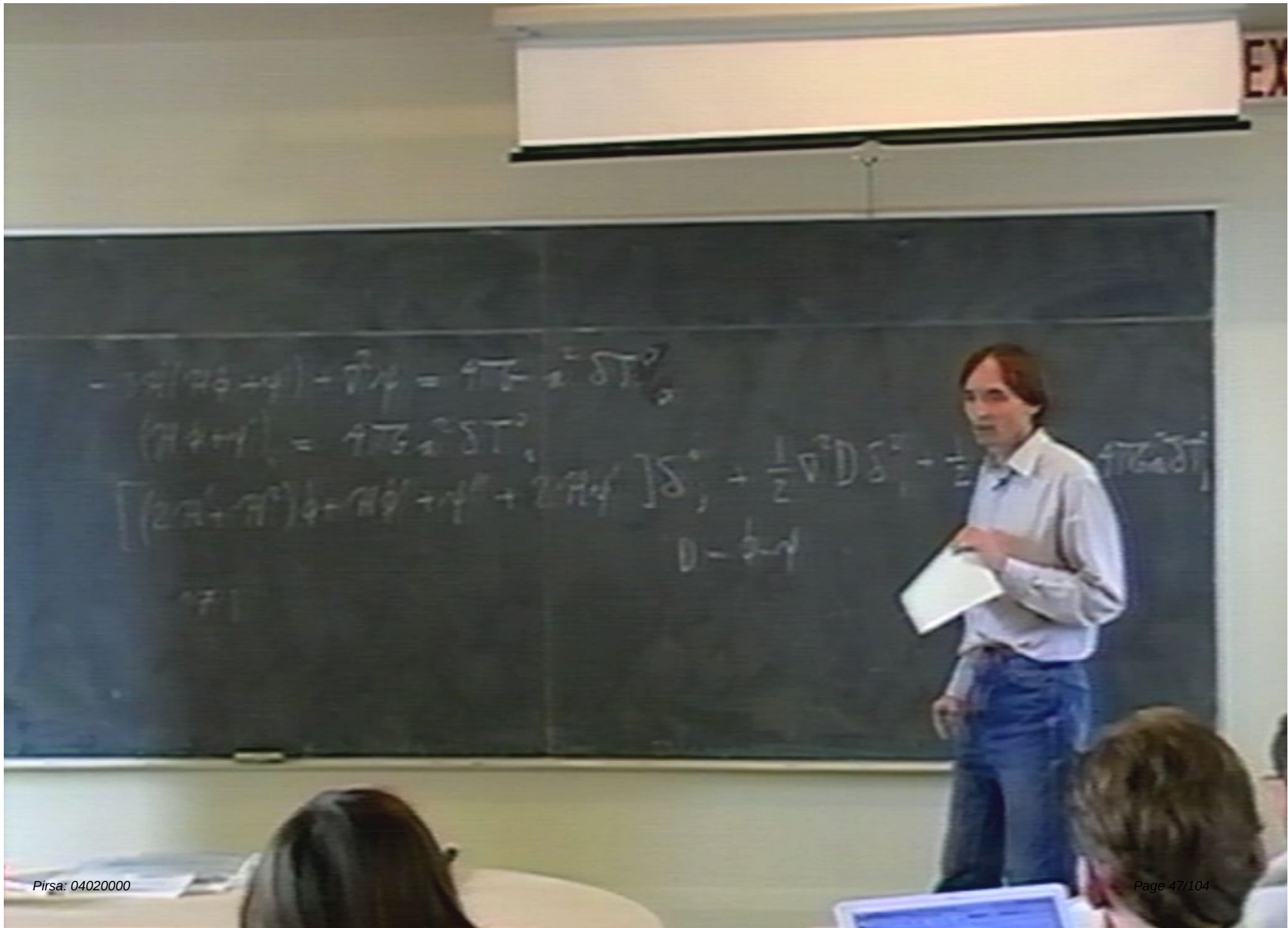
$$[\phi + H\phi' + \psi'' + 2H\psi'] \delta^i_i + \frac{1}{2} \nabla^2 D$$

$$-3H(\dot{H}\phi + \dot{\psi}) + \nabla^2 \psi = 4\pi G a^2 \delta T^0_0$$

$$(\dot{H}\phi + \dot{\psi})_i = 4\pi G a^2 \delta T^0_i$$

$$\left[(2\dot{H} + H^2)\phi + H\dot{\phi} + \ddot{\psi} + 2H\dot{\psi} \right] \delta^i_j + \frac{1}{2} \nabla^2 D \delta^i_j + \frac{1}{2}$$

$$D = \phi - \psi$$



$$\begin{aligned} -3\pi/(4\phi + \psi) + \nabla^2 \psi &= 4\pi G a^2 \delta T^0_0, \\ (4\phi + \psi) &= 4\pi G a^2 \delta T^0_0, \\ \left[(2\psi' + \phi')\dot{\phi} + 4\phi\dot{\phi}' + \psi'' + 2\psi\psi' \right] \delta^0_0 &+ \frac{1}{2} \nabla^2 D \delta^0_0 = \frac{1}{2} 4\pi G a^2 \delta T^0_0, \\ D &= \dot{\phi} - \dot{\psi} \end{aligned}$$

$$-3\pi(\pi\phi + \psi) + \nabla^2\psi = 4\pi G a^2 \delta T^0_0$$

$$(\pi\phi + \psi) = 4\pi G a^2 \delta T^0_0$$

$$\left[(2\pi + \pi^2)\phi + \pi\phi' + \psi'' + 2\pi\psi' \right] \delta_1 + \frac{1}{2}\nabla^2 D \delta_1 + \frac{1}{2}\delta_1 = -4\pi G a^2 \delta T^0_1$$

$$D = \phi - \psi$$

or

$$-3\pi(\dot{H}\phi + \dot{\psi}) + \dot{\tau}^2 \psi = 4\pi G a^2 \delta T^0_0$$

$$(\dot{H}\phi + \dot{\psi})' = 4\pi G a^2 \delta T^0_1$$

$$\left[(2\dot{H} + H^2)\phi + H\dot{\psi} + \psi'' + 2\dot{H}\dot{\psi} + \frac{1}{2}\nabla^2 D\delta^0_1 + \frac{1}{2}D''_1 \right] = -$$

$$\delta T^1_1 = 0$$

$$-3H(\dot{H}\phi + \dot{\psi}) + \dot{T}^0_0 = 4\pi G a^2 \delta T^0_0$$

$$(H\dot{\phi} + \dot{\psi})' = 4\pi G a^2 \delta T^0_0$$

$$[(2H' + H^2)\phi + H\dot{\phi}' + \psi'' + 2H\dot{\psi}] \delta^0_0 + \frac{1}{2} \nabla^2 D \delta^0_0 + \frac{1}{2} D''$$

$$\Rightarrow \delta T^0_0 = 0 \Rightarrow \phi = \psi \quad D = \phi - \psi$$

$$\begin{aligned}
 2(\mathcal{H}\dot{\phi} + \dot{\psi}) + \nabla^2 \psi &= 4\pi G a^2 \delta T^0_0 & 2 \text{ dy left } \phi, \delta\psi \\
 \mathcal{H}(\dot{\phi} + \dot{\psi}) &= 4\pi G a^2 \delta T^0_0 \\
 2(\mathcal{H}' + \mathcal{H}^2)\dot{\phi} + \mathcal{H}\dot{\phi}' + \dot{\psi}'' + 2\mathcal{H}\dot{\psi} &] \delta^0_0 + \frac{1}{2} \nabla^2 D \delta^0_0 + \frac{1}{2} \dot{D}^0_0 = -4\pi G a \delta T^0_0 \\
 \delta T^0_0 = 0 &\Rightarrow \dot{\phi} = 0 & D = \dot{\phi} - \dot{\psi} \\
 \uparrow & \text{adiabatic perturbation}
 \end{aligned}$$

coupled

$$3H(H\dot{\phi} + \dot{\psi}) + \ddot{\psi} = 4\pi G a^2 \delta T^0_0$$

$$(H\dot{\phi} + \dot{\psi}) = 4\pi G a^2 \delta T^0_0$$

$$[(2H + H^2)\dot{\phi} + H\dot{\phi}' + \ddot{\phi} + 2H\dot{\psi}] \delta^0_0 + \frac{1}{2} \nabla^2 D \delta^0_0 + \frac{1}{2} \dot{D}, = -4\pi G a^2 \delta T^0_0$$

2 deg left $\phi, \delta\psi$

$\delta T^0_0 = 0 \Rightarrow \dot{\phi} = 0$ D - 2nd

$\delta T^0_0 = 0 \Rightarrow \dot{\phi} = 0$ no monopole term

combination $\phi'' + 2\left(\mathcal{H} - \frac{\dot{\gamma}_0'''}{\dot{\gamma}_0'}\right)\phi' - \nabla^2\phi + 2(\mathcal{H}' -$

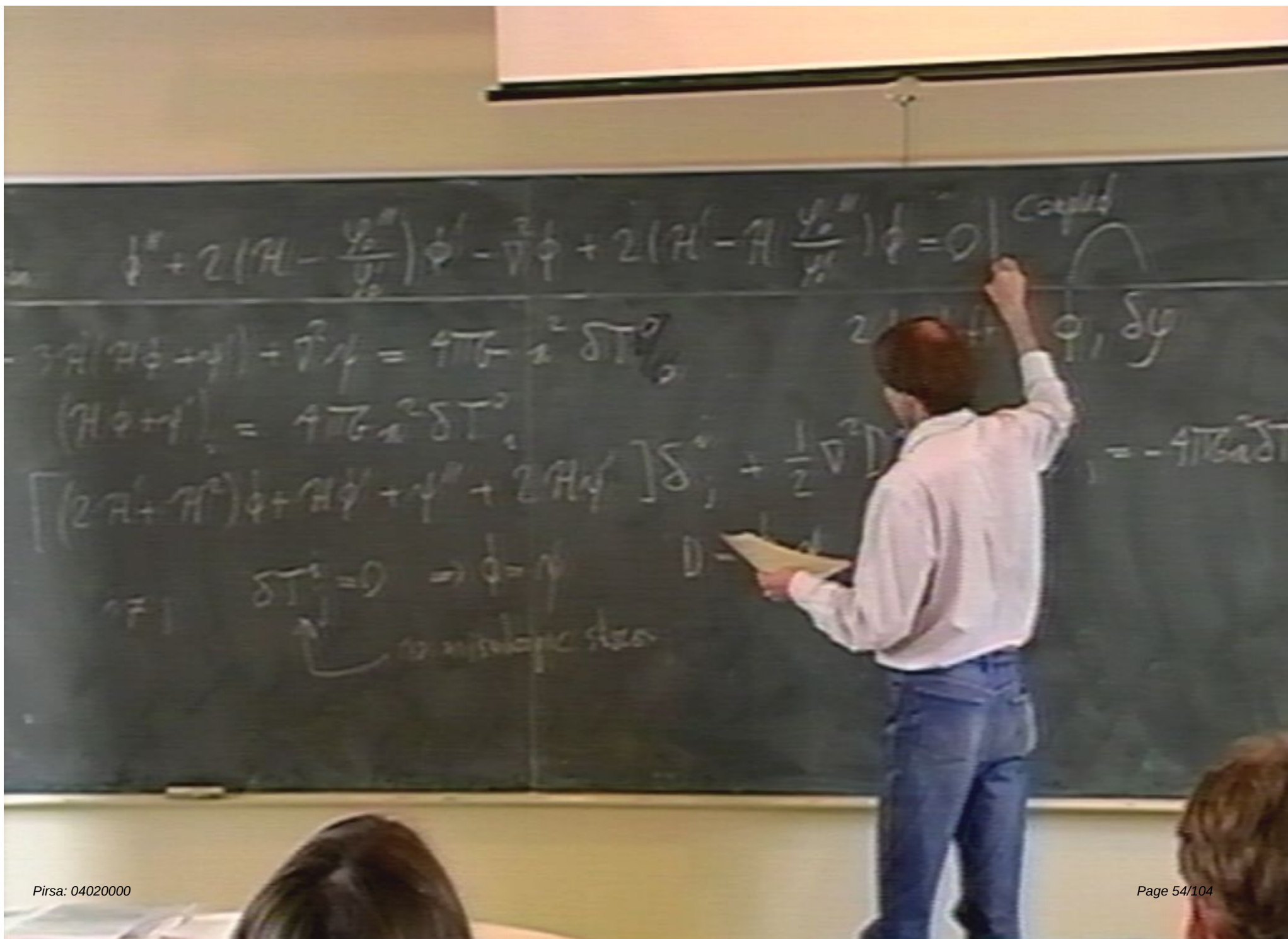
$$- 3\mathcal{H}(\mathcal{H}\phi + \psi') + \nabla^2\psi = 4\pi G a^2 \delta T^0_0$$

$$(\mathcal{H}\phi + \psi')_{,i} = 4\pi G a^2 \delta T^0_i$$

$$[(2\mathcal{H}' + \mathcal{H}^2)\phi + \mathcal{H}\phi' + \psi'' + 2\mathcal{H}\psi']\delta^i_j$$

$$\neq 1 \quad \delta T^i_j = 0 \Rightarrow \phi = \psi$$

no anisotropic stress



$$\phi'' + 2\left(\mathcal{H} - \frac{\dot{\mathcal{H}}}{\mathcal{H}}\right)\phi' - \nabla^2\phi + 2\left(\mathcal{H}' - \mathcal{H}\frac{\dot{\mathcal{H}}}{\mathcal{H}}\right)\phi = 0 \quad \text{coupled}$$

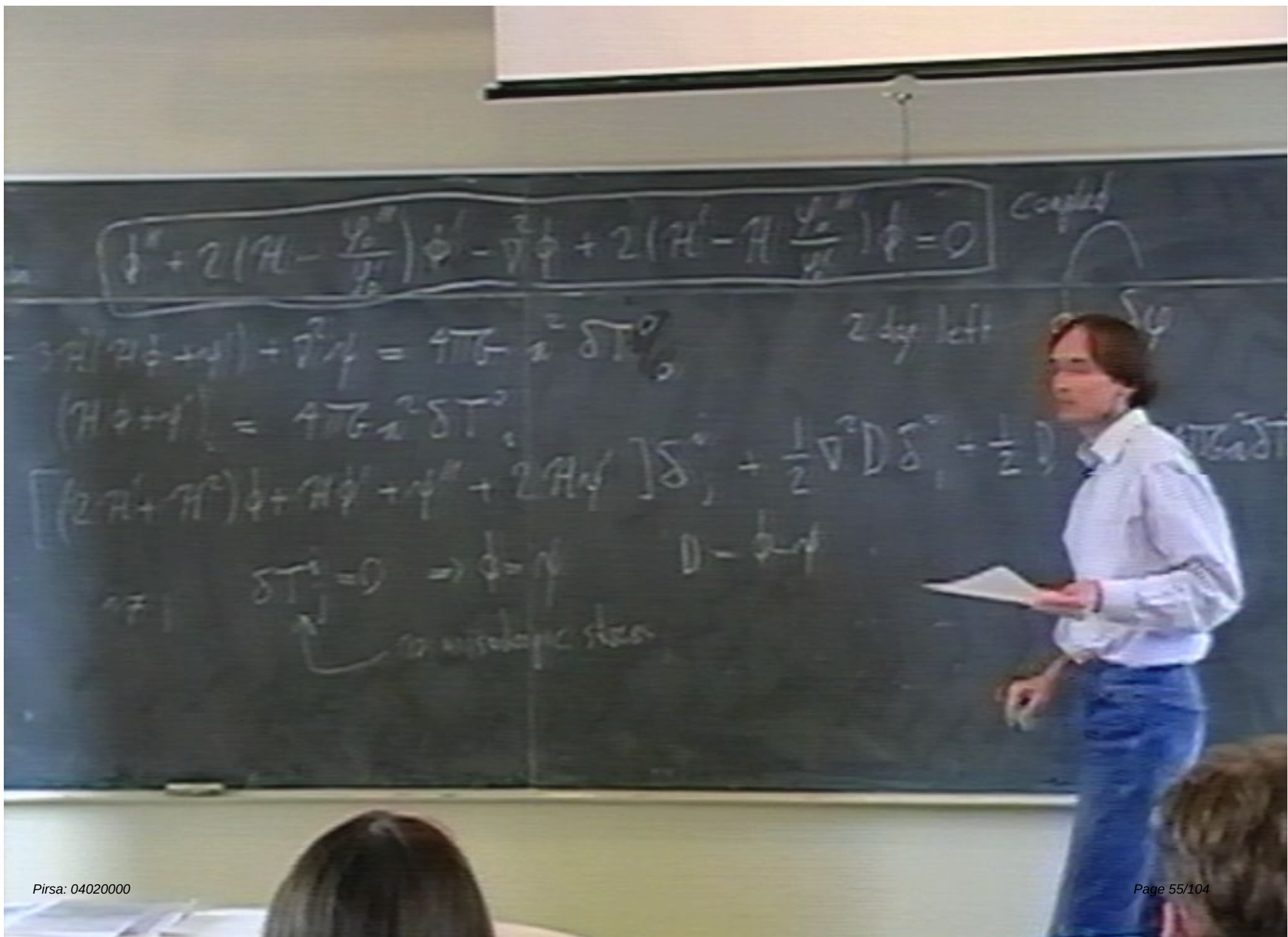
$$-3\mathcal{H}(\mathcal{H}\phi + \psi)' + \nabla^2\psi = 4\pi G a^2 \delta T^0_0$$

$$(\mathcal{H}\phi + \psi)' = 4\pi G a^2 \delta T^0_0$$

$$\left[(2\mathcal{H}' + \mathcal{H}^2)\phi + \mathcal{H}\phi' + \psi'' + 2\mathcal{H}\psi' \right] \delta^0_0 + \frac{1}{2}\nabla^2\psi = -4\pi G a^2 \delta T^0_0$$

$$\delta T^0_0 = 0 \Rightarrow \phi = \psi$$

no anisotropic stress



$$\boxed{\phi'' + 2\left(\mathcal{H} - \frac{\dot{\gamma}_a''}{\gamma_a'}\right)\phi' - \nabla^2\phi + 2\left(\mathcal{H}' - \mathcal{H}\frac{\dot{\gamma}_a''}{\gamma_a'}\right)\phi = 0} \quad \text{coupled}$$

$$3\mathcal{H}(\mathcal{H}\phi + \psi) + \nabla^2\psi = 4\pi G a^2 \delta T^0_0$$

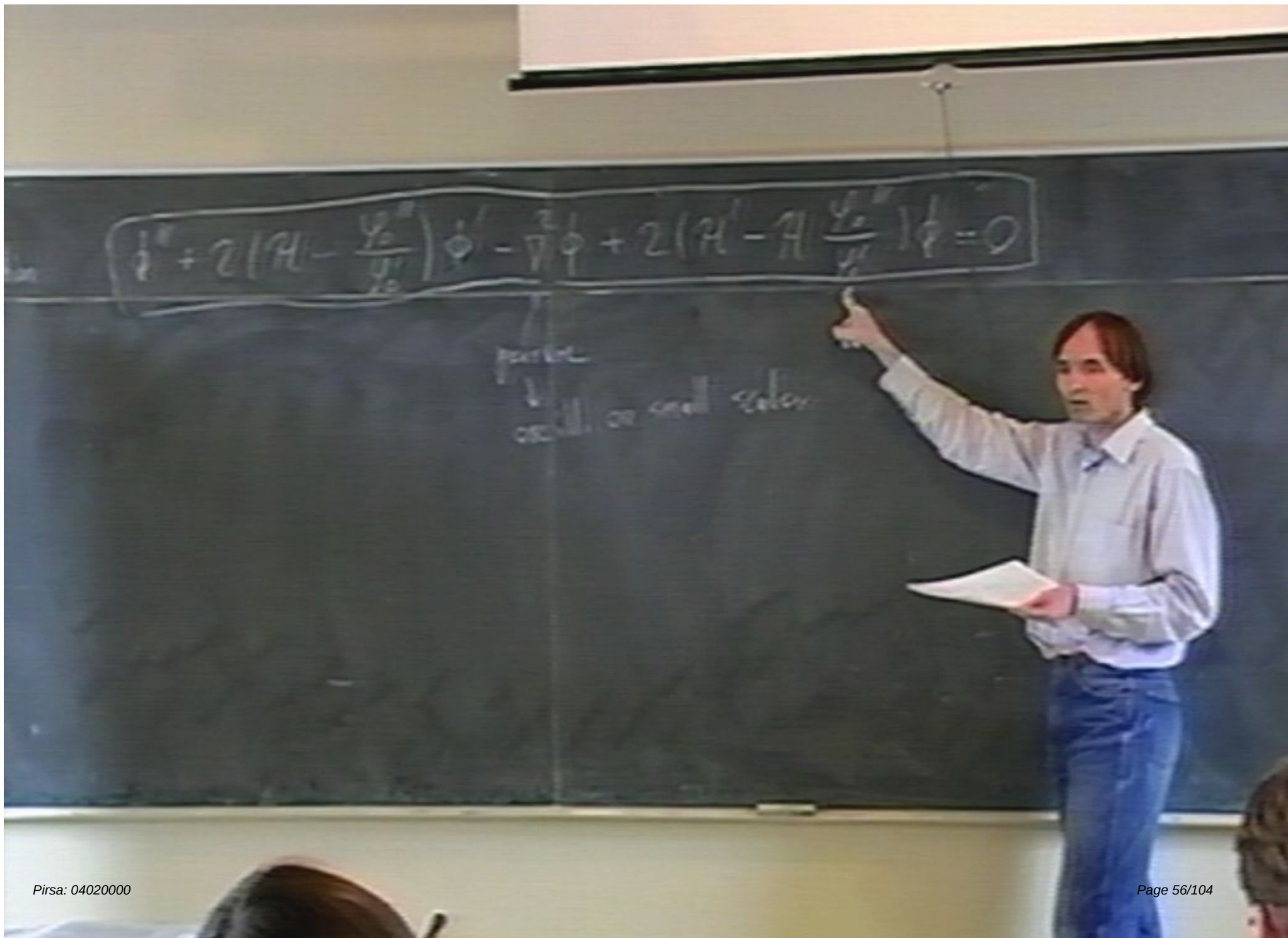
2 deg left

$$(\mathcal{H}\phi + \psi)' = 4\pi G a^2 \delta T^0_0$$

$$\left[(2\mathcal{H}' + \mathcal{H}^2)\phi + \mathcal{H}\phi' + \psi'' + 2\mathcal{H}\psi' \right] \delta^0_0 + \frac{1}{2}\nabla^2 D \delta^0_0 + \frac{1}{2}\nabla^2 \psi = 4\pi G a^2 \delta T^0_0$$

$\delta T^0_0 = 0 \Rightarrow \phi = \psi$ D - 2nd

$\uparrow$ or microscopic view



$$\ddot{\phi} + 2\left(\mathcal{H} - \frac{\dot{\psi}_0}{\psi_0}\right)\dot{\phi} - \frac{2}{\psi_0^2}\phi + 2\left(\mathcal{H}' - \mathcal{H}\frac{\dot{\psi}_0}{\psi_0}\right)\phi = 0$$

particle
↓
oscill. or small scalar

$$\ddot{\phi} + 2\left(\mathcal{H} - \frac{\dot{\psi}_0''}{\psi_0'}\right)\dot{\phi} - \nabla^2\phi + 2\left(\mathcal{H}' - \mathcal{H}\frac{\dot{\psi}_0''}{\psi_0'}\right)\phi = 0$$

perturb
 ↓
 oscill. or small scalar

self gravity

$$\delta \ddot{\rho} \sim -G \delta \rho$$



calculate

$$\left(\psi'' + 2 \left(H - \frac{\psi''}{\psi'} \right) \phi' - \frac{2}{\psi} \phi + 2 \left(H' - H \frac{\psi''}{\psi'} \right) \phi \right) = 0$$

Hecke algebra

particle

oscill or small vector

self adjoint

combination

$$\left(\phi'' + 2 \left(\mathcal{H} - \frac{\psi_0''}{\psi_0'} \right) \phi' - \nabla^2 \phi + 2 \left(\mathcal{H}' - \mathcal{H} \frac{\psi_0'}{\psi_0''} \right) \right) \phi = 0$$

Multiscale Juggling

pressure

oscill. or small scalar

special case

eq. at state

constant

$$\mathcal{H}' = 0$$

$$\frac{\psi_0''}{\psi_0'} = 0$$

$$\left(-\frac{Y_0''}{Y_0'}\right)\phi' - \nabla^2\phi + 2(\mathcal{H}' - \mathcal{H}\frac{Y_0''}{Y_0'})\phi = 0$$

de Sitter

at state

constant

$$\mathcal{H}' = 0, \quad \frac{Y_0''}{Y_0'} = 0$$

dominant mode of ϕ constant

pressure

oscill. or small scalar

self-gravity

$$\delta g \sim G \delta$$



$$\left[\phi'' + 2\left(\mathcal{H} - \frac{\dot{\psi}_0''}{\dot{\psi}_0'}\right)\phi' - \nabla^2\phi + 2\left(\mathcal{H}' - \mathcal{H}\frac{\dot{\psi}_0'''}{\dot{\psi}_0''}\right)\phi = 0 \right]$$

Hubble dragging

pressure

↓
oscill. or small scalar

special case

eq. of state

constant

$$\mathcal{H}' = 0, \quad \frac{\dot{\psi}_0'''}{\dot{\psi}_0''} = 0$$

dominant mode of

ϕ constant

$$\boxed{\phi' - \nabla^2 \phi + 2(\mathcal{H}' - \mathcal{H} \frac{\psi_0''}{\psi_0'}) \phi = 0}$$

$$\mathcal{H} = \frac{a'}{a}$$

pressure
 ↓
 oscill. or small scalar
 self gravity

$$\delta \ddot{g} \sim G \delta \rho$$

$\frac{\psi_0''}{\psi_0'} = 0$
 node of ϕ constant

$$\ddot{\phi} + 2\left(\mathcal{H} - \frac{Y_0''}{Y_0'}\right)\dot{\phi} - \nabla^2\phi + 2\left(\mathcal{H}' - \mathcal{H}\frac{Y_0''}{Y_0'}\right)\phi = 0 \quad \mathcal{H} =$$

Hubble dragging

pressure

self gravity

oscill. on small scales

$\delta g \sim$

case eq. of state

constant

$$\mathcal{H}' = 0, \quad \frac{Y_0''}{Y_0'} = 0$$

dominant mode of

ϕ constant on large scales

$$\boxed{\phi' - \frac{2}{\eta} \phi + 2(\mathcal{H}' - \mathcal{H} \frac{\psi_0''}{\psi_0'}) \phi = 0} \quad \mathcal{H} = \frac{a'}{a}$$

pressure
 $\downarrow$
 oscill. or small scalar

self gravity

$$\delta \ddot{g} \sim G \delta g$$

$\frac{\psi_0''}{\psi_0'} = 0$
 mode 2 ϕ constant on large scales

$$\left(\phi' - \nabla^2 \phi + 2(\mathcal{H}' - \mathcal{H} \frac{\psi_0''}{\psi_0'}) \phi = 0 \right)$$

$$\mathcal{H} = \frac{a'}{a}$$

pressure
 $\downarrow$
 oscill. or small scalar

self gravity

$$\delta g \sim G \delta \rho$$

$$t' = t + \delta t$$

$$\frac{\psi_0''}{\psi_0'} = 0$$

mode ϕ constant on large scales

$$\left[\phi' - \nabla^2 \phi + 2 \left(\mathcal{H}' - \mathcal{H} \frac{\psi_0''}{\psi_0'} \right) \phi = 0 \right] \quad \mathcal{H} = \frac{a'}{a}$$

pressure
 $\downarrow$
 oscill. or scalar
 self gravity

$$\delta g \sim G \delta \rho$$

$$t' = t + \delta t$$

$$\delta g_{\mu\nu}$$

$$\frac{\psi_0''}{\psi_0'} = 0$$

mode 4

large scalar

$$\left(\phi' - \nabla^2 \phi + 2 \left(\mathcal{H}' - \mathcal{H} \frac{\psi_0''}{\psi_0'} \right) \phi = 0 \right)$$

$$\mathcal{H} = \frac{a'}{a}$$

pressure
 ↓
 oscill. or small scalar

self gravity

$$\delta g \sim G \delta \rho$$

$$t' = t + \delta t$$

$$\delta g_{\mu\nu}$$

$\frac{\psi_0''}{\psi_0'} = 0$

$\psi_0' \neq 0$ ϕ constant or large scalar

$$\left(\frac{\psi_0'''}{\psi_0'} \right) \phi' - \nabla^2 \phi + 2(\mathcal{H}' - \mathcal{H} \frac{\psi_0'''}{\psi_0'}) \phi = 0$$

$$\mathcal{H} = \frac{a'}{a}$$

giving
 pressure
 oscill. or small scalar
 2nd special case
 inflation
 $\frac{\psi_0'''}{\psi_0'} = 0$
 not mode 2 ϕ constant on large scales
 self gravity

$$\delta g = \delta p$$

$$t' = t$$

$$\delta$$

$$\left(\frac{\psi_0'''}{\psi_0'} \right) \phi' - \frac{2}{\psi_0'} \phi + 2 \left(\mathcal{H}' - \mathcal{H} \frac{\psi_0'''}{\psi_0'} \right) \phi = 0$$

$$\mathcal{H} = \frac{a'}{a}$$

giving
 ate
 pressure
 oscill. on small scales
 self gravity
 2nd special case
 inflation

$$\frac{3}{2} \dot{\phi}$$

$\frac{\psi_0'''}{\psi_0'} = 0$
 not model of ϕ constant on large scales

$$\left(\frac{\psi_0'''}{\psi_0'}\right)\dot{\phi}' - \nabla^2\phi + 2(\mathcal{H}' - \mathcal{H}\frac{\psi_0'''}{\psi_0'})\phi = 0$$

$$\mathcal{H} = \frac{a'}{a}$$

giving
 ate
 -
)
 not mode of

positive
 ↓
 oscill. or small scalar
 2nd special case
 inflation

self gravity

$$\frac{3}{2}\dot{\mathcal{H}}(1+w) = 0 + \mathcal{O}(\nabla^2\phi)$$

$\frac{\psi_0'''}{\psi_0'} = 0$
 1st mode of ϕ constant on large scales

$$+ 2 \left(\mathcal{H}' - \mathcal{H} \frac{\psi_0''}{\psi_0'} \right) \phi = 0$$

$$\mathcal{H} = \frac{a'}{a}$$

self gravity

on small scales

2nd special case
inflation

constant on large scales

$$\frac{3}{2} \dot{\phi}^2 (1+w) = 0 + \mathcal{O}(w)$$

$$+ 2(\mathcal{H}' - \mathcal{H} \frac{\psi_0''}{\psi_0'}) \phi = 0$$

$$\mathcal{H} = \frac{a'}{a}$$

self gravity

on small scales

2nd special case
inflation

constant on large scales

$$\frac{3}{2} \dot{\mathcal{H}} (1+w) = 0$$

$$+ \mathcal{O}(\nabla^2 \phi)$$

$$w = \frac{p}{\rho}$$

$$\dot{\mathcal{H}} =$$

$$+ 2 \left(\mathcal{H}' - \mathcal{H} \frac{\psi_0''}{\psi_0'} \right) \phi = 0$$

$$\mathcal{H} = \frac{a'}{a}$$

self gravity

on small scales

2nd special case

inflation

$$\frac{3}{2} \dot{\epsilon} \mathcal{H} (1+w) = 0 + \mathcal{O}(\nabla^2 \phi)$$

$$w = \frac{p}{\rho}$$

constant on large scales

$$\zeta = \phi + \frac{2}{3} \left(\mathcal{H}^{-1} \dot{\phi} + \phi \right) - \frac{1}{6} \nabla^2 \phi$$

$$-\nabla^2 \phi + 2\left(\mathcal{H}' - \mathcal{H} \frac{\dot{\gamma}_0''}{\dot{\gamma}_0'}\right)\phi = 0$$

$$A = \frac{a}{x}$$

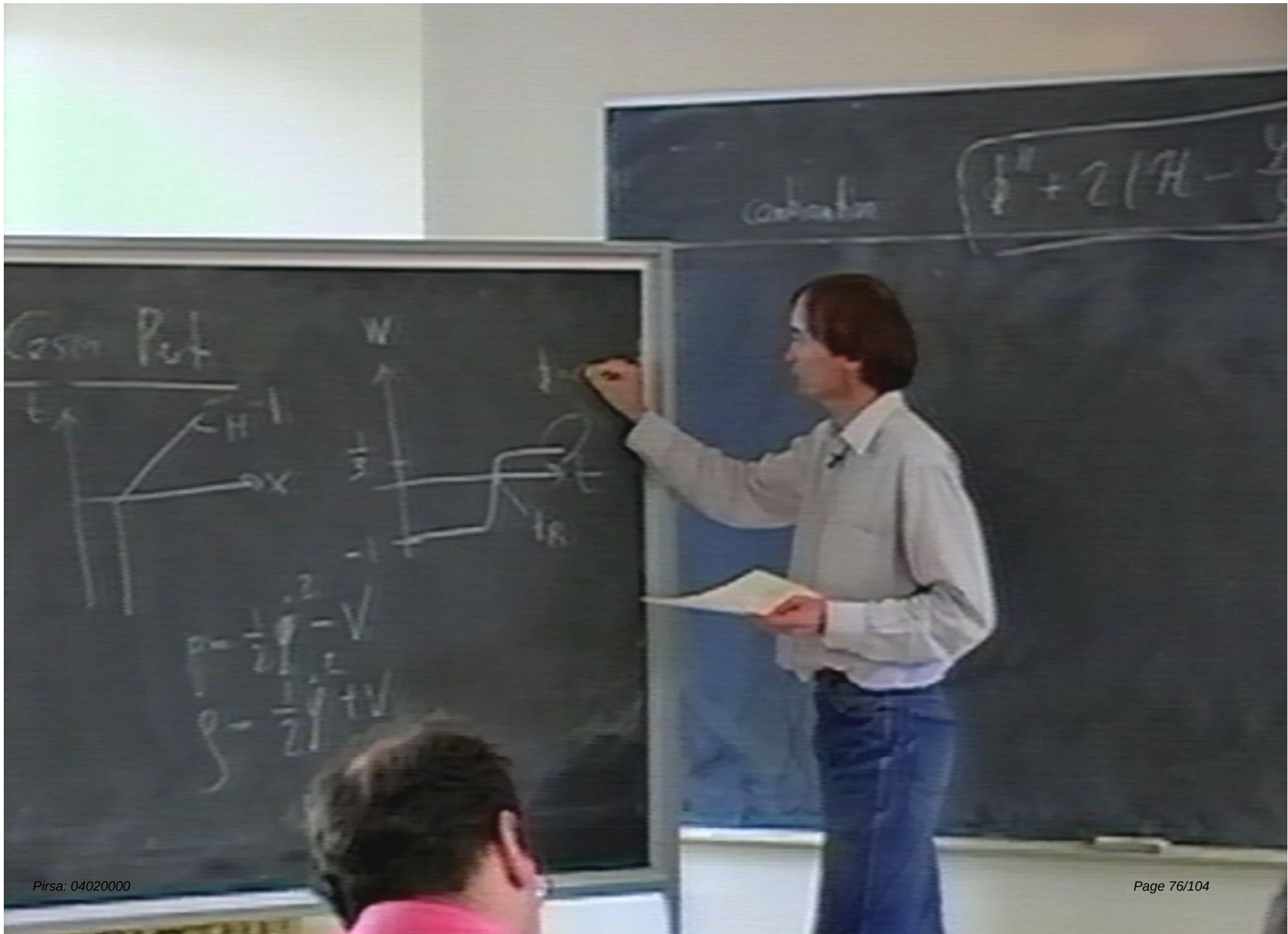
self gravity

$$\frac{3}{2} \int H(1+w) = 0 + O(\nabla^2 \phi)$$

$$W = \frac{P}{S}$$

$$f = \phi + \frac{\lambda}{2} \left(\frac{1}{H} \phi + \phi \right) \frac{1}{\Gamma H}$$

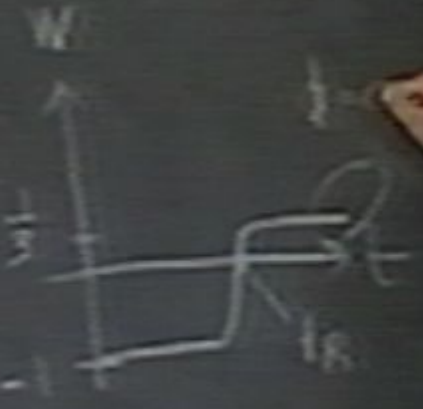
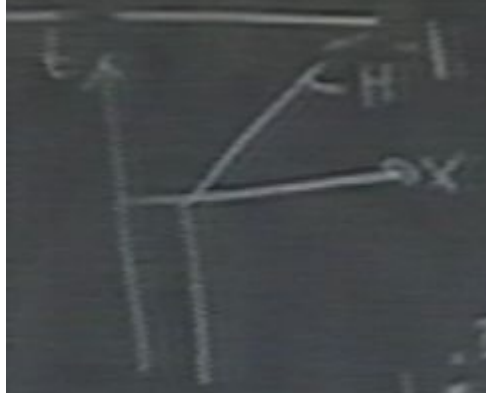




Combination

$$\phi'' + 2\mathcal{H}\phi - \frac{\phi}{a^2} = 0$$

Cosm. Pert



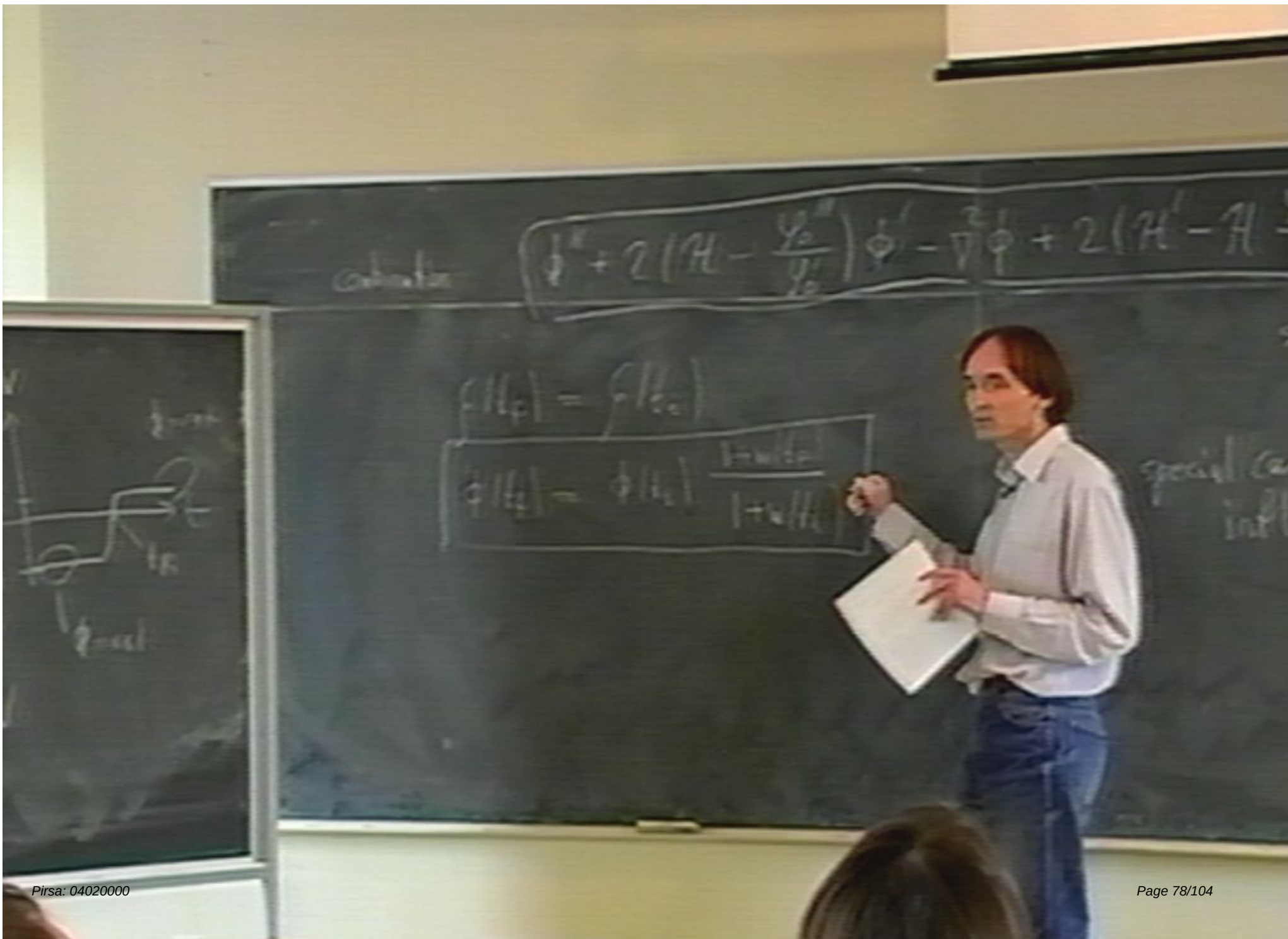
$$\dot{\phi}^2 = \frac{2}{3}(\dot{\phi}^2 - V)$$
$$\ddot{\phi} = -\frac{1}{2}\dot{\phi}^2 + V$$

Combination

$$\left(\phi'' + 2 \left(\mathcal{H} - \frac{\psi_0''}{\psi_0'} \right) \phi' - \nabla^2 \phi + \right.$$

$$f(t_f) = f(t_i)$$

$$\phi(t_f) = \phi(t_i) \frac{1+w(t_f)}{1+w(t_i)}$$



$$\nabla^2 \phi + 2 \left(\mathcal{H}' - \mathcal{H} \frac{\dot{\phi}''}{\dot{\phi}'} \right) \phi = 0$$

$$\mathcal{H} = \frac{a'}{a}$$

self gravity

$$\frac{3}{2} \dot{\mathcal{H}} H (1+w) = 0 + \mathcal{O}(\partial^2 \phi)$$

special case
inflation

$$w = \frac{p}{\rho}$$

$$\zeta = \phi + \frac{2}{3} \left(\mathcal{H}^{-1} \dot{\phi} + \frac{1}{1+w} \right)$$

$$\nabla^2 \phi + 2(\mathcal{H}' - \mathcal{H} \frac{\psi_0''}{\psi_0'}) \phi = 0$$

$$\mathcal{H} = \frac{a'}{a}$$

self gravity

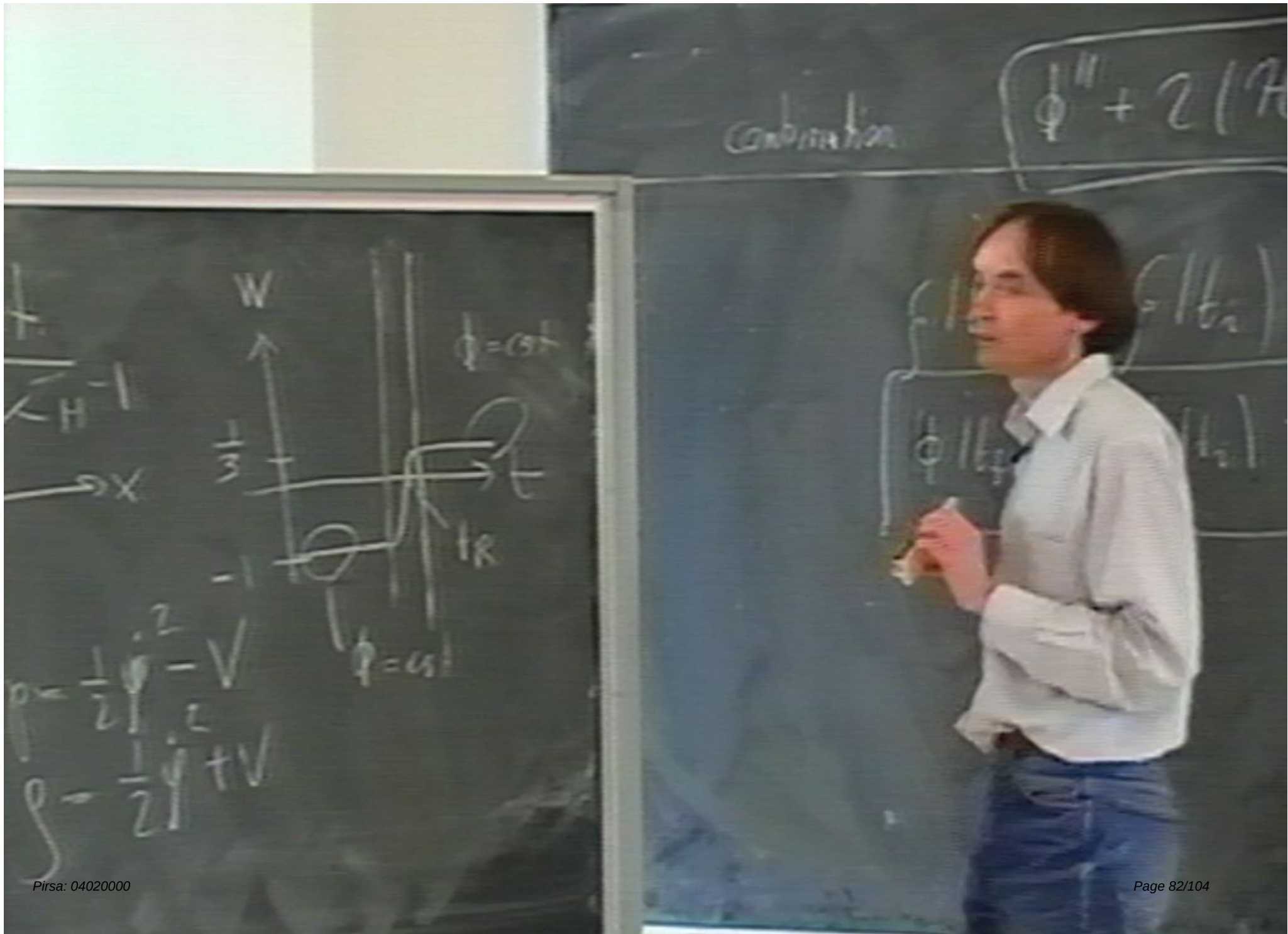
$$\frac{3}{2} \dot{\zeta} \mathcal{H} (1+w) = 0 + \mathcal{O}(\nabla^2 \phi)$$

special case
inflation

$$w = \frac{p}{\rho}$$

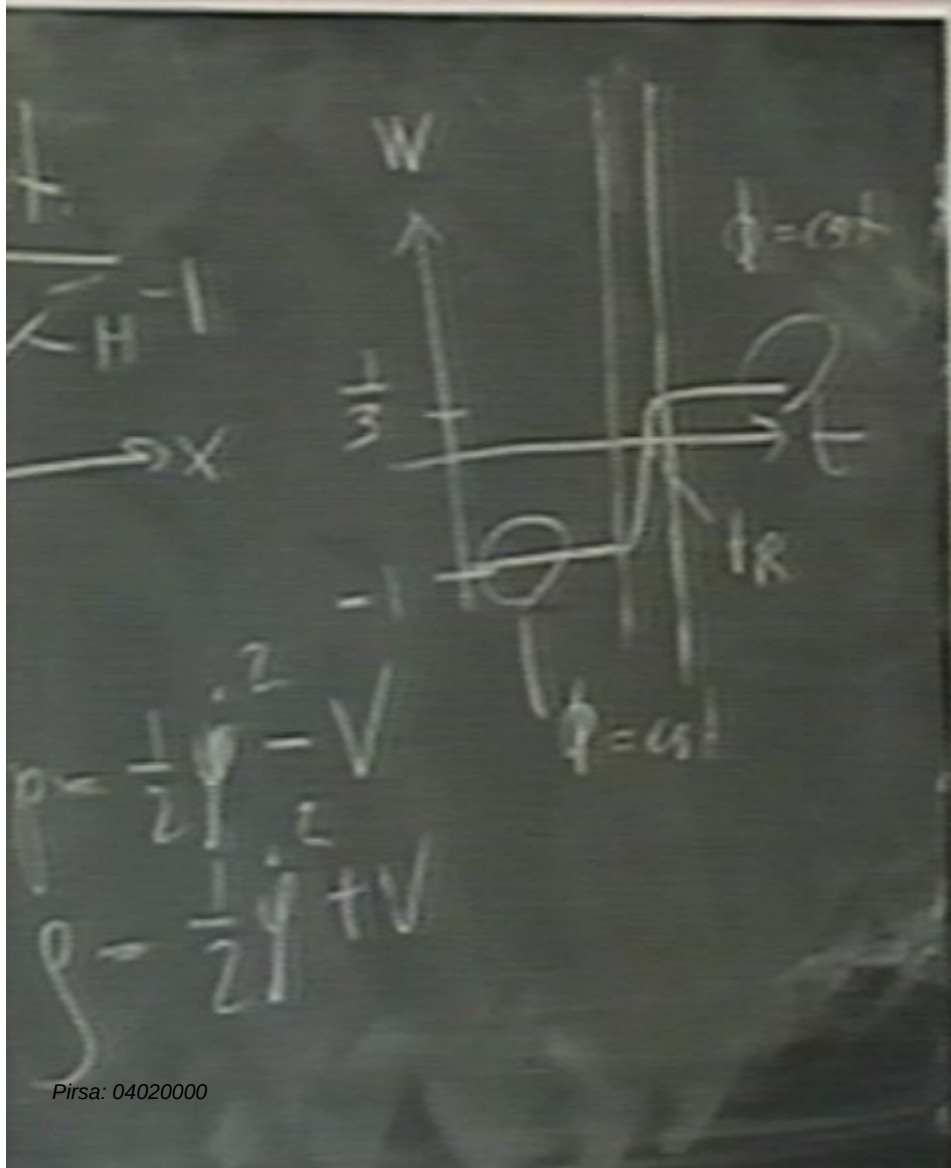
curvature pert.
in comoving gauge

$$\zeta = \phi + \frac{1}{3} \left(\mathcal{H}^{-1} \dot{\phi} + \phi \right) \frac{1}{1+w}$$



Combination

$$\boxed{\phi''' + 2\left(\frac{\phi''}{t}\right)}$$



Equations on the right:

$$\left\{ \begin{array}{l} \phi(t_1) \\ \phi(t_2) \end{array} \right\}$$

$$\left\{ \begin{array}{l} \phi(t_1) \\ \phi(t_2) \end{array} \right\}$$

$$\left[\phi'' + 2\left(\mathcal{H} - \frac{\psi_0''}{\psi_0'}\right)\phi' - \nabla^2\phi + 2\left(\mathcal{H}' - \mathcal{H}\frac{\psi_0''}{\psi_0'}\right)\phi = 0 \right]$$

self gravity

$$\frac{3}{2}$$

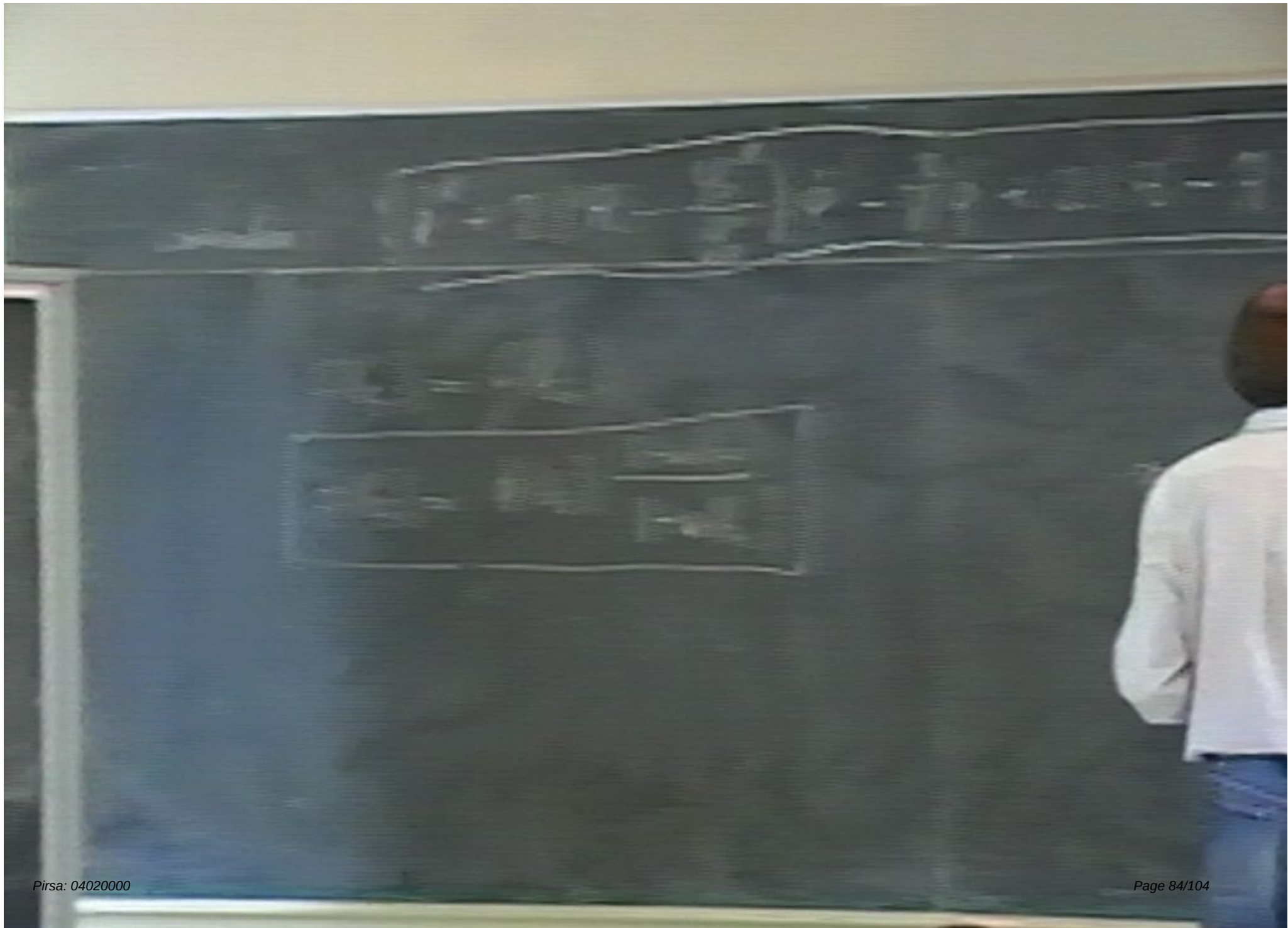
special case
inflation

w

curvature pert.
in comoving gauge

$$\left[\zeta = \right]$$

$$\begin{aligned} |t_f| &= f(t_i) \\ \phi(t_f) &= \phi(t_i) \frac{1+w(t_i)}{1+w(t_f)} \end{aligned}$$



continuation

$$\left(\phi'' + 2 \left(\mathcal{H} - \frac{\psi_0''}{\psi_0'} \right) \phi' - \nabla^2 \phi + 2 \left(\mathcal{H}' - \frac{\psi_0'''}{\psi_0'} \right) \phi \right)$$

$$f(t_f) = f(t_i)$$

$$\phi(t_f) = \frac{3}{5} \phi(t_i) \frac{1 + w(t_f)}{1 + w(t_i)}$$

Relativistic Theory of Cosm

Classification

Gauge Transformation

Equations

Quantum Theory



$$\phi + 2\left(\mathcal{H}' - \mathcal{H} \frac{\psi_0''}{\psi_0'}\right)\phi = 0 \quad \mathcal{H} = \frac{a'}{a}$$

$$S = \int d\tau \left[R + \frac{1}{2} g_{\mu\nu} \dot{\psi}^\mu \dot{\psi}^\nu - V(\psi) \right]$$

$$\square \phi + 2(\mathcal{H}' - \mathcal{H} \frac{\psi''}{\psi'}) \phi = 0 \quad \mathcal{H} = \frac{a'}{a}$$

$$S = \int d^4x \sqrt{-g} \left[R + \frac{1}{2} g_{\mu\nu} \partial^\mu \psi \partial^\nu \psi - V(\psi) \right]$$

$$\phi + 2(\mathcal{H}' - \mathcal{H} \frac{\psi_0'''}{\psi_0'}) \phi = 0$$

$$\mathcal{H} = \frac{a'}{a}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{8\pi G} + \frac{1}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi - V(\psi) \right]$$

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \delta g_{\mu\nu}$$

$$\psi = \psi^{(0)} + \delta\psi$$

$$S = S^{(0)}$$

$$\phi + 2(\mathcal{H}' - \mathcal{H} \frac{\psi_0''}{\psi_0'}) \phi = 0$$

$$\mathcal{H} = \frac{a'}{a}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{8\pi G} + \frac{1}{2} g_{\mu\nu} \partial^\mu \psi \partial^\nu \psi - V(\psi) \right]$$

$$\left. \begin{aligned} g_{\mu\nu} &= g_{\mu\nu}^{(0)} + \delta g_{\mu\nu} \\ \psi &= \psi^{(0)} + \delta\psi \end{aligned} \right\} \uparrow$$

$$S = S^{(0)} + S^{(2)}$$

$$-\frac{1}{8\pi\alpha'} \int d^4x \left[R + \frac{1}{2} g_{\mu\nu} \partial^\mu \psi \partial^\nu \psi - V(\psi) \right]$$

$\delta g_{\mu\nu}$
 $\delta \psi$

1. fix gauge completely
2. expand & quantize

+ S⁽²⁾

$$-\nabla^2 \phi + 2\left(\mathcal{H}' - \mathcal{H} \frac{\psi_0''}{\psi_0'}\right)\phi = 0$$

$$\mathcal{H} = \frac{a'}{a}$$

$$S = \int \frac{1}{8\pi G} \left[R + \frac{1}{2} g_{\mu\nu} \partial^\mu \psi \partial^\nu \psi - V(\psi) \right]$$

1. fix gauge completely

2. expand & quantize

$\phi, \delta\psi$

Combination

$$\left(\phi'' + 2 \left(\mathcal{H} - \frac{\psi_0'''}{\psi_0'} \right) \phi' - \nabla^2 \phi + 2 \left(\mathcal{H} \right) \right)$$

$$S^{(H)} = \int d^4x \left(V'^2 - V_{\mu\nu} V^{\mu\nu} \right)$$

$$S = \int d^4x$$

$$\frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial x^\mu}$$

$$\psi = \psi$$

$$S =$$

Combination

$$\left(\phi'' + 2 \left(\mathcal{H} - \frac{\psi_0'''}{\psi_0'} \right) \phi' - \nabla^2 \phi + 2 \left(\mathcal{H} \right) \right)$$

$$S^{hl} = \int d^4x \left[V'^2 - V_{\mu\nu} V^{\mu\nu} - \frac{Z''}{Z} V^2 \right] \quad S = \int d^4x$$

Combination:

$$\left(\phi'' + 2 \left(\mathcal{H} - \frac{\dot{\phi}_0'''}{\dot{\phi}_0'} \right) \phi' - \nabla^2 \phi + 2 \left(\mathcal{H} \right) \right)$$

$$S^{(h)} = \int d^4x \left[\dot{V}^2 - V_{,i} V_{,i} - \frac{z''}{z} V^2 \right]$$

$$S = \int d^4x$$

$$\gamma_{\mu\nu} = \eta_{\mu\nu}$$

$$\psi = \psi$$

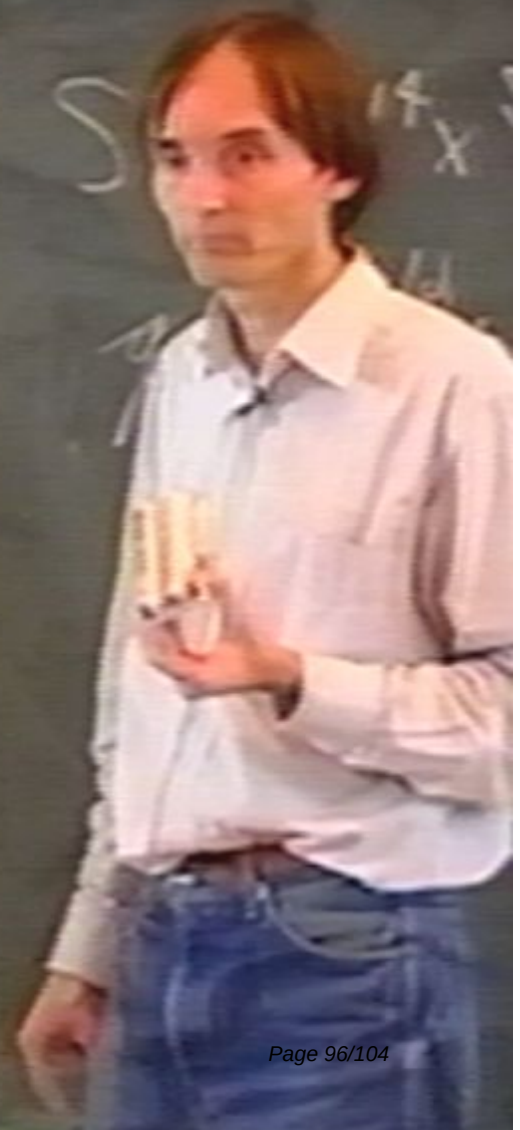
$$S =$$

Combination

$$\left(\phi'' + 2 \left(\mathcal{H} - \frac{\psi_0'''}{\psi_0'} \right) \phi' - \nabla^2 \phi + 2 \left(\mathcal{H}' - \frac{\psi_0'''}{\psi_0'} \right) \phi \right)$$

$$S^{\text{H}} = \int d^4x \left[V'^2 - V V_{\mu\nu} - \frac{z''}{z} V^2 \right]$$

$$S = \int d^4x$$



Combination

$$\left(\phi''' + 2 \left(\mathcal{H} - \frac{\psi_0'''}{\psi_0'} \right) \phi' - \nabla^2 \phi + 2 \left(\frac{\psi_0'''}{\psi_0'} \right) \phi \right)$$

$$S^{(H)} = \frac{1}{2} \int d^4x \left[V'^2 - V_{,\mu} V_{,\mu} - V^2 \right]$$

$$Z = \frac{a \psi_0'}{\mathcal{H}}$$

$$V = a \left[\delta \psi - \frac{1}{\mathcal{H}} \delta \mathcal{H} \right]$$

$$S = \int$$

$$\gamma_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$$

$$S =$$

Combination

$$\left(\phi'' + 2 \left(\mathcal{H} - \frac{\psi_0'''}{\psi_0'} \right) \phi' - \nabla^2 \phi + 2 \right)$$

$$S^{\mathcal{H}} = \frac{1}{2} \int d^4x \left[V'^2 - V_{\mu\nu} V^{\mu\nu} - \frac{z''}{z} V^2 \right]$$

$$z = \frac{a \psi_0'}{\mathcal{H}}$$

$$V = a \left[\delta\varphi + \frac{\psi_0'}{\mathcal{H}} \phi \right]$$

combination

$$\left(\phi'' + 2 \left(\mathcal{H} - \frac{\psi_0'''}{\psi_0'} \right) \phi' - \nabla^2 \phi + 2 \right)$$

$$S^H = \frac{1}{2} \int d^4x \left[V'^2 - V V_{,\mu} V_{,\mu} - \frac{z''}{z} V^2 \right]$$

$$z = \frac{a \psi_0'}{\mathcal{H}}$$

$$V = a \left[\delta\psi + \frac{\psi_0'}{\mathcal{H}} \phi \right]$$

$$= z R$$

curv pert is comoving

$$S =$$

$$\frac{1}{\mu}$$

$$\psi =$$

$$S =$$

condition

$$\left(\phi'' + 2\left(\mathcal{H} - \frac{\dot{\psi}_0''}{\dot{\psi}_0'}\right)\phi' - \frac{z}{\dot{\psi}_0'}\phi + 2\left(\mathcal{H}' - \mathcal{H}\frac{\dot{\psi}_0''}{\dot{\psi}_0'}\right)\phi = 0 \right)$$

$$S^{(H)} = \frac{1}{2} \int d^4x \left[\dot{V}^2 - V \dot{V} - \frac{z}{2} V^2 \right]$$

$$z = \frac{2\dot{\psi}_0'}{\mathcal{H}}$$

$$V = a \left[\delta\psi + \frac{\dot{\psi}_0'}{\mathcal{H}} \phi \right]$$

$$= z R$$

cor. p. in covariant gauge

$$S = \int d^4x \left[\frac{1}{2} \dot{\psi}^2 + \frac{1}{2} \dot{\phi}^2 \right]$$

$$\begin{aligned} \mathcal{L}_{\text{can}} &= \mathcal{L}_{\text{can}} + \delta\mathcal{L} \\ \psi &= \psi + \delta\psi \end{aligned}$$

$$S = S$$

combination

$$\left(\phi'' + 2(\mathcal{H} - \frac{\dot{\psi}_0''}{\dot{\psi}_0'}) \phi' - \frac{z''}{z} \phi + 2(\mathcal{H}' - \mathcal{H} \frac{\dot{\psi}_0'}{\dot{\psi}_0'}) \right)$$

$$S^{(h)} = \frac{1}{2} \int d^4x \left[V'^2 - V V'' - \frac{z''}{z} V^2 \right]$$

$$z = \frac{a \dot{\psi}_0'}{\mathcal{H}}$$

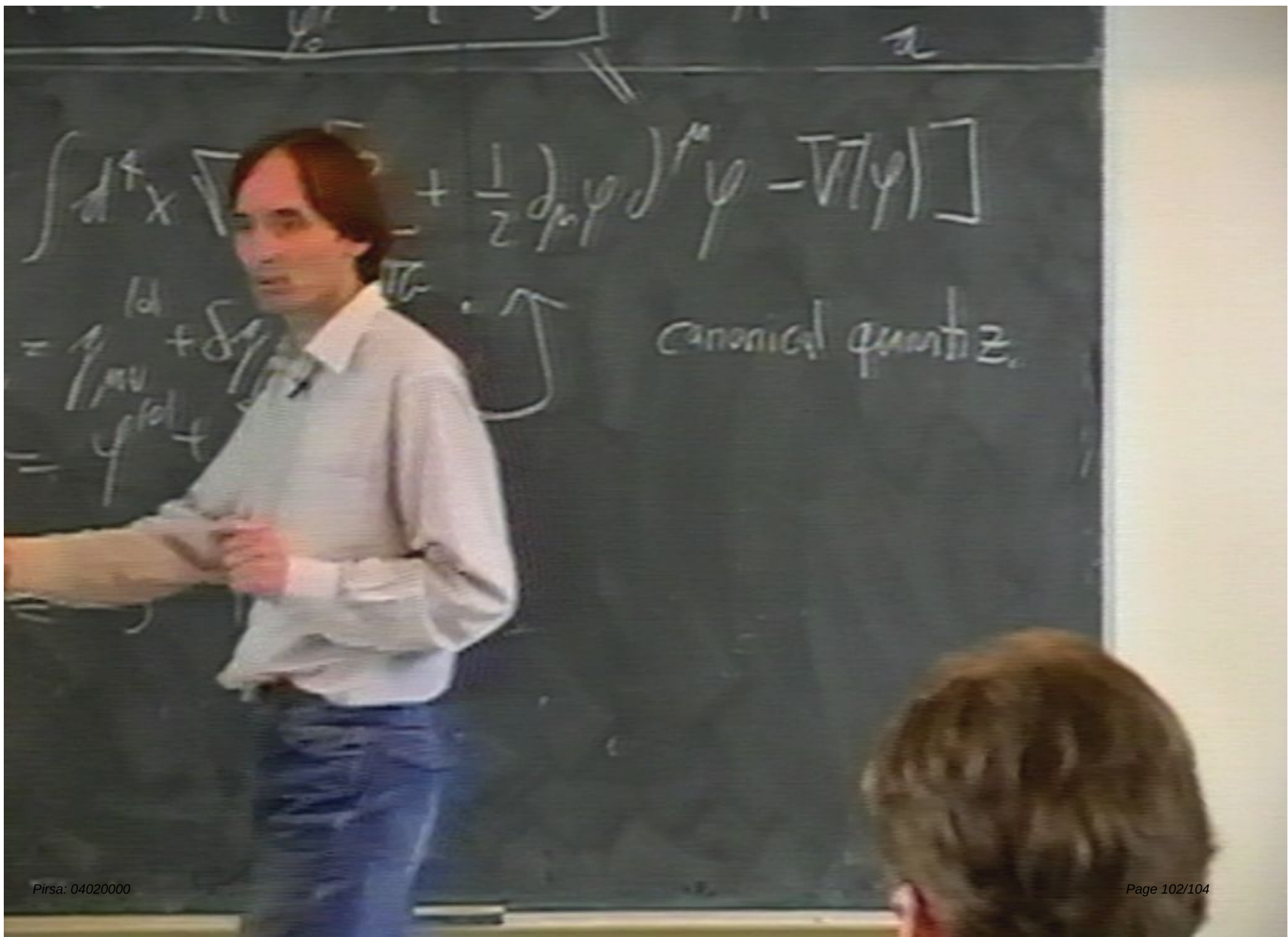
$$d^4x = d\eta d^3x$$

$$V = a \left[\delta\psi + \frac{\dot{\psi}_0'}{\mathcal{H}} \phi \right]$$

$$= z R$$

corr. pot. is growing

$$\int d^4x \sqrt{-g} \left[\frac{1}{2} (\partial_\mu \psi)^2 + \delta\mathcal{L}_{\text{m}} + \delta\psi \right]$$



$$S = \int d^4x \sqrt{-g} \left[\frac{1}{8\pi G} \left(R + \frac{1}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi - V(\psi) \right) \right]$$

$$\left. \begin{aligned} g_{\mu\nu} &= g_{\mu\nu}^{(0)} + \delta g_{\mu\nu} \\ \psi &= \psi^{(0)} + \delta\psi \end{aligned} \right\} \nearrow$$

canonical
initial state
choice

$$S = S^{(0)} + S^{(2)}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{8\pi G} + \frac{1}{2} g_{\mu\nu} \partial^\mu \psi \partial^\nu \psi - V(\psi) \right]$$

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \delta g_{\mu\nu}$$

$$\psi = \psi^{(0)} + \delta\psi$$

$$S = S^{(0)} + S^{(2)}$$

canonical quantize.

initial state

choice

at t_i vacuum